A Convex Decomposition Perspective on Dynamic Bandwidth Allocation and Applications

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Barcelona, 23rd September 2008

Outline

- Introduction
- Dynamic Bandwidth Allocation (DBA)
- Decomposition Framework in Convex Programming
- Cross-Layer DBA in DVB-RCS
- Distributed Algorithm for Uplink Scheduling in WiMAX Networks
- Conclusions and Future Work

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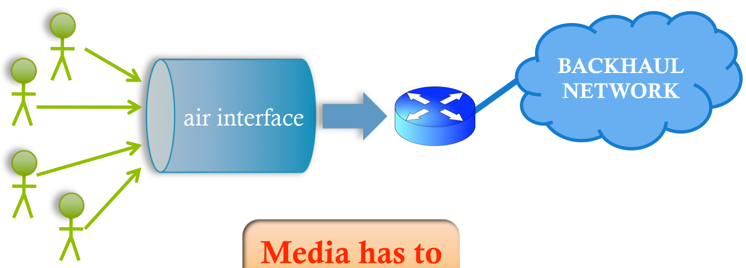
Introduction (1/4)

MOTIVATION

- Modern wireless systems: many users accessing the core network.

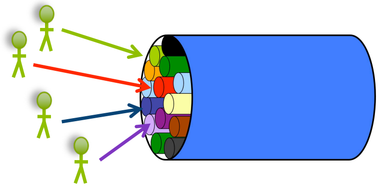


Media has to be shared

Introduction (2/4)

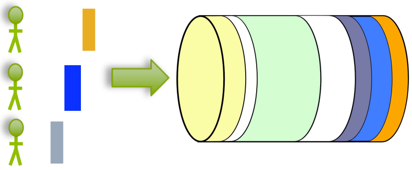
- Classical approaches:

Connection oriented



- Users distinguished by means of orthogonal interfaces

Contention oriented



- Multiplexing resorts to traffic nature: information is quantized or packetized

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Introduction (3/4)

- Pros and cons:

Connection oriented

- Dedicated connection: fixed QoS degree
- May lead to over-provisioning in case of multiple connections
- Suitable for continuous traffic
- Inefficient for traffic patterns with inactivity periods
- Fairness issues: blocked users
- Requires centralized control

Contention oriented

- Suits the nature of quantized traffic
- As the traffic volume grows, there is more conflict among users
- Low utilization of the multi-user channel
- Fairness and QoS issues not well stablished
- No need of a central controller

Can we do better?

Dynamic Bandwidth Allocation (DBA)

- Soft bandwidth distribution
- Fairness
- QoS

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Introduction (4/4)

- Our approach: implement DBA using the NUM framework
- It requires to find out the solution of an optimization problem
- Interest in
  - Algorithms that converge fast to the optimal solution
  - Distribute the optimization process among the network elements (horizontal decomposition) ⟺ **Signalling issues**
  - Distribute the optimization process among the OSI layers of a single network element (vertical decomposition) ⟺ **Cross-layer issues**

CONVEX DECOMPOSITION TECHNIQUES

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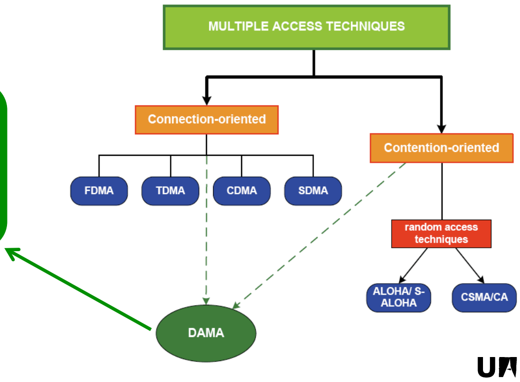
Dynamic Bandwidth Allocation (1/4)

- DBA: quite a general concept
- We focus on DBA application to the multiple access part
- Can be interpreted as half way between CONNECTION and CONTENTION oriented

IDEA:

•Users request resources (bandwidth) to some entity (BS)

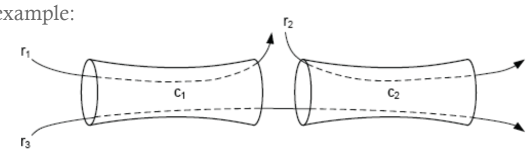
•It allocates the available bandwidth: QoS and fairness criteria



Dynamic Bandwidth Allocation (2/4)

FAIRNESS AND QoS

- Multiple fairness definitions (game theory)
  - Max-min fairness
  - Proportional fairness, etc.
- Can be found as the solution to certain optimization problems
- Simple example:



- NUM formulation:

$$\begin{aligned} \max_{\{r_i\}} \quad & \sum_{i=1}^{N} U_i(r_i) \\ s.t. \quad & Ar \le c \\ & r_i \ge 0 \end{aligned}$$

Utility functions

Dynamic Bandwidth Allocation (3/4)

- Assume $c_1=c_2=1$
- Max-min fairness: no increase in r_j if it implies a decrease in r_p ($r_p<r_j$)

  Solution: $r=\begin{bmatrix}1/2\\1/2\\1/2\end{bmatrix}$

- Proportional fairness: the sum of relative changes is not positive.

  Take two vectors and compute: $\sum_{i=1}^{N}\frac{r_i^{\ddagger}-r_i^{\dagger}}{r_i^{\dagger}}\leq 0$ **proportionally fair**

  Solution: $r=\begin{bmatrix}2/3\\2/3\\1/3\end{bmatrix}$

- Note the maximum network throughput solution: $r=\begin{bmatrix}1\\1\\0\end{bmatrix}$ ← **unfair**

- **Proportional fairness is known to provide a nice tradeoff fairness ⟷ throughput**

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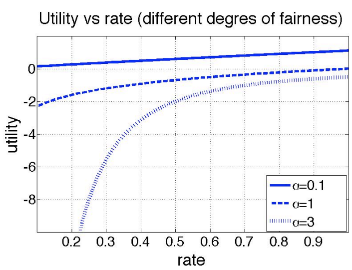
Dynamic Bandwidth Allocation (4/4)

- Generalization: (p,α)-proportional fairness criterion

$$\sum_{i=1}^{N}p_i\frac{r_i^{\ddagger}-r_i^{\dagger}}{(r_i^{\dagger})^{\alpha}}\leq 0$$

(p,α)-proportionally fair

- Mapping with utility functions: $U_i(r_i;p_i,\alpha)=\begin{cases}p_i\log(r_i), & \alpha=1\\ p_i\frac{r_i^{(1-\alpha)}}{1-\alpha}, & \alpha\neq 1\end{cases}$
  - α ⟶ 0: maximum network throuhput
  - α=1: proportional fairness
  - α ⟶ ∞: max-min fairness
- The concavity of the function fixes the degree of fairness
- QoS distinction



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Decomposition Framework in Convex Programming (1/16)

FUNDAMENTALS

- Convex problem: convex objective function with convex domain

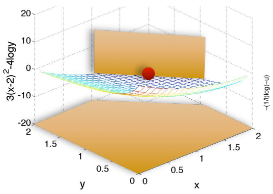
$$p^* = \min_{x} \; f_0(x) \quad s.t. \; f_i(x) \le 0 \;\; 1 \le i \le m, \quad h_i(x) = 0 \;\; 1 \le i \le p$$

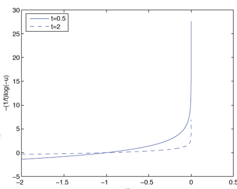
convex functions

affine functions

- Convex optimization: well-established theory to analyze and solve convex problems
- Options (depending on the problem)
  - Numerical approach
  - (Semi-) analytical approach: KKT conditions

- Numerical approach: interior-point methods (e.g. barrier method)





- Duality: alternative representation of a convex problem

LAGRANGIAN $L(x,\lambda,\nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x)$

DUAL FUNCTION $g(\lambda,\nu) = \inf_{x \in D} L(x,\lambda,\nu) = \inf_{x \in D} \left(f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x) \right)$

DUAL PROBLEM $d^* = \max_{\lambda,\nu} \; g(\lambda,\nu) \quad s.t. \; \lambda \succeq 0$

BEST UNDER-ESTIMATOR OF p*

Decomposition Framework in Convex Programming (2/16)

- Central result: under some conditions (e.g. Slater's condition), it holds:

$$\boxed{p^*=d^*}$$

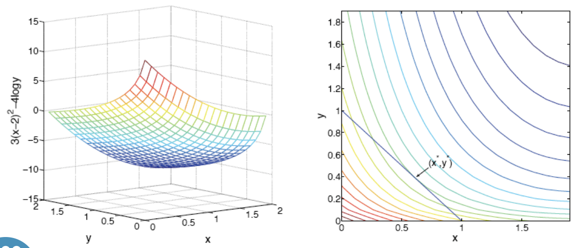
- Two different perspectives of the same problem:
  - Primal
  - Dual
- … and two different formulations to solve it!

Example:

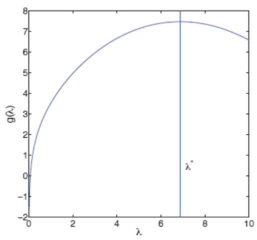
$$\begin{array}{ll} \min & 3(x-2)^2 - \log y \\ s.t. & x+y \leq 1 \end{array}$$

$$g(\lambda) = \frac{-\lambda^2}{12} + \log\lambda + \lambda + 1$$

PRIMAL



DUAL



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Decomposition Framework in Convex Programming (3/16)

- KKT conditions for convex problems:

$$\begin{array}{rcl} f_i(\boldsymbol{x}^*) & \leq & 0, \quad i=1,\dots,m \\ h_i(\boldsymbol{x}^*) & = & 0, \quad i=1,\dots,p \\ \lambda_i^* & \geq & 0, \quad i=1,\dots,m \\ \lambda_i^* f_i(\boldsymbol{x}^*) & = & 0, \quad i=1,\dots,m \\ \nabla f_0(\boldsymbol{x}^*) + \sum_{i=1}^m \lambda_i^* \nabla f_i(\boldsymbol{x}^*) + \sum_{i=1}^p \nu_i^* \nabla h_i(\boldsymbol{x}^*) & = & 0. \end{array}$$

✓ Gradient of the Lagrangian: must vanish given the optimal Lagrange multipliers

✓ Connection: PRIMAL ⟷ DUAL

Slackness constraints: Lagrange multipliers can be non-zero at active constraints

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Decomposition Framework in Convex Programming (4/16)

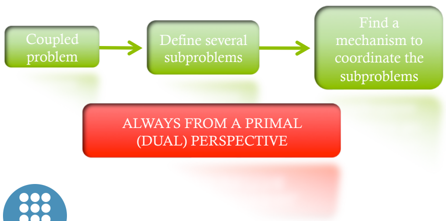
KNOWN DECOMPOSITION METHODS

- Classical decomposition approaches:
  - Primal decomposition
  - Dual decomposition

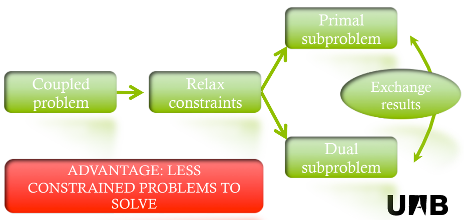
  } → **Same idea, different perspective (primal problem / dual problem)**
- We also have:
  - Mean Value Cross (MVC) decompositions method } → **Distinct approach**

PHILOSOPHY

PRIMAL/DUAL DECOMPOSITION



MVC-DECOMPOSITION



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Decomposition Framework in Convex Programming (5/16)

PRIMAL DECOMPOSITION

- Problem formulation:

$$\begin{array}{lll} \min\limits_{\{\boldsymbol{y}_j,\boldsymbol{x}_j\}} & \sum_{j=1}^{J} f_j(\boldsymbol{x}_j) & \\ s.t. & \boldsymbol{x}_j \in \mathcal{X}_j, & j=1,\dots,J \\ & \boldsymbol{A}_j\boldsymbol{x}_j \preceq \boldsymbol{y}_j, & j=1,\dots,J \\ & \sum_{j=1}^{J} \boldsymbol{y}_j \preceq \boldsymbol{b} & \end{array}$$

- It decouples for fixed $\mathbf{y}_j$ values (Subproblems):

$$p(\boldsymbol{y}_j) = \begin{array}{ll} \min & f_j(\boldsymbol{x}_j) \\ s.t. & \boldsymbol{A}_j\boldsymbol{x}_j \preceq \boldsymbol{y}_j, \quad \boldsymbol{x}_j \in \mathcal{X}_j \end{array}$$

p($\mathbf{y}_j$) is convex on $\mathbf{y}_j$

$-\lambda_j^*(\mathbf{y}_j)$

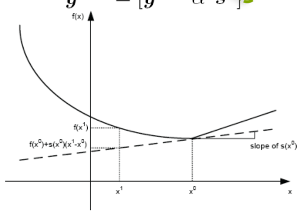
- Using subproblems, the general problem is (Master Problem):

$$\begin{array}{ll} \min & \sum_{j=1}^{J} p_j(\boldsymbol{y}_j) \\ s.t. & \sum_{j=1}^{J} \boldsymbol{y}_j \preceq \boldsymbol{b}, \ \boldsymbol{y}_j \in \mathcal{Y}_j, \quad j=1,\dots,J \end{array}$$

The MP is convex

- Solution: projected subgradient method
- Idea: move opposite to the (sub)gradient taking into account the feasible set

$$\boldsymbol{y}^{k+1} = [\boldsymbol{y}^k - \alpha^k \boldsymbol{s}^k]^{\dagger}$$



- Advantage: subgradient obtained at no cost from subproblems (Lagrange multipliers)
- Problem: user-defined step-size (speed of convergence)

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Decomposition Framework in Convex Programming (6/16)

DUAL DECOMPOSITION

- Problem formulation:

$$\begin{array}{lll} \min\limits_{\{\boldsymbol{x}_j\}} & \sum_{j=1}^{J} f_j(\boldsymbol{x}_j) & \\ s.t. & \boldsymbol{x}_j \in \mathcal{X}_j, & j = 1, \dots, J \\ & \sum_{j=1}^{J} \boldsymbol{h}_j(\boldsymbol{x}_j) \preceq \boldsymbol{b}. & \end{array}$$

- Dual function relaxing the coupling constraint:

$$q(\boldsymbol{\mu}) = \sum_{j=1}^{J} \min_{\boldsymbol{x}_j \in \mathcal{X}_j} \{f_j(\boldsymbol{x}_j) + \boldsymbol{\mu}^T \boldsymbol{h}_j(\boldsymbol{x}_j)\} - \boldsymbol{\mu}^T \boldsymbol{b}$$

- It decouples for a given μ (Dual Subproblems):

$$q_j(\boldsymbol{\mu}) = \min_{\boldsymbol{x}_j \in \mathcal{X}_j} \{f_j(\boldsymbol{x}_j) + \boldsymbol{\mu}^T \boldsymbol{h}_j(\boldsymbol{x}_j)\}$$

$q_j(\mu)$ is concave on μ

- And the Dual Master is:

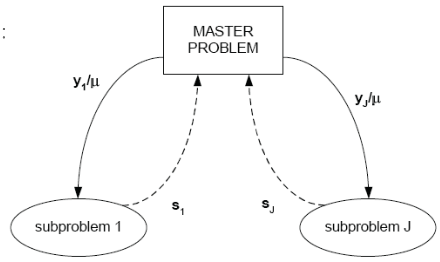
$$\begin{array}{ll} \max\limits_{\boldsymbol{\mu}} & q(\boldsymbol{\mu}) = \sum_{j=1}^{J} q_j(\boldsymbol{\mu}) - \boldsymbol{\mu}^T \boldsymbol{b} \\ s.t. & \boldsymbol{\mu} \succeq \boldsymbol{0} \end{array}$$

- Solution: as before

$$\boldsymbol{\mu}^{k+1} = [\boldsymbol{\mu}^k + \alpha^k \boldsymbol{s}^k]^+$$

- Pros and cons: as before

BLOCK DIAGRAM PRIMAL/DUAL



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Decomposition Framework in Convex Programming (7/16)

MVC DECOMPOSITION

- Problem formulation:

$$\begin{array}{ll} \min\limits_{\boldsymbol{x},\boldsymbol{y}} & c(\boldsymbol{x}) + d(\boldsymbol{y}) \\ s.t. & \boldsymbol{A}_1(\boldsymbol{x}) + \boldsymbol{B}_1(\boldsymbol{y}) \preceq \boldsymbol{b}_1 \\ & \boldsymbol{A}_2(\boldsymbol{x}) + \boldsymbol{B}_2(\boldsymbol{y}) \preceq \boldsymbol{b}_2 \\ & \boldsymbol{x} \in \mathcal{X} \\ & \boldsymbol{y} \in \mathcal{Y} \end{array}$$

- 'Dual function' relaxing first coupling constraint:

$$K(\boldsymbol{y}, \boldsymbol{\mu}) = \begin{array}{ll} \min\limits_{\boldsymbol{x}} & L(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\mu}) \\ s.t. & \boldsymbol{A}_2(\boldsymbol{x}) \leq \boldsymbol{b}_2 - \boldsymbol{B}_2(\boldsymbol{y}) \\ & \boldsymbol{x} \in \mathcal{X} \end{array}$$

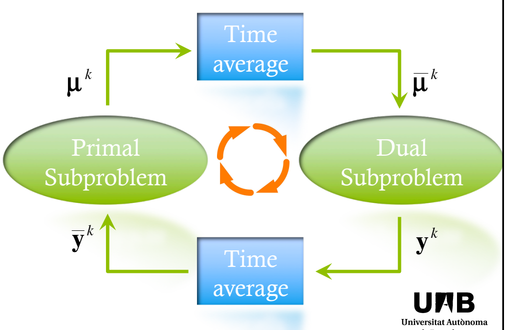
- Definition of primal and dual subproblem (fixing y or μ)

$$p(\boldsymbol{y}) = \begin{array}{ll} \max\limits_{\boldsymbol{\mu}} & K(\boldsymbol{y}, \boldsymbol{\mu}) \\ s.t. & \boldsymbol{\mu} \succeq \boldsymbol{0} \end{array} \qquad d(\boldsymbol{\mu}) = \begin{array}{ll} \min\limits_{\boldsymbol{y}} & K(\boldsymbol{y}, \boldsymbol{\mu}) \\ s.t. & \boldsymbol{y} \in \mathcal{Y} \end{array}$$

- Option to solve it: via primal/dual master problem

$$p^* = \begin{array}{ll} \min\limits_{\boldsymbol{y}} & p(\boldsymbol{y}) \\ s.t. & \boldsymbol{y} \in \mathcal{Y} \end{array} \qquad d^* = \begin{array}{ll} \max\limits_{\boldsymbol{\mu}} & d(\boldsymbol{\mu}) \\ s.t. & \boldsymbol{\mu} \succeq \boldsymbol{0} \end{array}$$

- Key in MVC: get rid of master problems



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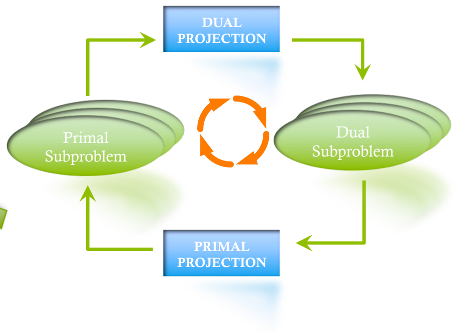
Decomposition Framework in Convex Programming (8/16)

PROPOSED METHOD: COUPLED-DECOMPOSITION

- Problem formulation:

$$\begin{array}{lll} \min\limits_{\{\boldsymbol{x}_j\},\boldsymbol{y}} & \sum_{j=1}^{J} f_j(\boldsymbol{x}_j) & \\ s.t. & \boldsymbol{x}_j \in \mathcal{X}_j, & j = 1,\ldots,J \\ & h_j(\boldsymbol{x}_j) \le y_j,\ (\lambda_j) & j = 1,\ldots,J \\ & \boldsymbol{A}\boldsymbol{y} \preceq \boldsymbol{c}\ (\mu) & \\ & \boldsymbol{y} \in \mathcal{Y}, & \mathcal{Y} = \mathcal{Y}_1 \times \ldots \times \mathcal{Y}_J \end{array}$$

suits primal and dual decomposition



- Question: how can we use both decompositions at the same time?

Decomposition Framework in Convex Programming (9/16)

Primal and Dual Subproblems

- The primal subproblems are:

$$\begin{array}{ll} \min\limits_{\boldsymbol{x}_j} & f_j(\boldsymbol{x}_j) \\ s.t. & \boldsymbol{x}_j \in \mathcal{X}_j, \\ & h_j(\boldsymbol{x}_j) \le y_j\ (\lambda_j) \end{array}$$

- The dual subproblems are (after relaxation of the coupling constraint):

$$q_j(\boldsymbol{\mu}) = \begin{array}{ll} \min\limits_{\boldsymbol{x}_j, y_j} & f_j(\boldsymbol{x}_j) + y_j([\boldsymbol{A}^T]_j\, \boldsymbol{\mu}) \\ & \boldsymbol{x}_j \in \mathcal{X}_j \\ & h_j(\boldsymbol{x}_j) \le y_j \quad (\lambda_j) \\ & y_j \in \mathcal{Y}_j \end{array}$$

- Subgradients no longer used as directions.
- The goal now is to be able to coordinately solve the KKT conditions.
- Subproblems are simply viewed as mappers that link primal and dual domains.
- Coordination itself is done at projections.

- From the KKT conditions point of view, some observations are derived:

Slackness:

$\lambda_j(h_j(\boldsymbol{x}_j^{p*}) - y_j) = 0$ → ACTIVE / NON-ACTIVE

Assuming local constraints non-active: $\lambda_j = [\boldsymbol{A}^T]_j\, \boldsymbol{\mu}$

If $\lambda_j > 0 \Longrightarrow y_j^* = h_j(\boldsymbol{x}_j^*)$

Decomposition Framework in Convex Programming (10/16)

- Primal-dual relationship in subproblems:
  - Consider the following one-dimensional problem

$$\begin{aligned} \min_{t_j} \quad & f_j(\boldsymbol{x}_j^*(t_j)) \\ s.t. \quad & h_j(\boldsymbol{x}_j^*(t_j)) \leq y_j \end{aligned}$$

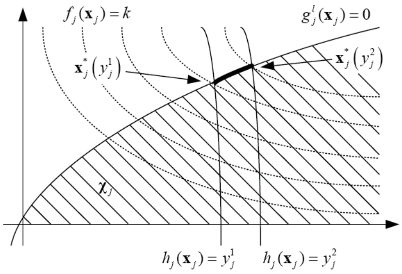
  - Just one constraint (under study)
  - The Lagrangian vanishes at

$$\frac{\partial L(t_j,\lambda_j)}{\partial t_j} = \frac{\partial f_j(\boldsymbol{x}_j^*(t_j))}{\partial t_j} + \lambda_j \frac{\partial h_j(\boldsymbol{x}_j^*(t_j))}{\partial t_j} = 0$$

- **Result:**

An increase (decrease) on y_j ⟺ A decrease (increase) on λ_j

- Observation: since Strong duality is assumed,
  - $\lambda_j > (<)\ \lambda_j^* \rightarrow y_j < (>)\ y_j^*$ (dual → primal)
  - $y_j > (<)\ y_j^* \rightarrow \lambda_j > (>)\ \lambda_j^*$ (primal → dual)



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Decomposition Framework in Convex Programming (11/16)

PRIMAL PROJECTION

- Observation: $\boldsymbol{A}\boldsymbol{y} \preceq c$ (μ)

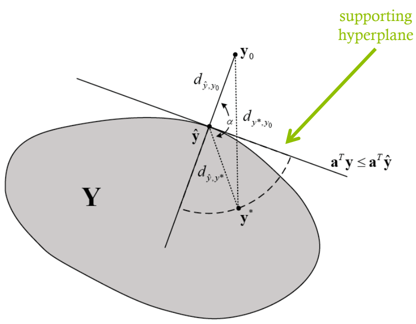
Slackness: $\mu_k > 0 \Rightarrow \boldsymbol{a}_k^T \boldsymbol{y} = c_k$

- Goal: get closer to y^*
- It is accomplished by solving:

$$\begin{aligned} \min_{\hat{\boldsymbol{y}}} \quad & \|\boldsymbol{y}_0 - \hat{\boldsymbol{y}}\|_2^2 \\ s.t. \quad & \boldsymbol{a}_k^T \hat{\boldsymbol{y}} = c_k, \quad k|\mu_k > 0 \\ & \boldsymbol{a}_k^T \hat{\boldsymbol{y}} \leq c_k, \quad \text{other } k \\ & \hat{\boldsymbol{y}} \in \mathcal{Y} \end{aligned}$$

dual subproblems output

corrected primal variables



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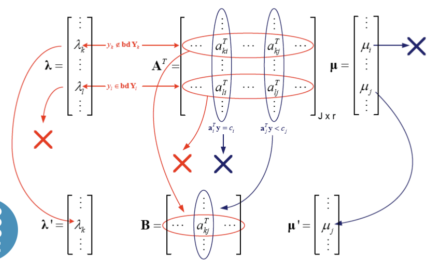
Decomposition Framework in Convex Programming (12/16)

DUAL PROJECTION

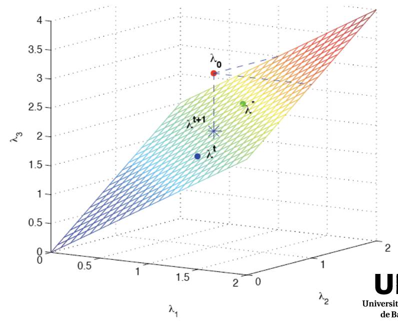
- Observations:
  - **Slackness:** $a_k^T y < c_k \Longrightarrow \mu_k = 0$
  - **Relation in dual variables:** $\lambda_{0_j} = [A^T]_j \, \mu$ (active subset)
- Over-determined linear system:



- Idea:
  - Project onto the hyperplane described by A^T…
  - … but close to the previous $\lambda^t = A^T\mu^t$ values

$$\min_{\mu'} \; \|\lambda'_{0\mathcal{D}} - B_{\mathcal{D}}\mu'\|^2$$
$$s.t. \quad \mu' \succeq 0$$

Selection of the closest candidates to the previous corrected values



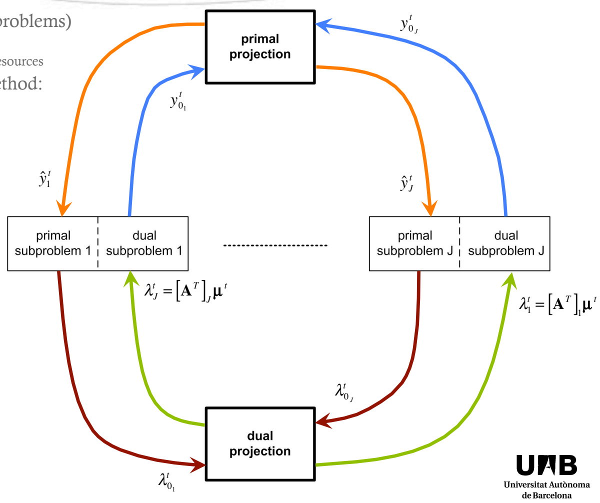
Decomposition Framework in Convex Programming (13/16)

SUMMARY / RESOURCE-PRICE INTERPRETATION

- It is possible to interpret: (e.g. subproblems)
  - Primal variables as resources
  - Dual variables as prices to be paid to use the resources
- Block diagram of the proposed method:

1. For fixed prices μ, the dual subproblems compute their own prices λ.
2. Dual subproblems take resources (grouped in y_0) depending on the price allocating internal resources x_j.
3. A potential unfeasible resource allocation is corrected.
4. The distribution proposed by primal projection may be unfair (some users paying more than others). Dual projection forces $\lambda = A^t\mu$ by modifying the prices the least possible amount.

It prevents strategic changes



Decomposition Framework in Convex Programming (14/16)

GEOMETRIC INTERPRETATION

- Consider the problem:

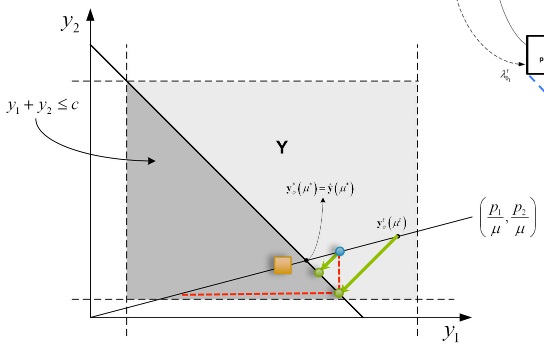
$$\begin{array}{llr} \min\limits_{x_1,x_2,y_1,y_2} & -p_1 \log x_1 - p_2 \log x_2 & \\ s.t. & m_i \le x_i \le d_i, & i = 1,2 \\ & x_i \le y_i & i = 1,2 \\ & y_1 + y_2 \le c & \\ & m_i \le y_i \le d_i, & i = 1,2 \end{array}$$

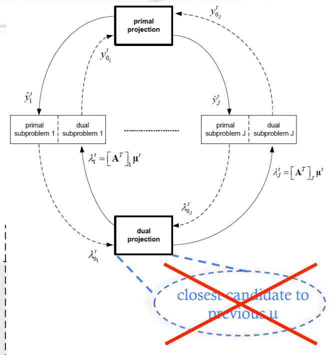
- Solution:

$$\boldsymbol{x}_i = \left.\frac{p_i}{\lambda_i}\right]_{m_i}^{d_i}$$

$$\mu = \lambda_1$$
$$\mu = \lambda_2$$

$$\mathbf{x}(\mu) = \mathbf{y}(\mu) = \left[\frac{p_1}{\mu}, \frac{p_2}{\mu}\right]_{\mathbf{m}}^{\mathbf{d}}$$





$$\begin{array}{ll} \min\limits_{\mu} & \|\boldsymbol{\lambda}_0 - \mathbf{1}\mu\|^2 \\ s.t. & \mu \ge 0 \end{array}$$

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Decomposition Framework in Convex Programming (15/16)

NUMERICAL EXAMPLE

- Consider the problem:

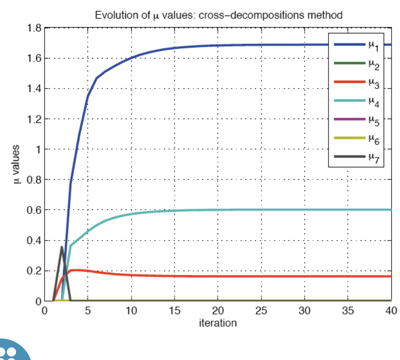
$$\begin{array}{lll} \min\limits_{\boldsymbol{x}} & -\sum_{i=1}^{20} p_i \log x_i & \\ s.t. & m_i \le x_i \le d_i, & i = 1, \ldots, 20 \\ & \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{c} & \end{array}$$

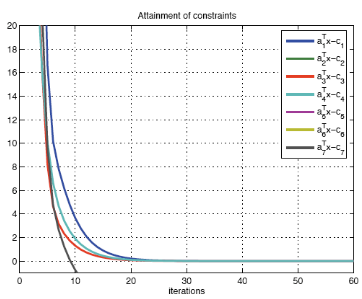
$$\boldsymbol{c} = [78.064, 342.67, 144.95, 60.464, 388.53, 353.98, 113.22]^T$$

$$\boldsymbol{m} = [4.63, 0.58, 3.66, 1.04, 2.69, 4.32, 0.48, 1.65, 3.24, 2.16, \ldots \\ \ldots, 0.78, 1.97, 2.47, 0.65, 1.35, 1.63, 4.76, 4.34, 4.56, 3.47]^T$$

$$\boldsymbol{d} = [38.48, 29.51, 33.83, 29.33, 57.56, 32.40, 20.57, 19.82, 32.23, 8.76, \ldots \\ \ldots, 38.04, 51.56, 45.19, 11.80, 46.46, 39.73, 35.09, 10.15, 59.96, 4.54]^T$$

Randomly generated values (the same for **A**)





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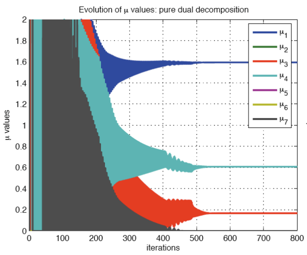
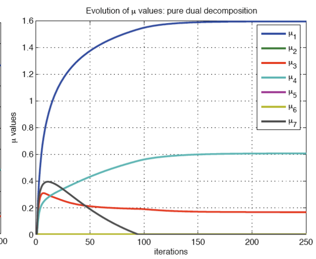
Decomposition Framework in Convex Programming (16/16)

- Comparison with dual-decomposition:

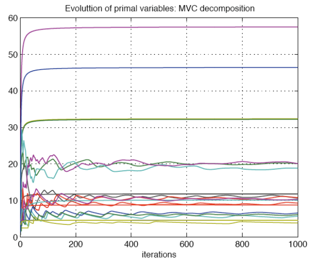


Diminishing step-size α=5/k



Constant step-size α=0.001

- Comparison with MVC decomposition:



Convergence properties fairly better using coupled-decomposition

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Outline

- **Introduction**
- **Dynamic Bandwidth Allocation (DBA)**
- **Decomposition Framework in Convex Programming**
- **Cross-Layer DBA in DVB-RCS**
- **Distributed Algorithm for Uplink Scheduling in WiMAX Networks**
- **Conclusions and Future Work**

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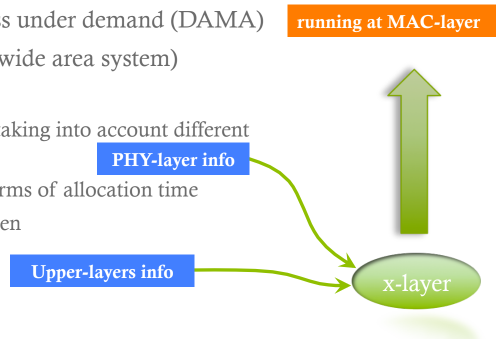
Cross-Layer DBA in DVB-RCS (1/8)

MOTIVATION

- DVB-RCS is designed to support multimedia IP communications
- Work focused on the multiple access under demand (DAMA)
- Maybe high number of users (very wide area system)
- Goals
  - Efficient usage of the air interface (taking into account different propagation conditions of users)
  - Efficient management of users in terms of allocation time
  - Include the desired balancing between
    - Fairness among users
    - QoS defined at upper layers



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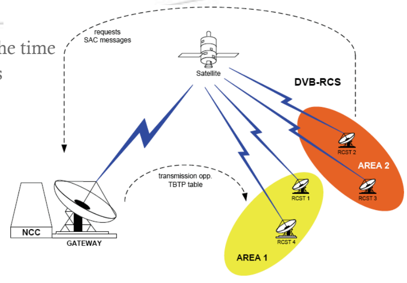
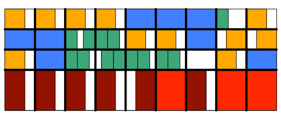
Cross-Layer DBA in DVB-RCS (2/8)

MULTIPLE ACCESS & PROPOSED FRAMEWORK

- Three types of request over a MF-TDMA resource division
  - Constant Rate Assignment (CRA): constant rate all the time
  - Rate Based Dynamic Capacity (RBDC): rate requests remain active until updated or timed out
  - Volume Based Dynamic Capacity (VBDC): volume requests, no constant rate needed

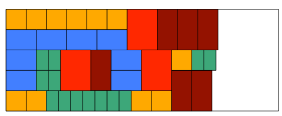


- Two key aspects in resource allocation:
  - DBA algorithm itself (fairness, QoS, …)
  - Structure imposed to the MF-TDMA (bandwidth usage)



complexity vs. efficiency

trade-off



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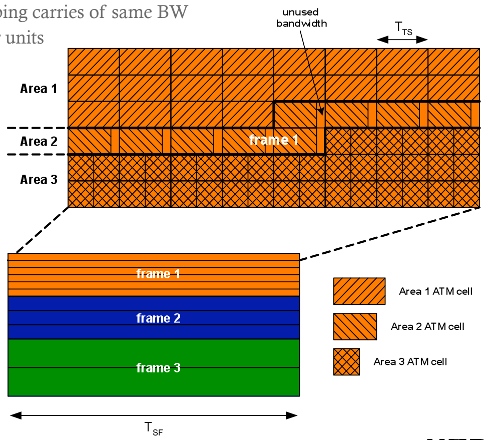
Cross-Layer DBA in DVB-RCS (3/8)

- Our choice: fixed structure optimizing bandwidth efficiency
  - Superframe is divided into frames grouping carries of same BW
  - TS duration allowing 1, 2, 4 MAC-layer units

Fairness in allocation, sustainment of QoS defined at upper layers and different propagation conditions is left to the DBA algorithm



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Cross-Layer DBA in DVB-RCS (4/8)

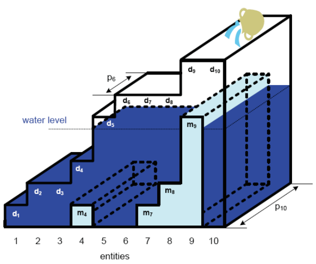
X-LAYER DBA ALGORITHMS

- Consider the following NUM formulation:

$$\max_{x_1,\ldots,x_N} \prod_{i=1}^{N} x_i^{p_i} \quad s.t. \quad \sum_{i=1}^{N} x_i \le P, \quad m_i \le x_i \le d_i$$

- Semi-analytical solution:

$$x_i = \left[\frac{p_i}{\lambda}\right]_{m_i}^{d_i} \triangleq \begin{cases} \frac{p_i}{\lambda}, & m_i \le \frac{p_i}{\lambda} \le d_i \\ m_i, & \frac{p_i}{\lambda} \le m_i \\ d_i, & \frac{p_i}{\lambda} \ge d_i \end{cases}$$



- Reformulation to match the DVB-RCS situation (fixed TS duration):

$$\max_{\{x_{i,j}\}} \prod_{i,j} (x_{i,j} \cdot K_i)^{p_{i,j}} \quad s.t. \quad \sum_{i,j} x_{i,j} \le P, \quad \left\lceil \frac{m_{i,j}}{K_i} \right\rceil \le x_{i,j} \le \left\lceil \frac{d_{i,j}}{K_i} \right\rceil, \ \forall i,j$$

duration of 1 ATM frame in the i^th area

$$K(T_{TS}, t_{a(i)}) = \left\lfloor \frac{T_{TS}}{t_{a(i)}} \right\rfloor \qquad P(C, T_F, T_{TS}) = \left\lfloor \frac{T_F}{T_{TS}} \right\rfloor$$

Number of ATM cells per TS — Number of TS in a frame

✓ TS's are allocated
✓ Utility is measured in ATM cells

- Practical computation:
  - Bisection
  - Coupled-decomposition (centrally computed)

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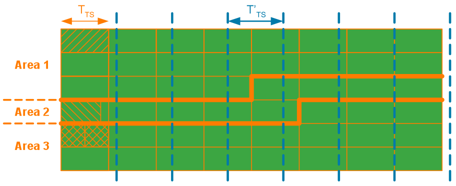
Cross-Layer DBA in DVB-RCS (5/8)

- Joint DBA and frame optimization:

$$\begin{aligned} \max_{T_{TS},\{x_{i,j}\}} \quad & \prod_{i,j} \left(x_{i,j} \cdot K(T_{TS}, t_{a(i)})\right)^{p_{i,j}} \\ s.t. \quad & \sum_{i,j} x_{i,j} \leq P(C, T_F, T_{TS}) \\ & \left\lceil \frac{m_{i,j}}{K(T_{TS}, t_{a(i)})} \right\rceil \leq x_{i,j} \leq \left\lceil \frac{d_{i,j}}{K(T_{TS}, t_{a(i)})} \right\rceil \\ & T_{min} \leq T_{TS} \leq T_{max} \end{aligned}$$



- Stopping criterion in coupled-decompositions:

$$B^t = \frac{\frac{1}{\mu^{t+1}} - \frac{1}{\mu^t}}{\frac{1}{\mu^t} - \frac{1}{\mu^{t-1}}}$$

When it does not change, we are in optimal zone

- ✓ Key property: Only multiple values of $t_{a(i)}$ are candidates to the optimal T_{TS}.
- ✓ Few candidates ⇨ exhaustive search over T_{TS} is reasonable.
- ✓ T_{TS} does not need updating on a superframe basis:
  - ✓ Area dynamics can be assumed slow-varying (an area is an aggregation)
  - ✓ Optimal T_{TS} is not very sensitive to slight request changes

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Cross-Layer DBA in DVB-RCS (6/8)

NUMERICAL RESULTS

- Allocation example:

Area identifier	RCST	ATM cells per TS	Requests	Minimums	Priorities
1	1-2	1	[15, 16]	[2, 0]	[1.75, 1.25]
2	3-6	1	[9, 19, 14, 5]	[0, 1, 2, 0]	[1.5, 2, 1.25, 1.75]
3	7-13	2	[17, 13, 4, 5, 13, 13, 8]	[1, 2, 2, 2, 0, 1, 1]	[1.75, 2, 1.5, 2, 1.25, 1.5, 1.25]
4	14-20	2	[12, 10, 2, 2, 7, 1, 8]	[1, 0, 2, 2, 2, 1, 2]	[1.5, 1.5, 1.75, 1.75, 2, 1.25, 1]
5	21-24	2	[14, 3, 2, 13]	[0, 1, 2, 3]	[1.5, 2, 1, 1.75]

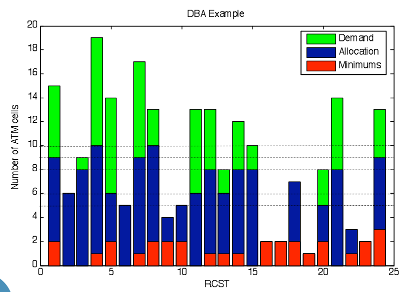
web browsing

telnet gaming

VoIP

FTP SMTP

streaming



- ✓ Resources are effectively balanced function of priority
  - ☆ Enables mapping of QoS at MAC layer

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Cross-Layer DBA in DVB-RCS (7/8)

- Overall system performance:
  - System
    - 26.5ms of SF duration
    - 111 carriers of 540kHz (60MHz in total)
    - Roll-off factor: 0,35
    - Areas and coding rates

Area identifier	Coding rate	ATM cell duration
1	$r_1 = 1/2$	$t_1 = 1.06ms$
2	$r_2 = 2/3$	$t_2 = 0.795ms$
3	$r_3 = 3/4$	$t_3 = 0.707ms$
4	$r_4 = 5/6$	$t_4 = 0.636ms$
5	$r_5 = 7/8$	$t_5 = 0.606ms$

- Scenario generation:
  - Number of terminals per area $\mathcal{V}_k \sim \mathcal{U}[0, 2 \cdot v_k]$
  - Requests per terminal $d_i \sim \mathcal{U}[0, 2\frac{D_{tot}}{\mathbf{1}^T \boldsymbol{v}}]$
  - ADR_{ref} = 4662 ATM_{cells}/26.5ms = 74.59Mbps

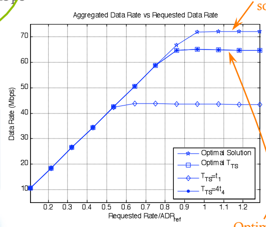
mean system load

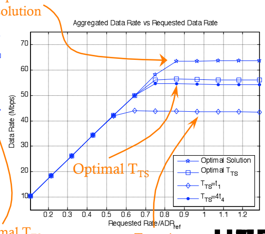
averaged distribution of RCSTs among areas (e.g. v=[5,10,10,30,45])

Scenario 1
v=[5,10,10,30,45]

Scenario 2
v=[20,20,20,20,20]

Optimal solution





Optimal T_{TS}

Optimal T_{TS}
T_{TS}= 4 t_4

T_{TS}= 4 t_4

- ✓ T_{TS} optimization reports significant gains in bandwidth usage (and thus in aggregated data rate) with respect to the original design
- ✓ Slight performance degradation with scenario variations (robust behavior)

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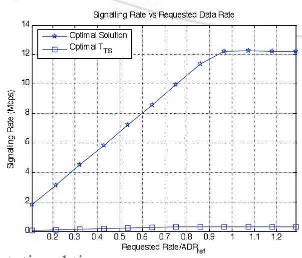
Cross-Layer DBA in DVB-RCS (8/8)

- Signaling



- ✓ Signaling can be kept very small in the DVB-RCS case when compared to a non-structured solution

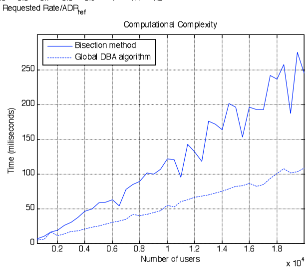
- Computational time

$d_i \sim \mathcal{U}[1, 20]$

$m_i \sim \mathcal{U}[0, 3]$

$K_i \sim \mathcal{U}[1, 2]$

$p_i \in \{1, 1.25, 1.5, 1.75, 2\}$



- ✓ With half the computation time, the proposed method is able to manage around 2x number of users
- ✓ More predictable computation time

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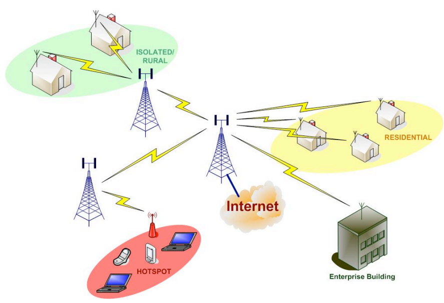
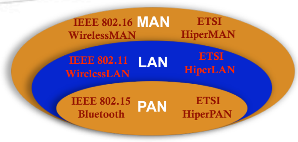
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (1/7)

MOTIVATION

- WiMAX: Metropolitan Area Network (MAN) standard
- Possible applications:





- WiMAX used as backhaul
- Uses DAMA in uplink
- Tree-deployed network
- Centralized scheduling requires a distributed solution

- The standard defines several physical layers (single carrier, OFDM, OFDMA)
- From a MAC perspective: fixed link capacity
- 3-way handshake mechanism to request and grant resources
- Scheduling services: UGS, rtPS, nrtPS, BE, ertPS

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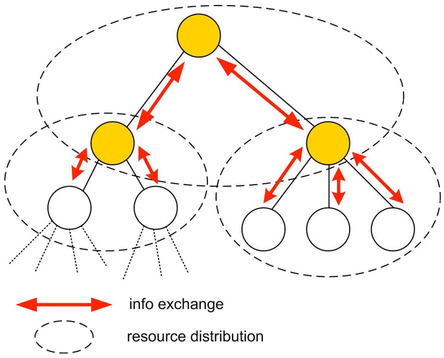
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (2/7)

BANDWIDTH REQUEST & ALLOCATION MODELLING

- Problem formulation within the NUM context
  - Advantage: fairness & QoS issues

$$
\begin{aligned}
\max_{\{\boldsymbol{r}_i\}} \quad & \sum_{i=1}^{N} U_i(\boldsymbol{r}_i) \\
s.t. \quad & \boldsymbol{r}_i \in \mathcal{R}_i \qquad i = 1 \dots N \\
& \sum_{i=1}^{N} h_i(\boldsymbol{r}_i) \leq c
\end{aligned}
$$

- Distributed solution: resources allocated inside each PMP cluster
- Global optimization: equal treatment of flows
- Attainable with decomposition methods
  - Multi-level decomposition



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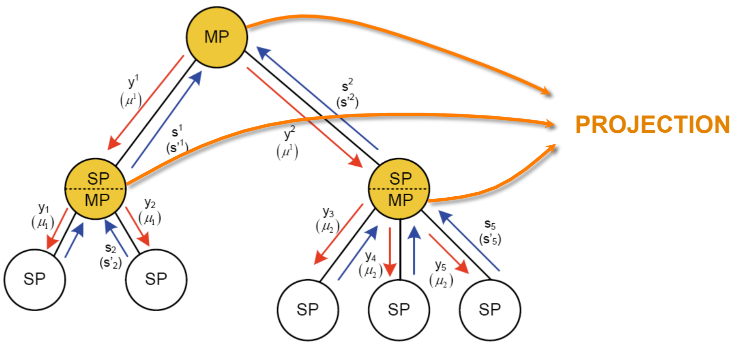
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (3/7)

DECOMPOSITION STRATEGIES

- System view with primal (dual) decomposition:



- Problem: iterations required to converge
  - User-defined step-size
  - Tree levels

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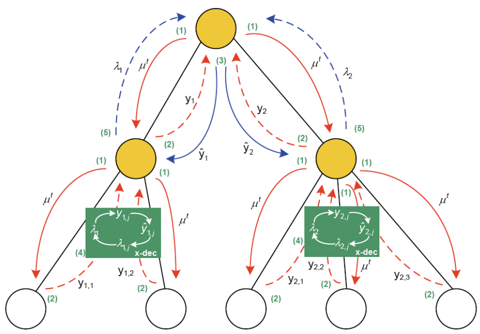
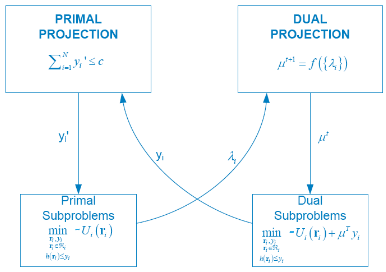
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (4/7)

- System view with coupled-decomposition:





- Advantages:
  - No user-defined step-size
  - Fast convergence

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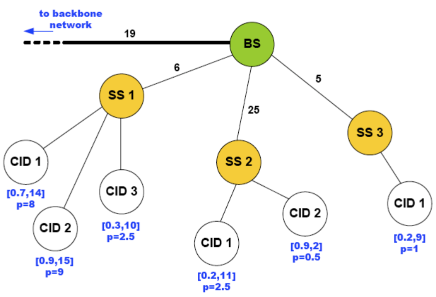
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (5/7)

NUMERICAL RESULTS

- Network example under test:

$$\max_{\{\boldsymbol{r}_i\}} \quad \sum_{i=1}^{N} U_i(\boldsymbol{r}_i)$$
$$s.t. \quad \boldsymbol{r}_i \in \mathcal{R}_i \qquad i = 1 \ldots N$$
$$\sum_{i=1}^{N} h_i(\boldsymbol{r}_i) \le c$$



- Highest level:

$$U_i(c_i) = \left\{ \begin{array}{ll} \max\limits_{\{r_i^j\}} & \sum_j U_i^j(r_i^j) \\ s.t. & \sum_j r_i^j \le c_i \end{array} \right\}$$

$$h_i(\boldsymbol{r}_i) = \sum_j r_i^j$$

- Lowest level:

$$U_i^j = p_i^j \log r_i^j$$

In order to achieve a proportionally fair solution

$$h_i^j(r_i^j) = r_i^j$$

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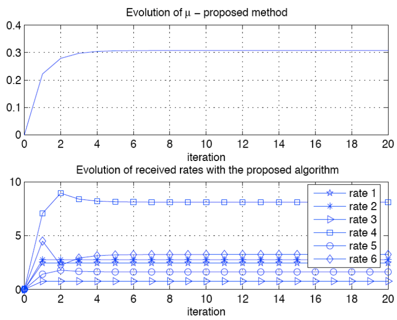
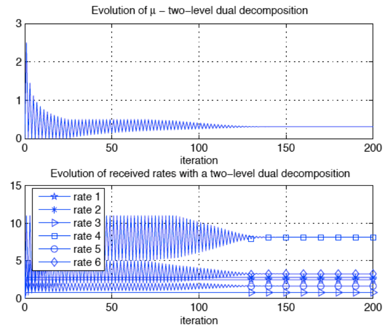
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (6/7)

- Comparison among solutions:

Coupled-Decomposition



2-level Dual Decomposition



- Differences:
  - Step-size
  - Iterations to converge

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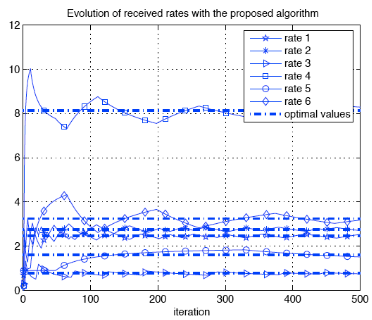
Distributed Algorithm for Uplink Scheduling in WiMAX Networks (7/7)

- MVC Decomposition:
  - Computed centrally for comparison purposes
  - The MVC Decomposition method does not provide an explicit distributed solution



- Convergence is still faster with the proposed method

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Conclusions and Future Work (1/2)

CONCLUSIONS

- Proposal of a novel method with advantages in
  - Convergence properties
  - No tuning required
- Application to DBA in:
  - DVB-RCS
  - WiMAX (uplink)
- Cross-layer DBA solution (centralized) in DVB-RCS including:
  - PHY layer of RCS Terminals
  - Upper-layer requirements (QoS)
  - Time-efficiency (impact on the number of users)
- DBA solution in WiMAX uplink
  - Mesh tree-deployed network
  - Attaining centralized scheduling requirements (without sacrificing optimality)
  - Time-efficiency (impact on increasing tree levels)

Conclusions and Future Work (2/2)

FUTURE WORK

- In chapter 3:
  - Proof the convergence of the method (general case) + refinement + extensions
  - Find efficient computations of primal/dual projections
  - Characterize the speed of convergence of the method
  - Other applications
- In chapter 4:
  - Simple strategies to tune priorities
  - Test our work in a DVB-RCS simulation platform
  - Join design of request/allocation
  - Include mobility and channel variation issues (statistical NUM)
- In chapter 5:
  - Other network topologies + adequate decompositions
  - Include PHY cross-layer solutions (subcarrier allocation, adaptive coding & modulation or subcarrier allocation)

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Related Research Contributions

- Chapter 3

- A. Morell, G. Seco-Granados, M.A. Vázquez-Castro, "Computationally Efficient Cross-Layer Algorithm for Fair Dynamic Bandwidth Allocation", IEEE ICCCN'07.
- A. Morell, G. Seco-Granados, J.L. Vicario "Distributed Algorithm for Uplink Scheduling in WiMAX Networks", IEEE Broadnets 2008.
- G. Seco-Granados, M.A. Vázquez-Castro, A. Morell and F. Vieira, "Algorithm for Fair Bandwidth Allocation with QoS Constraints in DVB-S2/RCS", IEEE GLOBECOM'06.

- Chapter 4

- T. Pecorella, G. Mennuti (chapter editors), N. Celandroni, F. Davoli, E. Ferro, A. Gotta, S. Karapantazis, A. Morell, G. Seco-Granados, P. Todorova and M.A. Vázquez-Castro (authors in alphabetical order), "Dynamic Bandwidth Allocation", chapter in Resource Management in Satellite Networks: Optimization and Cross-Layer Design, Springer
- A. Morell, G. Seco-Granados and M.A. Vázquez-Castro, "Cross-Layer Design of Dynamic Bandwidth Allocation in DVB-RCS", IEEE Systems Journal.
- A. Morell, G. Seco-Granados and M.A. Vázquez-Castro, "Enhanced Dynamic Resource Allocation for DVB-RCS: a Cross-Layer Operational Framework", IEEE MILCOM'07.
- A. Morell, G. Seco-Granados, M.A. Vázquez-Castro, "Joint Time Slot Optimization and Fair Bandwidth Allocation for DVB-RCS Systems", IEEE GLOBECOM'06.

- Chapter 5

- A. Morell, G. Seco-Granados, J.L. Vicario "Distributed Algorithm for Uplink Scheduling in WiMAX Networks", IEEE Broadnets 2008.
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THANK YOU!

Questions?

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