Optimal Noncoherent Detector for HS-GNSS Receivers

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***Abstract*—This paper addresses the problem of optimal signal detection in the context of high-sensitivity (HS) global navigation satellite system (GNSS) receivers. Contrary to the traditional belief that second-order statistics are optimal for signal detection/estimation under the Gaussian assumption, this paper shows that optimal signal detection turns out to be based on the absolute moments of the input signal. This result opens the door for the introduction of fractional lower-order statistics (FLOS), a wider family of statistics with links to algebraic-tailed processes such as the ones affected by abnornals or outliers. In that sense, the proposed detection strategy and its extension to FLOS seems to be the best candidate for application onto harsh working scenarios such as the ones involved in HS-GNSS receivers.**

I. Introduction

One of the main contributions to extend the usage of Global Navigation Satellite Systems (GNSS) in scenarios other than clear outdoor ones would be the proposal of reliable signal detection algorithms. This is the case of indoor and dense urban scenarios, where signal attenuations on the order of 20 to 70dB can be introduced because of the existence of blocking obstacles (e.g. walls, floors, etc.). In these circumstances high-sensitivity (HS) receivers become essential to cope with such a severe impairment while still providing a reasonable performance.

In the context of spread spectrum GNSS such as GPS and Galileo, the most critical part of the receiver is related to code acquisition for initial synchronization. This initial acquisition provides a coarse code epoch estimation that is refined later on by subsequent modules to produce the final user position estimate. For the particular case of HS-receivers, stringent requirements are imposed on the receiver sensitivity with the aim of reliable code acquisition at very low C/N_0 values. A proof of this statement is that while traditional receivers have their operational threshold around $C/N_0 = 33 - 35$ dBHz, HS-GNSS receivers are expected to operate within the range from $10 - 25$ dBHz.

For such a weak received power, a well-known result from detection theory states that the only way to achieve better detection sensitivity passes through the adoption of long correlation intervals [1]. For this to be possible, HS receivers have to overcome the presence of modulated data and/or phase uncertainties which restrict the maximum length of coherent correlation. This problem is circumvented by implementing noncoherent combinations of partial coherent correlation terms, thus leading to an overall extended correlation with superior detection performance. Traditional approaches for implementing noncoherent integration are often based on second-order statistics. Two popular approaches for this are the sum of squared envelopes of coherent correlation samples [2, Ch. 7], [3], [4] and the sum of differential correlation samples [5]. Combinations of both approaches can also be found in the literature such as the one in [6], or based on some optimality criteria such as the maximum likelihood post-detection method in [7].

At this point it is interesting to remark that traditional second-order statistics have been found to be optimal for parameter estimation of Gaussian input signals as the SNR tends to zero [8]. However, operation under very low SNR values and the presence of external impairments may introduce ouliers that cause an increase in the tails of the input data statistical distribution. In these circumstances, adopting the traditional approach of using the squared envelope of pre-detected samples may lead to noise enhancement that degrades the total energy capture. This is a rather well-known concept in the field of RADAR detection but not widely adopted within the communications community [9].

To circumvent noise enhancement in very low SNR scenarios, second-order statistics have also been proposed in the literature but in a differential manner. That is, by cross-correlating a piece of incoming signal with the subsequent one. This approach asymptotically reduces the noise contribution due to the fact that cross-correlation noise terms tend to vanish. Nevertheless, this noise reduction can directly be translated into improved detection sensitivity as long as the symbol transition patter does not degrade the total accumulated metric. The dependence of differential correlation methods on the symbol transition density is thus one of its main drawbacks. In the recent years, this problem has motivated the development of hybrid approaches where both differential correlation and squared envelopes are considered. This is the case, for instance, of the so-called dirty template approaches proposed in the field of ultra-wideband communications [10], [11], [12]. A similar result is also proposed in [13] under the optimal framework of maximum likelihood post-detection integration, where both squared and differentially correlated samples are combined to provide an optimal detection rule that is shown to outperform traditional noncoherent (second-

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oder based) integration.

In any case, the ultimate limiting factor of second-order statistics seems to be the significant degradation in the presence of severe noise. Since the presence of severe noise can be associated with an input distribution with large tails, the analysis of detection techniques for heavy tailed (i.e. non-Gaussian) distributions attains a relevant interest.

Based on the above assumptions, the main goal of this paper is to show that a clear link exists between the optimal decision rule for detecting an unknown constant immersed in Gaussian noise and FLOS usually adopted in non-Gaussian signal processing. The exploitation of this link leads us to the proposal of a generalized detection rule based on the noncoherent metric given by the absolute value of pre-detected samples in the form $|\cdot|^{\alpha}$, with the power α a constant within the range $(0,1]$. By analyzing the receiver operating characteristics (ROC), simulation results show a significant improvement when given values of the power α are adopted. About a 15% increase in probability of detection is observed for a given probability of false alarm compared to conventional noncoherent integration. The tests have been performed in a HS-GNSS software receiver implementing the double-FFT acquisition method proposed in [14]. This acquisition method efficiently implements the coherent integration of whole bit durations and leads to an overall receiver structure with superior performance in harsh environments such as indoor and urban scenarios. Finally, experimental results are also compared with theoretical bounds to assess the possible degradation loss when operating in real working conditions.

The paper is organized as follows. The signal model and working assumptions are presented in Section II. The formal derivation of the optimal detection rule is presented in Section III based on the generalized likelihood ratio test. The corresponding detection threshold is presented and an extension is proposed which links the resulting detection rule with fractional lower-order statistics. In Section IV, the detection performance of the optimal detection rule is introduced where both theoretical bounds and experimental results are compared. Next, simulation results are provided in Section V for realistic working conditions by using a HS-GNSS software receiver. Finally, conclusions are drawn in Section VI.

II. Signal Model

The problem to be addressed in this paper arises in the signal acquisition stage of a HS-GNSS receiver. As for any spread-spectrum system, the main purpose of this stage is to cross-correlate the incoming signal with the local code replica. The result is then evaluated for deciding whether the signal is present or not. This cross-correlation involves a time-frequency search since both code phase and Doppler corrections must be determined. However, unlike traditional receivers, a rather long correlation interval must be selected in HS receivers to compensate for the very weak power of received signals. This is especially true when operating in indoor environments, urban canyons or any other scenario where visibility conditions and surrounding obstacles degrade

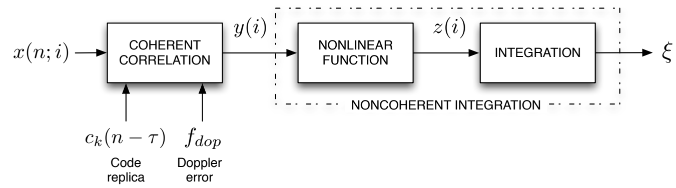

Fig. 1. Block diagram of the GNSS acquisition module under analysis.

the quality of the received signal. In these circumstances the long correlation interval is implemented as a two-step procedure. First, coherent cross-correlation is performed between the incoming signal and the local replica. Second, a set of N_i coherent correlations are noncoherently combined to provide the output detection metric [3, Ch.10]. Finally, this output is compared with a given detection threshold γ in order to determine whether the signal of interest is present or not. An illustration of this procedure is depicted in Fig. 1.

Let the discrete-time baseband equivalent of the incoming signal be denoted by $x(n)$. For coherent cross-correlation, let us assume that the incoming signal is splitted into pieces of N_x samples. Each of these pieces correspond to an observation interval of T_{coh} seconds, and pieces are indicated as $x(n;i)$. For the case of GPS L1, and assuming no assistance information, values for this coherent correlation interval range from 10ms in traditional outdoor receivers to 20ms in advanced HS receivers [14]. In any case, the output of coherent cross-correlation with the i-th segment of input data is represented herein by $y(i)$. Then noncoherent integration is defined as,

$$\xi = \sum_{i=0}^{N_i-1} z(i) \tag{1}$$

where $z(i) \doteq g\left(y(i)\right)$ is a nonlinear transformation of $y(i)$ and ξ is the resulting metric to be compared with a detection threshold. For a given code phase and Doppler trial two hypotheses are to be considered $\{\mathcal{H}_1, \mathcal{H}_0\}$ depending on whether the trial is successful or unsuccessful, respectively. In these circumstances, the signal model for the coherent correlation output $y(i)$ can be expressed as follows:

$$y(i) = \begin{cases} A \cdot e^{j\theta(i)} + n(i) & : \quad \mathcal{H}_1 \\ n(i) & : \quad \mathcal{H}_0 \end{cases} \tag{2}$$

with A a real-valued unknown constant affected by unknown complex rotation $\theta(i)$ and $n(i)$ being the noise contribution. Since the input signal $x(n;i)$ is assumed to be Gaussian distributed and coherent correlation is nothing but a linear filtering of the input signal, the noise contribution $n(i)$ at the correlation output can also be assumed to Gaussian. In particular it will be assumed that $n(i)$ is zero-mean Gaussian distributed with variance σ_n^2. That is, $n(i) \sim \mathcal{N}(0, \sigma_n^2)$.

III. Optimal Noncoherent Detection

Noncoherent integration allows the extension of the correlation interval while reducing degradations introduced by residual frequency errors and bit transitions. As previously mentioned, extending the correlation interval is essential for HS-GNSS receivers in order to improve their detection sensitivity and compensate the very weak power of the received signal.

The traditional approach for noncoherent integration is based on using the second order law for the nonlinear function $g(\cdot)$ to be applied onto the coherent correlation output samples. This is a widely adopted criterion whose optimality is often supported on the sufficiency of second-order statistics for parameter estimation/detection. In particular, for the case of Gaussian input signals in the low SNR regime. However, and in the context of HS-GNSS receivers, it is the main purpose of this paper to show that contrary to the traditional belief second order statistics are no longer optimal for noncoherent integration. To proof this statement, the optimal decision statistics are to be derived using standard tools from detection theory.

A. GLRT detector for HS-GNSS acquisition

Let us consider the signal detection problem to be faced by HS-GNSS receivers. This problem involves detecting the presence of signal with unknown amplitude at the output of coherent correlation. To this end, let a set of N_i coherent correlation output samples be stacked into the vector $\mathbf{y} \doteq [y(0), y(1), \ldots, y(N_i-1)]^T$. Similarly, let the set of unknown parameters be stacked into the vector $\mathbf{\Omega} \doteq [A, \{\theta(i)\}]$. Then, the probability density function for $\mathcal{H}_1$ is denoted by $f_{\mathbf{y}}(\mathbf{y}; \widehat{\mathbf{\Omega}}, \mathcal{H}_1)$ whereas for $\mathcal{H}_0$ we have $f_{\mathbf{y}}(\mathbf{y}; \mathcal{H}_0)$. Based on these premises, optimal detection passes through the formulation of the generalized likelihood ratio test (GLRT) [1],

$$\Lambda(\mathbf{y}) = \frac{f_{\mathbf{y}}(\mathbf{y}; \widehat{\mathbf{\Omega}}, \mathcal{H}_1)}{f_{\mathbf{y}}(\mathbf{y}; \mathcal{H}_0)} \begin{array}{c} > \\ < \end{array} \gamma \tag{3}$$

where the set of unknown parameters must be substituted by its maximum likelihood estimate $\widehat{\mathbf{\Omega}}$. Based on the GLRT in (3), the detection criterion is to decide $\mathcal{H}_1$ when $\Lambda(\mathbf{y}) > \gamma$ and $\mathcal{H}_0$ otherwise. The threshold γ is set so that a given probability of false alarm $P_{FA} = prob\{\Lambda(\mathbf{y}) > \gamma | \mathcal{H}_0\}$ is achieved.

Since the signal model in (2) is Gaussian distributed, the likelihood ratio test is nothing but a quocient of a noncentral and a central Gaussian probability density function. Substituting into (3) and taking the logarithm, the log-likelihood ratio $L(y) \doteq \log \Lambda(y)$ results in

$$L(\mathbf{y}) = -\frac{1}{\sigma_n^2} \sum_{i=0}^{N_i-1} \left| y(i) - \hat{A} \cdot e^{j\hat{\theta}(i)} \right|^2 + \frac{1}{\sigma_n^2} \sum_{i=0}^{N_i-1} \left| y(i) \right|^2. \tag{4}$$

By expanding quadratic terms and applying some simplifications, the equivalent log-likelihood ratio can be expressed as follows,

$$L(\mathbf{y}) = \frac{2\hat{A}}{\sigma_n^2} \sum_{i=0}^{N_i-1} \mathrm{Re}\left[y(i) \cdot e^{-j\hat{\theta}(i)} \right] - \frac{N_i \cdot \hat{A}^2}{\sigma_n^2}. \tag{5}$$

The expression in (5) can be reformulated by absorbing all irrelevant constants into a new detection threshold given by $\gamma' = \frac{\sigma_n^2}{2\hat{A}} \left[\log \gamma + N_i \hat{A}^2 / \sigma_w^2 \right]$. Proceeding in this way, the GLRT results in

$$L(\mathbf{y}) = \sum_{i=0}^{N_i-1} \mathrm{Re}\left[y(i) \cdot e^{-j\hat{\theta}(i)} \right] \begin{array}{c} > \\ < \end{array} \gamma'. \tag{6}$$

Note that once $\hat{A}$ is absorbed into the detection threshold γ', the GLRT does not depend on $\hat{A}$ anymore. This is because γ' is nothing but a constant to be determined later on according to some requirements in terms of probability of false alarm.

The final expression for the GLRT in (6) requires the substitution of $\hat{\theta}(i)$ with its maximum likelihood estimate. That is, the value $\hat{\theta}_{\mathrm{ML}}(i)$ for which $\hat{\theta}_{\mathrm{ML}}(i) = \arg\max_\theta f_y(\mathbf{y}; \{\theta(i)\}, \mathcal{H}_1)$. Taking the logarithm of $f_y(\mathbf{y}; \{\theta(i)\}, \mathcal{H}_1)$ and solving for $\theta(i)$, it is not difficult to see that the maximum likelihood estimate of $\theta(i)$ becomes,

$$\hat{\theta}_{\mathrm{ML}}(i) = \arg\{y(i)\}. \tag{7}$$

At this point it is important to recall that for any complex number x, it is always true that $|x| = x \cdot e^{-j\arg\{x\}}$. With this observation in mind and substituting (7) into (6), the final expression for the GLRT results in the simple decision rule,

$$L(\mathbf{y}) = \sum_{i=0}^{N_i-1} |y(i)| \begin{array}{c} > \\ < \end{array} \gamma' \tag{8}$$

which is based on the *absolute moments* of the input signal rather than on second-order statistics.

As a result of these derivations, and coming back to the HS-GNSS block diagram in Fig. 1, the optimal nonlinear function $g(\cdot)$ for noncoherent integration turns out to be given by the absolute value, $g(\cdot) = |\cdot|$. Then, the optimal HS-GNSS detection metric in Fig. (1) becomes

$$\xi = \sum_{i=0}^{N_i-1} |y(i)|. \tag{9}$$

B. Detection threshold

The detection threshold γ' in (8) can be determined based on a target probability of false alarm P_{FA} as follows,

$$P_{FA} = prob\{\xi > \gamma' | \mathcal{H}_0\}. \tag{10}$$

For the traditional case of second-order noncoherent integration, determining the probability of false alarm is rather simple. The detection metric ξ becomes central chi-square distributed and γ' can be obtained from its right tail probability function.

Unfortunately, determining γ' for the absolute value detection metric in (9) is far more complicated. The reason is that ξ is composed by the sum of N_i iid Rayleigh random variables, and a closed-form expression for the resulting probability density function is only available for the case of $N_i = 2$. For an arbitrary sum of N_i terms, numerical evaluations and approximations have been proposed in the literature. For instance, a well known approach is to use infinite series approximations to precisely obtain the cumulative density function [15]. Recently, accurate simple closed-form approximations have also been proposed based on small argument approximations (SAA) [16]. In any case, and for the problem under study, a simpler approach will be adopted by invoking the central limit theorem. By doing so the detection metric ξ can be assumed to asymptotically converge (when $N_i \gg 1$) to Gaussian distributed with mean and variance given by

$$\mu_\xi = N_i \frac{\sigma_n}{\sqrt{2}} \cdot \mathrm{E}\left[v\right], \qquad (11)$$

$$\sigma_\xi^2 = N_i \frac{\sigma_n^2}{2} \cdot var\left[v\right], \qquad (12)$$

with v a normalized Rayleigh random variable. Since v is Rayleigh distributed, we have that for $\mathcal{H}_0$, $E\left[v\right] = \sqrt{\pi/2}$ and $var\left[v\right] = 2 - \pi/2$ so that,

$$\mu_{\xi|\mathcal{H}_0} = N_i \sigma_n \sqrt{\pi}/2, \qquad (13)$$

$$\sigma^2_{\xi|\mathcal{H}_0} = N_i \sigma_n^2 \left[1 - \pi/4\right]. \qquad (14)$$

Based on the above assumptions, the detection threshold can asymptotically be assumed to be the value γ' such that

$$P_{FA} \overset{N_i >> 1}{\longrightarrow} Q\left(\frac{\gamma' - \mu_{\xi|\mathcal{H}_0}}{\sigma_{\xi|\mathcal{H}_0}}\right). \qquad (15)$$

The goodness of this approach will be evaluated later on when analyzing the performance of the proposed GLRT detector.

C. Extension to FLOS Noncoherent Detection

Signal detection in HS-GNSS receivers must often undergo severe working conditions where both impulsive noise and/or interference sources may be present. In these circumstances the input signal may fail to be Gaussian distributed and thus the optimal detection criterion should be reformulated. When this is the case, it is important to have in mind that one of the main characteristics of non-Gaussian processes is the different decay rate of their distribution tails compared to the Gaussian ones. For instance, this is the case of algebraic-tailed distributions for which their right tail probability can be approximated by[1] $prob\left(X > x\right) \sim c \cdot x^{-\beta}$, for some constant c and $\beta > 0$. This is in contrast with the Gaussian distribution for which tails decay faster as $prob\left(|X| > x\right) \sim \sqrt{2/\pi} x^{-1} e^{-x^2/2}$.

The advantage of introducing algebraic-tailed distributions is the possibility to model the presence of outliers or abnormals in the input signal, as it occurs when operating in harsh working conditions. This is one of the reasons why heavy tailed distributions are of great interest in many fields such as astrophysics, economics or engineering [17]. For this family of heavy-tailed distributions, an important results states that finite absolute moments do exist for orders less than β. That is, $\mathrm{E}\left[|X|^k\right] < \infty$ as long as $k < \beta$. However the important part of this statement is the converse one, which states that absolute moments do not converge for orders $k \geq \beta$. Therefore, traditional second-order moments associated with Gaussian distributions no longer exist for algebraic-tailed distributions with $\beta < 2$, as it often occurs [18]. In that case, statistical analyses must be based on fractional low-order statistics (FLOS) defined as,

$$\mathrm{E}\left[|X|^\alpha\right] \doteq \int_\infty^\infty |x|^\alpha f_x(x) dx \qquad (16)$$

with $0 < \alpha < \beta$. As it can be seen, a clear link can be established between FLOS and the absolute value GLRT detector in (9). In this sense, an extended FLOS version of this detector is proposed to be adopted as follows,

$$\xi_\alpha = \sum_{i=0}^{N_i - 1} |y(i)|^\alpha \qquad (17)$$

with $\alpha \in (0, 1]$. The resulting detection metric would be expected to inherit some of the robustness properties of FLOS for impulsive processes and thus, provide some performance gain when operating in harsh scenarios. This intuition will be evaluated later on in Section V.

[1]The parameter β corresponds to the inverse impulsiveness factor in the sense that increased impulsiveness corresponds to small values of β.

IV. Detection Performance

In this section, the detection performance of the GLRT detector in (9) is analyzed and compared to that of the traditional second-order based detector. For the sake of clarity, and adopting the notation in (17), let us denote the GLRT detector metric by ξ_1 and the second-order based detector by ξ_2.

For the GLRT detection metric ξ_1, probability of false alarm P_{FA} was shown to be given by (15) based on the mean and variance of Rayleigh random variables. Unfortunately, obtaining the probability of detection P_D for this metric is significantly more difficult. To illustrate this statement note that ξ_1 becomes the sum of iid Rician distributed random variables for the signal present hypothesis $\mathcal{H}_1$. The problem there is that since no closed-form expression is available for the Rician probability density function, there is no choice but to resort to numerical evaluation for determining P_D from its cumulative density function.

A first step to circumvent these limitations is to proceed similarly to what already proposed for the detection threshold determination in Section III-B. That is, by invoking the central limit theorem. The reason again is that for $N_i \gg 1$ it seems reasonable to assume that ξ_1 asymptotically becomes Gaussian distributed with mean and variance given by (11) and (12). Consequently, the probability of detection P_D becomes,

$$P_D \overset{N_i >> 1}{\longrightarrow} Q\left(\frac{\gamma' - \mu_{\xi|\mathcal{H}_1}}{\sigma_{\xi|\mathcal{H}_1}}\right). \qquad (18)$$

The main difference with respect to $\mathcal{H}_0$ is that the random variable v in (11), (12) is now Rician distributed with probability density function,

$$f_v(v) = \frac{2v}{\sigma_n^2} e^{-\frac{1}{\sigma_n^2}\left(v^2 + A^2\right)} I_0\left(\frac{2Av}{\sigma_n^2}\right) \qquad (19)$$

being $I_0\left(x\right) = \frac{1}{2\pi}\int_0^{2\pi} e^{x\cos\theta} d\theta$ the modified Bessel function of the first kind and order 0. The required mean and variance in

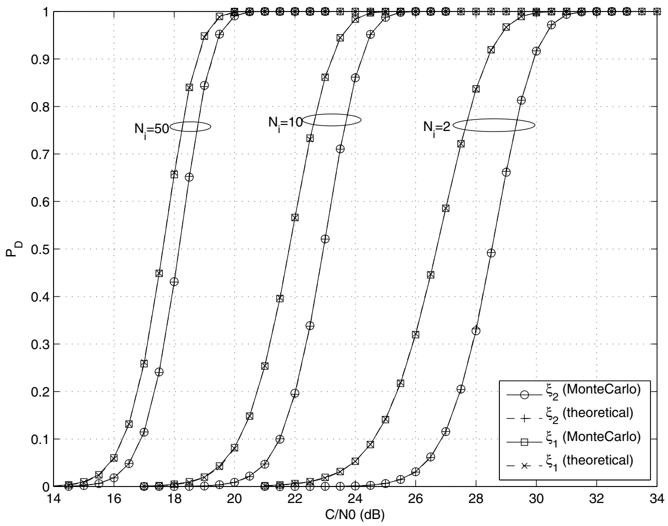

Fig. 2. Probability of detection vs C/N_0 with $N_i = \{2, 10, 50\}$ and $P_{FA} = 0.05$ for the proposed GRLT ξ_1 and the second-order based detector ξ_2.

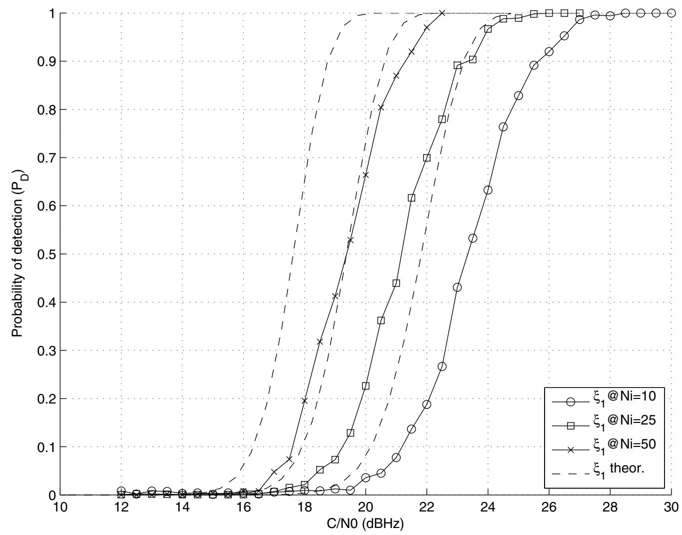

Fig. 3. Probability of detection vs C/N_0 for a HS-GNSS receiver based on the double-FFT acquisition method and $P_{FA} = 10^{-6}$.

(18) can be obtained from the first and second order moments of v, being the expression for the k-th moment given by

$$E\left[v^k\right] = \sigma_n^k \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{\pi}} e^{-\frac{A^2}{\sigma_n^2}} \cdot F_1\left(\frac{k+1}{2}; \frac{1}{2}; \frac{A^2}{\sigma_n^2}\right) \quad (20)$$

with $F_1(\alpha, \beta, \gamma)$ the confluent hypergeometric function [19].

Since no closed-form expression is available for these moments, there is no choice but to resort to numerical evaluation for determining P_D. By doing so, the goodness of the expression in (18) is shown in Fig. 2. This figure compares P_D obtained by numerically evaluating (18) and the one obtained from MonteCarlo simulations. From the results shown in this figure, two main conclusions should be drawn. First, the results show a very tight match between the approximation in (18) and the true P_D obtained via MonteCarlo. Second, the absolute value GLRT decision metric in (9) provides a significant gain with respect to the traditional second-order based noncoherent detector, especially for the low values of N_i. Therefore, these results confirm the superior performance of the proposed GLRT detector in front of the traditional second-order based approach.

V. HS-GNSS Simulation Results

In this section performance results are presented for both the GLRT detector in (9) and the extended FLOS detector in (17). The simulation set up is based on a GPS L1 high-sensitivity software receiver whose acquisition module is based on the double-FFT algorithm [14]. This acquisition technique allows an efficient implementation while providing the maximum coherent correlation length in the absence of bit knowledge. That is, $T_{coh} = 20$ ms. The goal of this module is to detect the presence of satellites based on some assistance information including coarse estimations of their Doppler errors.

The first experiment compares the theoretical probability of detection derived in Section IV with the one obtained in realistic working conditions. It should be noted that in practice, some degradation losses are always incurred due to the presence of uniform code phase and residual Doppler errors. This causes the receiver to observe the incoming signal with a lower C/N_0 than the one expected. For the case under consideration with $N_{sc} = 4$ samples per chip and residual frequency error within the ± 500 Hz range, this loss may range from 0.8 to 1.8 dB. The results of this first experiment are shown in Fig. 3 for different noncoherent integrations $N_i = \{10, 25, 50\}$. As it can seen, curves for the empirical probability of detection exhibit a C/N_0 shift which is in line with the expected losses. It should be noted, however, that this shift seems to increase with the probability of detection for all tested configurations.

The second experiment compares the empirical performance of the GLRT detector in (9) with traditional second-order based and FLOS detectors with $\alpha = \{0.5, 0.7, 0.8\}$. The simulation setup was configured with an input $C/N_0 = 23$ dBHz for a short integration interval of 100 ms (i.e. $N_i = 5$). Results are shown in Fig. 4 where the focus has been placed on analyzing the region of low probability of false alarm. From the observation of this figure it is interesting to note that two regions of operation can be distinguished. For the very low probability of false alarm, the GLRT detection in (9) is found to provide the best performance. However, for the region of $P_{FA} > 0.05$, FLOS based on $\alpha = 0.7$ and $\alpha = 0.8$ outperform the GLRT detector. This results confirms that using FLOS for signal detection may provide some gain, and it opens the door for a more detailed analysis on the applicability of FLOS for signal detection in harsh scenarios.

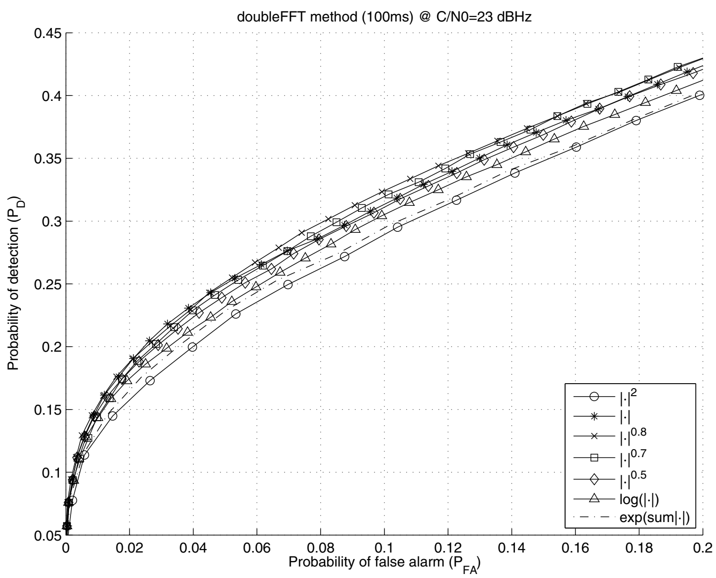

Fig. 4. ROC curves for different FLOS detection metrics for $N_i = 5$, $P_{FA} = 10^{-6}$ and $C/N_0 = 23$ dBHz.

VI. Conclusion

This paper has shown that contrary to the traditional belief, absolute moments, and not second-order moments, are the optimal statistics for detecting a constant with unknown time-varying phase immersed in noise. In addition to this, the resulting GLRT detector is linked with the theory of non-Gaussian fractional lower-order statistics for which an extended detector is proposed. Based on realistic simulation results, the proposed extended detector is found to provide some performance gain with respect to the Gaussian GLRT detector for harsh working conditions.

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