Low-Complexity Multi-User Non-Line-of-Sight Channel Estimation

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***Abstract*—Future radio access networks are expected to rely on large-aperture antenna arrays, for which an increasing portion of the coverage region may fall within the radiative near-field. In this regime, conventional far-field models become inaccurate, and the received spatial signature depends jointly on the propagation range and azimuth, making reliable uplink channel acquisition a key physical-layer challenge. Many existing near-field estimation works rely on simplified line-of-sight-dominant or single-path channel models, which fail to capture practical non-line-of-sight (NLoS) multipath environments. In contrast to prior single-path near-field partial relaxation (PR) formulations, in this paper, we develop a PR-based framework for near-field NLoS uplink channel estimation by modeling each user channel as a superposition of multiple spherical-wave components characterized by their ranges and azimuths. For the known-pilot case, we develop a greedy PR-based maximum likelihood estimator that iteratively extracts dominant propagation paths while mitigating multi-user interference. For the unknown-symbol case, we propose a PR-based rank-adaptive covariance-fitting approach that captures the multipath structure. We further derive the corresponding Cramér–Rao bounds for both cases. Numerical results show that the proposed multipath PR-based methods achieve strong estimation performance, remain close to the corresponding bounds, and outperform near-field two-dimensional multiple signal classification across the considered scenarios, supporting their relevance for future large-aperture uplink systems.**

***Index Terms*—Cramér–Rao bounds, multipath, near-field channel estimation, non-line-of-sight, partial relaxation.**

I. Introduction

Multiple input multiple output (MIMO) systems have become a key enabler of modern wireless networks by exploiting spatial degrees of freedom through multi-antenna transmission and reception. In the evolution toward 6G and IMT-2030, future radio access networks are expected to support more demanding communication, localization, and sensing capabilities, for which large antenna arrays are among the key enabling technologies [1]. In addition to improving spectral efficiency via beamforming and spatial multiplexing, large antenna arrays also enable emerging capabilities such as localization and sensing [2]–[4]. These functionalities, however, rely critically on accurate channel state information (CSI). As a result, designing channel estimation techniques that provide reliable CSI while maintaining reasonable pilot overhead and computational complexity remains a central challenge in contemporary and future wireless systems [2].

As antenna arrays continue to grow in size and aperture, the underlying propagation characteristics of wireless channels change significantly. In particular, users located close to the base station may fall within the radiative near-field (NF), where the wavefront curvature across the array becomes non-negligible [5]. In this regime, the received phase profile depends jointly on both the propagation distance and the angle of arrival, which makes conventional far-field planar-wave models inaccurate. Consequently, channel estimation techniques must explicitly account for this NF structure in order to achieve reliable performance, especially in multi-user uplink scenarios where interference and limited snapshot availability further complicate the estimation task.

Subspace-based methods are widely used for parameter estimation in array processing. In NF scenarios, the two-dimensional MUltiple SIgnal Classification (2D-MUSIC) algorithm has been extended to spherical-wave models to jointly estimate range and angle [6]. However, these approaches rely on the estimation of the sample covariance matrix and may suffer from subspace leakage when the number of snapshots is limited or the signal-to-noise ratio (SNR) is low. Their performance can further degrade in multi-user uplink scenarios due to strong interference and closely spaced users.

To better handle multi-user interference, the partial relaxation (PR) principle has been proposed for array parameter estimation. In this framework, the desired user is modeled parametrically, while the remaining users are absorbed into a relaxed nuisance component [7]. Prior studies have shown that PR-based estimators can outperform classical subspace methods [7]–[9]. Recently, this framework was extended to radiative NF channel estimation under a spherical-wave model [10]. However, that work considered a simplified single-path channel model, leaving practical NF NLoS multipath propagation insufficiently explored.

This limitation is significant because, in NLoS near-field

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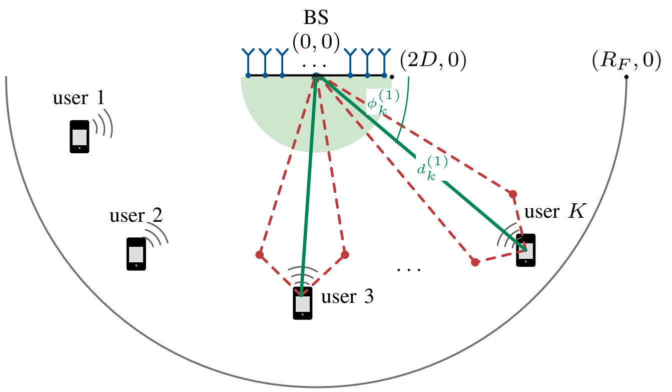

Fig. 1: NF multi-user uplink system. Users are located in the radiative NF region, i.e., between $2D$ and R_F, where $D = (N-1)\lambda/2$ and $R_F = 2D^2/\lambda$. For the p-th path of user k, $d_k^{(p)}$ denotes the path range from the array reference point, and $\phi_k^{(p)}$ is the corresponding azimuth angle measured with respect to the array axis.

channels, the desired-user channel may contain multiple spherical-wave components with different ranges, azimuths, and complex gains. This increases the parameter dimension and introduces path-selection ambiguities that are absent in the single-path formulation. Therefore, a clear gap remains in extending PR-based near-field estimation from the single-path setting to practical NLoS multipath channels.

Motivated by this gap, this paper extends the partial relaxation (PR) framework to NF NLoS multipath channel estimation. The desired-user channel is modeled as a superposition of NF paths, each parameterized by its range and azimuth under the spherical-wave model. Compared with the single-path PR formulation in [10], the proposed formulation explicitly accounts for multiple propagation paths and adaptively extracts the dominant path components of the desired user while relaxing the remaining multi-user interference. Based on this model, we develop partial-relaxation maximum likelihood (PR-ML) for known pilots and partial-relaxation covariance fitting (PR-CF) for unknown symbols, and derive the corresponding performance bounds, namely Partial-Relaxation Cramér–Rao Bound for Known Pilots ($\mathrm{PR\text{-}CRB_p}$) and Partial-Relaxation Cramér–Rao Bound for Unknown Symbols ($\mathrm{PR\text{-}CRB_u}$).

II. System Model

We consider a multi-user uplink scenario where a base station (BS) equipped with an N-element uniform linear array (ULA) receives L snapshots from K NF users. The channel of each user is modeled as a superposition of multiple propagation paths, and the spacing of the elements is $\Delta = \lambda/2$. The position of the antenna element n is $x_n = \delta_n \Delta$, where $\delta_n = n - (N+1)/2$, $n = 1, \ldots, N$.

The p-th path of user k is characterized by $\boldsymbol{\theta}_k^{(p)} = (d_k^{(p)}, \phi_k^{(p)})$, for $p = 1, \ldots, P_k$, where $d_k^{(p)}$, $\phi_k^{(p)}$, and P_k denote the range, azimuth, and number of propagation paths of user k, respectively. Under the spherical-wave model, the distance between antenna n and path p of user k is

$$r_{n,k}^{(p)} = \sqrt{\left(d_k^{(p)}\right)^2 + (\delta_n \Delta)^2 - 2 d_k^{(p)} \delta_n \Delta \cos\left(\phi_k^{(p)}\right)}. \quad (1)$$

The corresponding NF steering vector associated with the parameter pair $\boldsymbol{\theta}_k^{(p)}$ is defined as

$$\mathbf{h}\left(\boldsymbol{\theta}_k^{(p)}\right) = \left[e^{-j\frac{2\pi}{\lambda} r_{1,k}^{(p)}}, \ldots, e^{-j\frac{2\pi}{\lambda} r_{N,k}^{(p)}}\right]^\mathsf{T} \in \mathbb{C}^{N\times 1}. \quad (2)$$

The effective channel vector of user k is modeled as the superposition of P_k propagation paths [11], i.e.,

$$\mathbf{h}_k = \sum_{p=1}^{P_k} \alpha_k^{(p)} \mathbf{h}\left(\boldsymbol{\theta}_k^{(p)}\right), \qquad \mathbf{h}_k \in \mathbb{C}^{N\times 1}, \quad (3)$$

where $\alpha_k^{(p)} \in \mathbb{C}$ denotes the complex gain of the p-th propagation path. Any user-specific phase offset can be absorbed into the complex path gains $\alpha_k^{(p)}$ and is therefore omitted for notational simplicity. Stacking the channels of all users yields the multi-user channel matrix $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \in \mathbb{C}^{N\times K}$. Let $\mathbf{s}[\ell] \in \mathbb{C}^{K\times 1}$ denote the transmitted symbol vector at snapshot ℓ, and let $\mathbf{y}[\ell] \in \mathbb{C}^{N\times 1}$ denote the received signal vector at the BS. The received signal model is given by

$$\mathbf{y}[\ell] = \sqrt{P_s}\,\mathbf{H}\mathbf{s}[\ell] + \mathbf{w}[\ell], \qquad \ell = 1, \ldots, L, \quad (4)$$

where P_s denotes the transmit power, and $\mathbf{w}[\ell] \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$ represents additive white Gaussian noise with variance σ^2. Collecting all snapshots gives $\mathbf{Y} = [\mathbf{y}[1], \ldots, \mathbf{y}[L]] \in \mathbb{C}^{N\times L}$. The transmitted symbols are i.i.d. circularly symmetric complex Gaussian.

III. PR-Based Multipath Channel Estimation

This section presents the proposed PR-based formulations for NF multipath channel estimation in the known-pilot and unknown-symbol cases, building on the PR framework in [10].

a) General PR formulation: PR estimates the desired component parametrically while absorbing the remaining interfering components into an unstructured nuisance term [7]. In the proposed NF multipath uplink setting, we apply this principle to estimate the desired-user multipath channel while relaxing the remaining multi-user interference.

The desired user's channel is represented as a superposition of P_1 propagation paths, where P_1 is not assumed known a priori. Let $\boldsymbol{\Theta}_1 = \{\boldsymbol{\theta}_1^{(p)}\}_{p=1}^{P_1}$ denote its path-parameter set, with $\boldsymbol{\theta}_1^{(p)} = (d_1^{(p)}, \phi_1^{(p)})$, and let $\mathbf{a}_1 = [\alpha_1^{(1)}, \ldots, \alpha_1^{(P_1)}]^\mathsf{T} \in \mathbb{C}^{P_1\times 1}$ collect the corresponding path gains. The resulting multipath steering matrix is

$$\mathbf{H}(\boldsymbol{\Theta}_1) = \left[\mathbf{h}(\boldsymbol{\theta}_1^{(1)}) \;\cdots\; \mathbf{h}(\boldsymbol{\theta}_1^{(P_1)})\right] \in \mathbb{C}^{N\times P_1}. \quad (5)$$

Let $\mathbf{s}_1 \in \mathbb{C}^{L\times 1}$ denote the signal sequence of the desired user, and $\mathbf{S}_B = [\mathbf{s}_2 \;\cdots\; \mathbf{s}_K] \in \mathbb{C}^{L\times(K-1)}$ collect the signal sequences of the other users. The resulting PR data model is

$$\mathbf{Y} = \underbrace{\mathbf{H}(\boldsymbol{\Theta}_1)\mathbf{a}_1\mathbf{s}_1^\mathsf{T}}_{\text{Desired user}} + \underbrace{\mathbf{B}\mathbf{S}_B^\mathsf{T}}_{\text{Other users (relaxed)}} + \underbrace{\mathbf{W}}_{\text{Noise}}. \quad (6)$$

where $\mathbf{B} \in \mathbb{C}^{N\times(K-1)}$ is an unstructured nuisance matrix representing the aggregate contribution of the remaining users under the PR formulation, with the transmit-power scaling and per-user phase offsets absorbed into $\mathbf{a}_1$ for the desired user and into $\mathbf{B}$ for the remaining users.

$$J(\mathbf{\Theta}_1, \mathbf{a}_1, \mathbf{B}) = \left\| \mathbf{Y} - \mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1\mathbf{s}_1^{\mathsf{T}} - \mathbf{B}\mathbf{S}_B^{\mathsf{T}} \right\|_F^2 . \tag{7}$$

Depending on whether $\mathbf{s}_1$ and $\mathbf{S}_B$ are known or unknown, this model yields a multipath PR-ML formulation for the known-pilot case and a rank-adaptive multipath PR-CF formulation for the unknown-symbol case. The proposed PR-based estimators reduce complexity by sequentially estimating the desired user's dominant paths and absorbing the remaining users into a relaxed nuisance term, while achieving better performance than the 2D-MUSIC baseline at comparable complexity.

b) PR-ML with known pilots: For known pilots, we exploit the knowledge of $\mathbf{s}_1$ and $\mathbf{S}_B$ and develop a greedy procedure to estimate the desired user's path parameters. Minimizing the cost with respect to the matrix $\mathbf{B}$ yields

$$J(\mathbf{\Theta}_1, \mathbf{a}_1) = \left\| \mathbf{R}(\mathbf{\Theta}_1, \mathbf{a}_1)\mathbf{P}_{\mathbf{S}_B}^{\perp} \right\|_F^2 , \tag{8}$$

where $\mathbf{R}(\mathbf{\Theta}_1, \mathbf{a}_1) = \mathbf{Y} - \mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1\mathbf{s}_1^{\mathsf{T}}$ and, assuming that $\mathbf{S}_B$ has full column rank, $\mathbf{P}_{\mathbf{S}_B}^{\perp} = \mathbf{I}_L - \mathbf{S}_B(\mathbf{S}_B^{\mathsf{H}}\mathbf{S}_B)^{-1}\mathbf{S}_B^{\mathsf{H}}$. The projected desired-user pilot must satisfy $\mathbf{P}_{\mathbf{S}_B}^{\perp}\mathbf{s}_1 \neq \mathbf{0}$, which holds for orthogonal or sufficiently well-conditioned pilot sequences. Further concentrating with respect to $\mathbf{a}_1$, and defining $\tilde{\mathbf{Y}} = \mathbf{Y}\mathbf{P}_{\mathbf{S}_B}^{\perp}$ and $\tilde{\mathbf{s}}_1 = \mathbf{P}_{\mathbf{S}_B}^{\perp}\mathbf{s}_1$, we obtain the compressed observation $\bar{\mathbf{y}} = \tilde{\mathbf{Y}}\tilde{\mathbf{s}}_1^*/\|\tilde{\mathbf{s}}_1\|_2^2$. This is the matched-filtered spatial observation after nuisance-pilot projection, and it preserves the least-squares solution for the desired-user channel. The LS estimate of $\mathbf{a}_1$ then leads to the multipath PR-ML score

$$S_{\text{PR-ML}}(\mathbf{\Theta}_1) = \bar{\mathbf{y}}^{\mathsf{H}}\mathbf{\Pi}_{\mathbf{H}(\mathbf{\Theta}_1)}\bar{\mathbf{y}}, \tag{9}$$

where $\mathbf{\Pi}_{\mathbf{H}(\mathbf{\Theta}_1)} = \mathbf{H}(\mathbf{\Theta}_1)\big(\mathbf{H}^{\mathsf{H}}(\mathbf{\Theta}_1)\mathbf{H}(\mathbf{\Theta}_1)\big)^{-1}\mathbf{H}^{\mathsf{H}}(\mathbf{\Theta}_1)$. We then employ a greedy path extraction procedure that sequentially identifies dominant paths and updates their gains, as summarized in Algorithm 1. The estimated number of paths is given by the final greedy iteration index, and therefore it may differ from the true number of physical paths when weak paths are below the noise floor or when noise/interference components are over-extracted. The stopping threshold ε and the upper bound $P_{\max}$ are used to control this underestimation–overestimation tradeoff.

c) PR-CF with unknown symbols: For unknown symbols, we adopt a rank-adaptive multipath PR-CF formulation based on the sample covariance matrix $\mathbf{R}_b \triangleq \mathbf{Y}\mathbf{Y}^{\mathsf{H}} \in \mathbb{C}^{N\times N}$. Building on the single-path PR-CF model in [9] and the near-field single-path formulation in [10], we consider a discretized range–azimuth grid $\mathcal{G} \subset \mathbb{R}_+ \times \mathbb{R}$. Here, $\mathbf{h}(\boldsymbol{\theta})$ denotes the NF steering vector associated with a generic parameter pair $\boldsymbol{\theta} = (d, \phi)$, as defined in Section II. We further assume that the dominant paths of different users lie in disjoint user-specific regions $\{\mathcal{G}_k\}_{k=1}^K$, with $\mathcal{G}_k \cap \mathcal{G}_\ell = \emptyset$ for $k \neq \ell$, so that the greedy search targets the multipath structure of a single user. Following the PR-CF formulation, the score of a candidate multipath steering matrix is computed from the matrices below. As in covariance-fitting PR methods, we assume that $\mathbf{R}_b$ is positive definite. This typically holds when $L \geq N$ and the observations are sufficiently rich. If $\mathbf{R}_b$ is singular or ill-conditioned, diagonal loading can be applied [7], [12]. Define

$$\mathbf{T}\Big(\mathbf{H}^{(p)}\Big) = \mathbf{H}^{(p)}\left(\mathbf{H}^{(p)\mathsf{H}}\mathbf{R}_b^{-1}\mathbf{H}^{(p)}\right)^{-1}\mathbf{H}^{(p)\mathsf{H}}, \tag{10}$$

and

$$\mathbf{M}\Big(\mathbf{H}^{(p)}\Big) = \mathbf{R}_b - \mathbf{T}\Big(\mathbf{H}^{(p)}\Big). \tag{11}$$

Let $\lambda_i(\cdot)$ denote the i-th largest eigenvalue. The resulting rank-adaptive multipath PR-CF objective is

$$f_{\text{PR-CF}}\Big(\mathbf{H}^{(p)}\Big) = \frac{1}{\sum_{i=K}^{N}\lambda_i^2\big(\mathbf{M}\big(\mathbf{H}^{(p)}\big)\big)}. \tag{12}$$

The eigenvalue-based objective follows from the low-rank covariance-fitting interpretation of PR: for a given candidate desired-user subspace, the relaxed nuisance component is represented by the dominant low-rank part of the residual covariance, while the remaining fitting error is captured by the sum of the squared residual eigenvalues [7].

The dominant paths are identified through a greedy search over the user-specific region $\mathcal{G}_k$. Starting from $\mathbf{H}^{(0)} = \emptyset$, each candidate location $\boldsymbol{\theta} \in \mathcal{G}_k$ defines the temporary steering matrix

$$\mathbf{H}^{(p)}(\boldsymbol{\theta}) = \big[\mathbf{H}^{(p-1)}\ \mathbf{h}(\boldsymbol{\theta})\big], \tag{13}$$

and the next path is selected as

$$\hat{\boldsymbol{\theta}}^{(p)} = \arg\max_{\boldsymbol{\theta}\in\mathcal{G}_k} f_{\text{PR-CF}}\Big(\mathbf{H}^{(p)}(\boldsymbol{\theta})\Big). \tag{14}$$

After selecting p paths, the steering matrix is updated as

$$\mathbf{H}^{(p)} = \big[\mathbf{h}(\hat{\boldsymbol{\theta}}^{(1)})\ \cdots\ \mathbf{h}(\hat{\boldsymbol{\theta}}^{(p)})\big] \in \mathbb{C}^{N\times p}. \tag{15}$$

The first path is always extracted. From the second iteration onward, the procedure stops when the relative improvement in the objective falls below ε or when $p = P_{\max}$. The complete estimation procedure is summarized in Algorithm 2.

Algorithm 1 PR-ML Multipath Estimation

Input: Received data $\mathbf{Y}$, pilot $\mathbf{s}_1$, pilot matrix $\mathbf{S}_B$, search grid $\mathcal{G}$, maximum number of paths $P_{\max}$, threshold ϵ.
Output: Estimated path set $\hat{\mathbf{\Theta}}_1$, gains $\hat{\mathbf{a}}_1$, channel $\hat{\mathbf{h}}_1$.

Compute projection matrix $\mathbf{P}_{\mathbf{S}_B}^{\perp} = \mathbf{I}_L - \mathbf{S}_B(\mathbf{S}_B^{\mathsf{H}}\mathbf{S}_B)^{-1}\mathbf{S}_B^{\mathsf{H}}$
Project received data $\tilde{\mathbf{Y}} = \mathbf{Y}\mathbf{P}_{\mathbf{S}_B}^{\perp}$, $\tilde{\mathbf{s}}_1 = \mathbf{P}_{\mathbf{S}_B}^{\perp}\mathbf{s}_1$
Compress snapshots $\bar{\mathbf{y}} = \tilde{\mathbf{Y}}\tilde{\mathbf{s}}_1^*/\|\tilde{\mathbf{s}}_1\|_2^2$
Initialize $p = 0$, $\mathbf{r}^{(0)} = \bar{\mathbf{y}}$, $\mathbf{H}^{(0)} = [\ ]$
repeat
    $p = p + 1$
    Select path $\hat{\boldsymbol{\theta}}_1^{(p)} = \arg\max_{\boldsymbol{\theta}\in\mathcal{G}} \frac{|\mathbf{h}(\boldsymbol{\theta})^{\mathsf{H}}\mathbf{r}^{(p-1)}|^2}{\|\mathbf{h}(\boldsymbol{\theta})\|_2^2}$
    Update steering matrix $\mathbf{H}^{(p)} = [\mathbf{H}^{(p-1)}\ \mathbf{h}(\hat{\boldsymbol{\theta}}_1^{(p)})]$
    Estimate gains $\hat{\mathbf{a}}_1^{(p)} = (\mathbf{H}^{(p)\mathsf{H}}\mathbf{H}^{(p)})^{-1}\mathbf{H}^{(p)\mathsf{H}}\bar{\mathbf{y}}$
    Update residual $\mathbf{r}^{(p)} = \bar{\mathbf{y}} - \mathbf{H}^{(p)}\hat{\mathbf{a}}_1^{(p)}$
until $\|\mathbf{r}^{(p)}\|_2^2/\|\mathbf{r}^{(0)}\|_2^2 \leq \epsilon$ or $p = P_{\max}$
Set $\hat{P}_1 = p$, $\hat{\mathbf{\Theta}}_1 = \{\hat{\boldsymbol{\theta}}_1^{(p)}\}_{p=1}^{\hat{P}_1}$, $\hat{\mathbf{a}}_1 = \hat{\mathbf{a}}_1^{(\hat{P}_1)}$, $\hat{\mathbf{h}}_1 = \mathbf{H}^{(\hat{P}_1)}\hat{\mathbf{a}}_1$.

Algorithm 2 Greedy PR-CF Multipath Estimation

Input: Received data $\mathbf{Y}$, user-specific search region $\mathcal{G}_k \subseteq \mathcal{G}$, maximum number of paths $P_{\max}$, threshold ε.
Output: Estimated path set $\hat{\mathbf{\Theta}}$, channel $\hat{\mathbf{h}}$.
Compute sample covariance $\mathbf{R}_b = \mathbf{Y}\mathbf{Y}^{\mathsf{H}}$
Initialize $p = 0$, $\mathbf{H}^{(0)} = \emptyset$, $\mathbf{\Theta}^{(0)} = \emptyset$, $f^{(0)} = 0$
repeat
    $p = p + 1$
    For each candidate $\boldsymbol{\theta} \in \mathcal{G}_k$
        Form steering matrix $\mathbf{H}^{(p)}(\boldsymbol{\theta}) = [\mathbf{H}^{(p-1)}\ \mathbf{h}(\boldsymbol{\theta})]$
        Compute $\mathbf{T}\big(\mathbf{H}^{(p)}(\boldsymbol{\theta})\big)$
        Compute $\mathbf{M}\big(\mathbf{H}^{(p)}(\boldsymbol{\theta})\big) = \mathbf{R}_b - \mathbf{T}\big(\mathbf{H}^{(p)}(\boldsymbol{\theta})\big)$
        Evaluate score $f_{\text{PR-CF}}\big(\mathbf{H}^{(p)}(\boldsymbol{\theta})\big)$
    Select path $\hat{\boldsymbol{\theta}}^{(p)} = \arg\max_{\boldsymbol{\theta}\in\mathcal{G}_k} f_{\text{PR-CF}}\big(\mathbf{H}^{(p)}(\boldsymbol{\theta})\big)$
    Update steering matrix $\mathbf{H}^{(p)} = [\mathbf{H}^{(p-1)}\ \mathbf{h}(\hat{\boldsymbol{\theta}}^{(p)})]$
    Update path set $\mathbf{\Theta}^{(p)} = \mathbf{\Theta}^{(p-1)} \cup \{\hat{\boldsymbol{\theta}}^{(p)}\}$
    Update score $f^{(p)} = f_{\text{PR-CF}}(\mathbf{H}^{(p)})$
Compute $\Delta f^{(p)} = \frac{f^{(p)} - f^{(p-1)}}{f^{(p-1)}}$.
until $(p \geq 2$ AND $\Delta f^{(p)} < \epsilon)$ or $p = P_{\max}$
Set $p_{\text{out}} = p - 1$ if $(p \geq 2$ AND $\Delta f^{(p)} < \epsilon)$; otherwise, set $p_{\text{out}} = p$.
Set $\hat{\mathbf{\Theta}} = \mathbf{\Theta}^{(p_{\text{out}})}$.
Form the corresponding steering matrix and reconstruct $\hat{\mathbf{h}}$.

d) 2D-MUSIC Baseline: As a baseline, we consider NF 2D-MUSIC [6], where each propagation path is treated as an independent source and the channel is reconstructed from the detected peaks. The pseudospectrum is

$$S_{\text{MUSIC}}(\boldsymbol{\theta}) = \frac{1}{\mathbf{h}(\boldsymbol{\theta})^{\mathsf{H}} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^{\mathsf{H}} \mathbf{h}(\boldsymbol{\theta})}, \tag{16}$$

where $\hat{\mathbf{U}}_n$ is the noise subspace of $\mathbf{R}_b$. Evaluating $S_{\text{MUSIC}}(\boldsymbol{\theta})$ over $\mathcal{G}$, dominant peaks are extracted sequentially using $S_{\text{MUSIC}}(\hat{\boldsymbol{\theta}}) \geq \varepsilon S_{\max}$, where $S_{\max} = \max_{\boldsymbol{\theta}\in\mathcal{G}} S_{\text{MUSIC}}(\boldsymbol{\theta})$, followed by non-maximum suppression (NMS). Let $\hat{Q} = |\hat{\mathbf{\Theta}}|$ with $\hat{\mathbf{\Theta}} = \{\hat{\boldsymbol{\theta}}_q\}_{q=1}^{\hat{Q}}$, and form $\hat{\mathbf{H}} = [\mathbf{h}(\hat{\boldsymbol{\theta}}_1)\ \cdots\ \mathbf{h}(\hat{\boldsymbol{\theta}}_{\hat{Q}})]$. The path gains are then estimated as $\hat{\mathbf{a}} = (\hat{\mathbf{H}}^{\mathsf{H}}\hat{\mathbf{H}})^{-1}\hat{\mathbf{H}}^{\mathsf{H}}\bar{\mathbf{y}}$.

IV. Cramér–Rao Bounds for Partial Relaxation

This section derives multipath Cramér–Rao bounds (CRBs) under the PR framework for NF channel estimation. In particular, we develop PR-CRB$_{\text{p}}$ for the known-pilot case and PR-CRB$_{\text{u}}$ for the unknown-symbol case.

a) PR-CRB$_{\text{p}}$ for known pilots: For the known-pilot case, we derive the multipath PR-CRB$_{\text{p}}$ under the PR framework. The received data follow

$$\mathbf{Y} = \mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1\mathbf{s}_1^{\mathsf{T}} + \mathbf{B}\mathbf{S}_B^{\mathsf{T}} + \mathbf{W}, \tag{17}$$

Let $\mathbf{y} = \operatorname{vec}(\mathbf{Y}) \in \mathbb{C}^{NL}$ and $\mathbf{b} = \operatorname{vec}(\mathbf{B}) \in \mathbb{C}^{N(K-1)}$. Then

$$\mathbf{y} = \underbrace{(\mathbf{s}_1 \otimes \mathbf{I}_N)\mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1 + (\mathbf{S}_B \otimes \mathbf{I}_N)\mathbf{b}}_{\triangleq \mathbf{m}(\boldsymbol{\eta})} + \mathbf{w}, \tag{18}$$

where $\mathbf{m} \triangleq \mathbf{m}(\boldsymbol{\eta}) \in \mathbb{C}^{NL}$ is the mean of $\mathbf{y}$, and $\boldsymbol{\eta} = [\boldsymbol{\vartheta}^{\mathsf{T}}, \boldsymbol{\xi}^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{R}^{4P_1+2N(K-1)}$, with $\boldsymbol{\vartheta} \in \mathbb{R}^{4P_1}$ and $\boldsymbol{\xi} \in \mathbb{R}^{2N(K-1)}$. Following the real-valued CRB parametrization for complex-valued array models in [13], we collect the real physical parameters together with the real and imaginary parts of the complex coefficients. Hence,

$$\boldsymbol{\vartheta} = [\mathbf{d}^{\mathsf{T}}, \boldsymbol{\phi}^{\mathsf{T}}, \Re\{\mathbf{a}_1\}^{\mathsf{T}}, \Im\{\mathbf{a}_1\}^{\mathsf{T}}]^{\mathsf{T}}, \quad \boldsymbol{\xi} = [\Re\{\mathbf{b}\}^{\mathsf{T}}, \Im\{\mathbf{b}\}^{\mathsf{T}}]^{\mathsf{T}}.$$

where $\mathbf{d}, \boldsymbol{\phi} \in \mathbb{R}^{P_1}$. Using the general Gaussian FIM in [14, Sec. 3.9, Eq. (3.31)], and noting that the covariance $\sigma^2\mathbf{I}$ is parameter independent, the FIM for the real-valued parameter vector $\boldsymbol{\eta}$ becomes

$$\mathbf{F}(\boldsymbol{\eta}) = \frac{2}{\sigma^2}\Re\left\{\left(\frac{\partial \mathbf{m}}{\partial \boldsymbol{\eta}}\right)^{\mathsf{H}}\left(\frac{\partial \mathbf{m}}{\partial \boldsymbol{\eta}}\right)\right\}, \quad \mathbf{F}(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{J}_{\vartheta\vartheta} & \mathbf{J}_{\vartheta\xi} \\ \mathbf{J}_{\xi\vartheta} & \mathbf{J}_{\xi\xi} \end{bmatrix}.$$

Eliminating the nuisance parameter vector $\boldsymbol{\xi}$ via the Schur complement yields $\mathbf{F}_{\text{eff}} = \mathbf{J}_{\vartheta\vartheta} - \mathbf{J}_{\vartheta\xi}\mathbf{J}_{\xi\xi}^{-1}\mathbf{J}_{\xi\vartheta}$, so that $\text{PR-CRB}_p(\boldsymbol{\vartheta}) = \mathbf{F}_{\text{eff}}^{-1}$. Let $\mathbf{h}_p = \mathbf{h}(\boldsymbol{\theta}_1^{(p)})$ and $\mathbf{S} = \mathbf{s}_1 \otimes \mathbf{I}_N$. The required Jacobian terms are

$$\frac{\partial \mathbf{m}}{\partial \Re\{\alpha_1^{(p)}\}} = \mathbf{S}\mathbf{h}_p, \qquad \frac{\partial \mathbf{m}}{\partial d_1^{(p)}} = \mathbf{S}\,\alpha_1^{(p)}\frac{\partial \mathbf{h}_p}{\partial d_1^{(p)}},$$
$$\frac{\partial \mathbf{m}}{\partial \Im\{\alpha_1^{(p)}\}} = j\,\mathbf{S}\mathbf{h}_p, \qquad \frac{\partial \mathbf{m}}{\partial \phi_1^{(p)}} = \mathbf{S}\,\alpha_1^{(p)}\frac{\partial \mathbf{h}_p}{\partial \phi_1^{(p)}}.$$

The derivatives of $\mathbf{h}_p$ follow from [6]. Let $\mathbf{J}_h = \partial(\mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1)/\partial\boldsymbol{\vartheta}^{\mathsf{T}}$. The channel-domain bound is

$$\mathbf{CRB}_{\mathbf{h}} = \mathbf{J}_h\,\mathbf{PR\text{-}CRB}_p(\boldsymbol{\vartheta})\,\mathbf{J}_h^{\mathsf{T}}. \tag{19}$$

b) PR-CRB$_{\text{u}}$ for unknown symbols: Based on the constrained PR-CRB$_{\text{u}}$ formulation in [10], [15], we derive the multipath PR-CRB$_{\text{u}}$ for the unknown-symbol case within the PR framework, as a benchmark for the multipath PR-CF estimator, where the desired-user channel is modeled by $\mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1$. In this setting, we represent the relaxed interference block $\mathbf{B}\mathbf{S}_B^{\mathsf{T}}$ through its singular-value decomposition (SVD) as $\mathbf{U}_r\mathbf{\Gamma}\mathbf{V}_r^{\mathsf{H}}$. The resulting received signal model is

$$\mathbf{Y} = \mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1\mathbf{s}_1^{\mathsf{T}} + \mathbf{U}_r\mathbf{\Gamma}\mathbf{V}_r^{\mathsf{H}} + \mathbf{W}. \tag{20}$$

Here, r denotes the rank of the relaxed interference model, $\mathbf{U}_r = [\mathbf{u}_1, \ldots, \mathbf{u}_r] \in \mathbb{C}^{N\times r}$, $\mathbf{V}_r = [\mathbf{v}_1, \ldots, \mathbf{v}_r] \in \mathbb{C}^{L\times r}$, and $\mathbf{\Gamma} = \operatorname{diag}(\boldsymbol{\gamma}) \in \mathbb{R}_+^{r\times r}$ is a diagonal matrix containing the singular values. In addition to the non-uniqueness of the relaxed-block factorization, the desired-user term $\mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1\mathbf{s}_1^{\mathsf{T}}$ is identifiable only up to a common complex scale/phase factor between $\mathbf{a}_1$ and $\mathbf{s}_1$. Without loss of generality, a set of constraints is defined to make the factorizations unique, and hence to obtain a finite bound. Specifically, the columns of $\mathbf{U}_r$ and $\mathbf{V}_r$ are constrained to having a unit norm, and the first element of each vector $\mathbf{u}_k$ is set to be real because any phase rotation can be included in the corresponding vector $\mathbf{v}_k$. As a result, the set of constraints is $\|\mathbf{u}_k\|_2 = \|\mathbf{v}_k\|_2 = 1$ and $\Im\{[\mathbf{u}_k]_1\} = 0$, $\forall k$, and $\Re\{[\mathbf{s}_1]_1\} = 1$, $\Im\{[\mathbf{s}_1]_1\} = 0$.

Using the same vectorized model and Fisher-information matrix as in the known-pilot case, the mean vector of $\mathbf{y}$ is

$$\boldsymbol{\mu}(\boldsymbol{\zeta}) = (\mathbf{s}_1 \otimes \mathbf{I}_N)\mathbf{H}(\mathbf{\Theta}_1)\mathbf{a}_1 + \operatorname{vec}(\mathbf{U}_r\mathbf{\Gamma}\mathbf{V}_r^{\mathsf{H}}), \tag{21}$$

where the real-valued parameter vector is

$$\boldsymbol{\zeta} = \big[\mathbf{d}^{\mathsf{T}}, \boldsymbol{\phi}^{\mathsf{T}}, \Re\{\mathbf{a}_1\}^{\mathsf{T}}, \Im\{\mathbf{a}_1\}^{\mathsf{T}}, \Re\{\mathbf{s}_1\}^{\mathsf{T}}, \Im\{\mathbf{s}_1\}^{\mathsf{T}}, \boldsymbol{\gamma}^{\mathsf{T}}, \\ \Re\{\mathrm{vec}(\mathbf{U}_r)\}^{\mathsf{T}}, \Im\{\mathrm{vec}(\mathbf{U}_r)\}^{\mathsf{T}}, \\ \Re\{\mathrm{vec}(\mathbf{V}_r)\}^{\mathsf{T}}, \Im\{\mathrm{vec}(\mathbf{V}_r)\}^{\mathsf{T}}\big]^{\mathsf{T}} \in \mathbb{R}^{n_\zeta}.$$

with $n_\zeta = 4P_1 + 2L + r + 2Nr + 2Lr$. The derivatives in $\mathbf{F}(\boldsymbol{\zeta}) \in \mathbb{R}^{n_\zeta \times n_\zeta}$ are taken with respect to the entries of $\boldsymbol{\zeta}$. The set of constraints can be represented as a function $\mathbf{g}(\boldsymbol{\zeta}) = \mathbf{0}$ with $\mathbf{g} \in \mathbb{R}^{3r+2}$. We define its Jacobian as $\mathbf{G} = \partial \mathbf{g}(\boldsymbol{\zeta})/\partial \boldsymbol{\zeta}^{\mathsf{T}} \in \mathbb{R}^{(3r+2)\times n_\zeta}$. If $\mathbf{N}_G \in \mathbb{R}^{n_\zeta \times (n_\zeta - 3r-2)}$ is an orthonormal basis for the nullspace of $\mathbf{G}$, then the constrained bound is

$$\mathbf{PR\text{-}CRB}_u^{(r)}(\boldsymbol{\zeta}) = \mathbf{N}_G \left(\mathbf{N}_G^{\mathsf{T}} \mathbf{F}(\boldsymbol{\zeta}) \mathbf{N}_G\right)^{-1} \mathbf{N}_G^{\mathsf{T}}. \tag{22}$$

The corresponding channel-domain bound is obtained as in the known-pilot case.

V. Numerical Results

This section assesses the proposed PR-based NF multipath channel estimation framework under two representative propagation regimes. We first consider a pure NLoS setting, where no dominant LoS component is present, and evaluate the impact of SNR and the number of users on the normalized mean squared error (NMSE) performance. We then consider a mixed LoS/NLoS setting, where the effects of SNR and LoS dominance are investigated through the Rician factor.

Channel and power model: We consider the NF multipath uplink model introduced in Section II, where all K users transmit simultaneously to the BS. The gain of the p-th path of user k is modeled as $\alpha_k^{(p)} = \sqrt{P_k^{(p)}} e^{j\psi_k^{(p)}}$, where $P_k^{(p)}$ and $\psi_k^{(p)}$ denote the corresponding path power and phase, respectively. For a fair comparison across users and channel realizations, the path powers of each user are normalized such that $\sum_{p=1}^{P_k} P_k^{(p)} = P_{\mathrm{tot}}$, with $P_{\mathrm{tot}} = 1$. The nominal SNR is defined as P_s/σ^2, where P_s is the transmit power and σ^2 is the noise variance. In the multi-user setting, increasing K also increases the aggregate contribution of the interfering users, which is treated by the proposed PR estimators as a relaxed nuisance component. Therefore, the user-scaling results reflect both the effect of multi-user interference and the ability of the proposed estimators to separate the desired-user multipath channel from the remaining users. In the simulations, channels are generated with $P_{\max}$ paths per user, whereas the number of estimated paths is determined adaptively by each algorithm. For pure NLoS channels, the path powers are randomly generated and then normalized. For Rician LoS/NLoS channels, the power allocation is controlled by $K_R = P_{\mathrm{LoS}}/P_{\mathrm{NLoS}}$, with $P_{\mathrm{LoS}} + P_{\mathrm{NLoS}} = P_{\mathrm{tot}}$.

Simulation setup: Azimuth angles are drawn uniformly from $[0°, 120°]$, while ranges are drawn from the radiative NF region, i.e., $[3.70, 68.45]$ m. The path-search-based estimators use an adaptive stopping threshold ε and a maximum number of extracted paths $P_{\max} = 10$, with $\varepsilon = 10^{-2}$ unless otherwise stated. Performance is evaluated in terms of the NMSE between the true and estimated channels. Comparisons also include the classical parametric CRB in [6]. The remaining simulation parameters are summarized in Table I.

TABLE I: Simulation Parameters

Parameter (Symbol)	Fig. 2	Fig. 3	Fig. 4	Fig. 5
Number of antennas (N)	38			
Carrier frequency (f_c)	3 GHz			
Number of users (K)	2	1–4	2	2
Number of snapshots (L)	80	80	40	40
Rician factor (K_R) (dB)	0	0	10	0–15
SNR (dB)	$[-10, 25]$	18	$[-10, 25]$	18

Results: Figs 2 and 3 first examine the pure NLoS case ($K_R = 0$), where the channel is composed only of scattered multipath components without a dominant LoS path. Fig 2 shows that the proposed PR-based estimators achieve a consistent NMSE reduction as the SNR increases and remain close to the corresponding CRBs. In contrast, 2D-MUSIC exhibits a noticeably larger performance gap, especially at moderate and high SNR. Fig.3 further evaluates the robustness of the estimators under increasing multi-user interference. Although the NMSE of all methods degrades as the number of users increases, the proposed PR estimators maintain the lowest NMSE across the considered values of K. This confirms that the proposed estimators can effectively relax the aggregate nuisance component while preserving accurate estimation of the desired-user multipath channel.

Figs 4 and 5 then consider the mixed LoS/NLoS case, where the channel contains both a dominant LoS component and scattered NLoS paths. Fig.4 shows that the presence of a dominant LoS component ($K_R = 10$) improves the estimation accuracy and brings the proposed methods closer to the corresponding bounds over the considered SNR range. Fig.5 further confirms this behavior by showing that increasing the Rician factor leads to a clear NMSE improvement, since the channel becomes more dominated by the structured LoS component. Overall, the results show that the proposed PR-based estimators consistently outperform 2D-MUSIC across all considered scenarios. As expected, PR-ML achieves the best performance because it exploits the known pilot sequence. More importantly, PR-CF also provides a clear performance advantage over 2D-MUSIC under the same unknown-symbol setting, demonstrating that the proposed covariance-fitting formulation can achieve accurate NF multipath channel estimation even without prior knowledge of the transmitted symbols.

VI. Conclusions

In this paper, we studied NF NLoS multipath channel estimation for multi-user uplink systems with large-aperture arrays. Within a PR framework, the desired-user channel was represented by a structured multipath model, while the remaining users were absorbed into a relaxed nuisance component. For known pilots, we developed a PR-ML estimator and derived the corresponding PR-CRB$_{\mathrm{p}}$. For unknown symbols, we proposed a rank-adaptive PR-CF formulation and derived the associated PR-CRB$_{\mathrm{u}}$. Numerical results showed that the resulting estimators remain close to their bounds and consistently outperform near-field 2D-MUSIC in both pure NLoS and mixed LoS/NLoS scenarios. In particular, PR-CF achieved

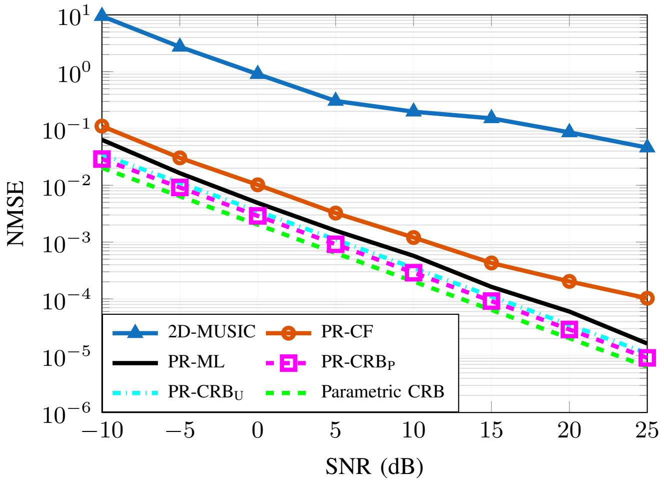

Fig. 2: Pure NLoS ($K_R = 0$): effect of SNR for $K = 2$ and $L = 80$.

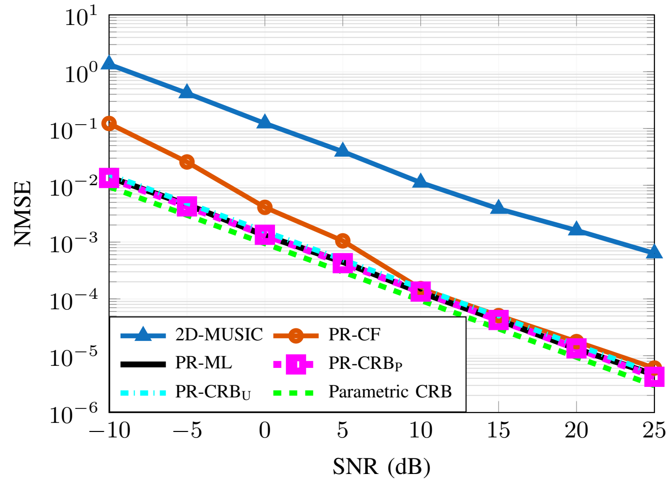

Fig. 4: LoS/NLoS: effect of SNR for $K_R = 10$ and $L = 40$.

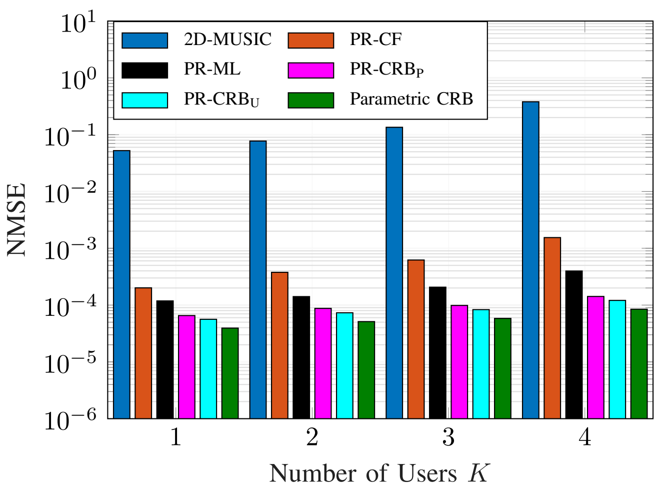

Fig. 3: Pure NLoS ($K_R = 0$): effect of K at SNR $= 18$ dB, $L = 80$.

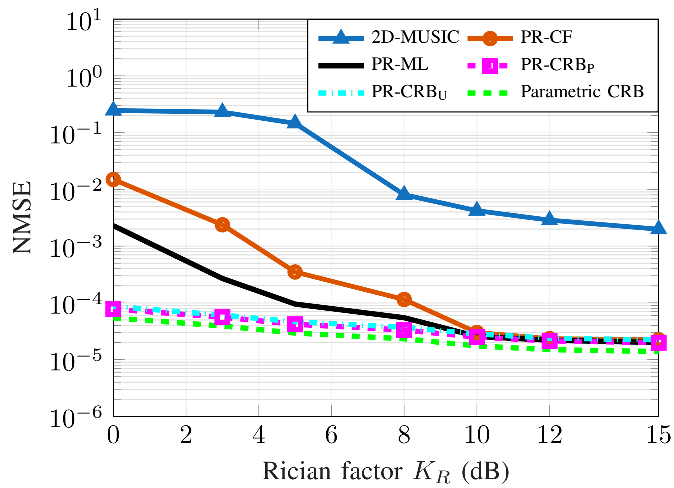

Fig. 5: LoS/NLoS: effect of K_R at SNR $= 18$ dB and $L = 40$.

a clear advantage in the unknown-symbol setting, highlighting its suitability for practical NF channel estimation when pilot or symbol knowledge is unavailable. These results indicate that the proposed framework can serve as an effective physical-layer solution for future large-aperture uplink systems.

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