

# Combining SAGE and LMS for Blind Multipath Mitigation in GNSS Receivers

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**Abstract**—Global Navigation Satellite Systems (GNSSs) face a difficult challenge when combating multipath reflections, since, due to their high correlation with the signal of interest, they pose a significant threat for GNSS receivers. The use of array processing techniques greatly helps receivers to combat these effects, typically exploiting available information for an effective mitigation. However, relying on external and possibly out-dated references might cause the receiver to perform poorly, and thus an approach that does not require any previously obtained parameters can significantly enhance the behaviour of the receiver. This paper presents a blind alternative to the reference-dependent existing techniques for multipath mitigation in GNSS receivers. Combining two well-known algorithms, SAGE and LMS, this approach allows to obtain large attenuation capacities while ensuring resilience and robustness against rapid changes in the operating environment.

**Index Terms**—GNSS, multipath, beamforming, blind, SAGE, LMS

## I. INTRODUCTION

Nowadays, the use of navigation and positioning technologies is widely extended across the globe. The number of applications exploiting these services is constantly growing, and the emergence of these new use cases calls for a refinement in the existing solutions, as well as for innovative approaches, able to meet the enhanced accuracy requirements while maintaining a low complexity.

In recent years, different trends have been evaluated with the purpose of meeting these new demands. The use of Low Earth Orbit (LEO) satellites, either through dedicated constellations or with opportunistic communication signals, as well as exploiting terrestrial cellular networks, such as 5G and its future generations, are two great examples of the new navigation tendencies [1]–[7]. However, in spite of these advances, Global Navigation Satellite Systems (GNSSs) still remain at the core of the navigation technologies, being the reference and standard solution. The advantages and reach

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of these systems is such, that efforts must still be put into combating its deficiencies and improving its capacities in terms of precision and robustness.

A major source of concern in this domain is the poor performance of the receivers in the presence of correlated interference. The use of code-division multiple access (CDMA) to share the medium makes GNSSs signals vulnerable to highly correlated contributions, since the despreading process implies computing the correlation between the received signal and the corresponding CDMA code. The presence of correlated sources distorts the expected output of the correlation, leading to large positioning errors or even to the inability to compute a navigation solution at all.

Although this situation may seem unusual, it is very common in urban areas, where the reflections of the line-of-sight signal (LOSS) coming directly from the satellite in nearby objects results in highly correlated replicas that may arrive at the receiver along with the desired LOSS. This phenomenon is commonly known as multipath, and it has been the focus of study in the GNSSs field for a long time [8]–[10].

A promising approach to combat multipath effects in GNSSs has been found in the use of antenna arrays, since the deployment of several receiving elements allows the analysis of the spatial domain [11], [12]. However, this introduces a new set of challenges and limitations. Most spatial diversity techniques effective against multipath require the receiver to have some reference, such as the direction of arrival (DoA) of the LOSS and its received power, which might be difficult to obtain accurately in certain scenarios [13], [14].

In view of this, the solution is to find a *blind* multipath mitigation approach where the receiver does not need any outside information. In this paper, an approach that meets these requirements is presented, relying on two well-known algorithms; the Space Alternating Generalised Expectation-Maximisation (SAGE) and the Least Mean Squares (LMS). The SAGE algorithm enables the receiver to accurately estimate the DoA and the reference model of the LOSS, including its received power. This information is then used as input for the LMS algorithm, which computes the optimal coefficients for the mitigation technique.

## II. PROBLEM STATEMENT

### A. Signal Model

Let us consider an antenna array formed by  $L$  elements receiving a LOSS along with  $M$  multipath contributions. After the despreading process, the resulting signal perceived by the  $L$  antennas can be expressed through an  $L \times 1$  vector,  $\mathbf{x}[n]$ , with the following complex baseband signal representation,

$$\mathbf{x}[n] = \alpha_0 \mathbf{a}(\theta_0) s[n; \tau_0] + \sum_{m=1}^M \alpha_m \mathbf{a}(\theta_m) s[n; \tau_m] + \mathbf{n}[n] \quad (1)$$

where  $s[n; \tau_m]$  is the output of the GNSS code correlation with a time delay  $\tau_m$ ,  $\alpha_m$  is the amplitude of the  $m$ -th contribution, and  $\mathbf{a}(\theta_m) \in \mathbb{C}^{L \times 1}$  defines the spatial signature introduced by the array for a signal with DoA  $\theta_m$ . Note that the subscript 0 in Eq. 1 is used to refer to the LOSS, whereas  $m$  indicates the different multipath reflections;  $\forall m \in \{1, 2, \dots, M\}$ . Lastly, the term  $\mathbf{n}[n] \in \mathbb{C}^{L \times 1}$  models the noise experienced at each antenna channel.

### B. The Beamforming Principle

Beamforming is defined as the linear combination of the samples at the different antenna elements following a certain criteria. This way, the incoming samples,  $\mathbf{x}[n]$ , are combined through a set of coefficients,  $\mathbf{w} \in \mathbb{C}^{L \times 1}$ , to result in a single output sample,  $y[n]$ , which is the input to the following stages of the GNSS receiver. This process is mathematically described in Eq. 2.

$$y[n] = \mathbf{w}^H \mathbf{x}[n] \quad (2)$$

The appeal of beamforming therefore lies in how the coefficients or *weights*,  $\mathbf{w}$ , are calculated, which is determined by the spatial diversity technique employed. It is in this computation that the receiver may require spatial information, i.e., the DoA of the LOSS, or temporal information, i.e., a reference of the LOSS, and the constraints under which the coefficients are derived define the behaviour of the algorithm.

### C. Limitations of the Existing Mitigation Techniques

Combating multipath reflections in the GNSS domain is a challenging task even when antenna arrays are deployed. Traditional beamforming techniques, such as the Minimum Variance Distortionless Response or Capon (CAP) beamformer [15], fail to eliminate the undesired contributions in the signal due to the high degree of correlation between the LOSS and the replicas. The CAP beamformer aims to minimise the total power at the output of the array while maintaining a distortionless response at the DoA of the LOSS, i.e.  $\mathbf{w}^H \mathbf{a}(\theta_0) = 1$ . After some mathematical manipulation, this results in the beamforming solution presented in Eq. 3, where  $\mathbf{R}_{\mathbf{x}} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}[n] \mathbf{x}^H[n]$  is the averaged spatial correlation matrix of the incoming samples,  $\mathbf{x}[n]$ .

$$\mathbf{w}_{\text{CAP}} = \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{a}(\theta_0) / (\mathbf{a}^H(\theta_0) \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{a}(\theta_0)) \quad (3)$$

The fact that the LOSS and the replica are correlated leads to a non-zero cross-correlation coefficient that the CAP beamformer mixes together with the remaining contributions to minimise the power, ultimately resulting in the cancellation of the desired signal [13].

In order to overcome the previous limitation, recent approaches propose to use additional information to estimate and eliminate these cross-correlation terms from the spatial correlation matrix, leading to a slightly modified version of the matrix with only uncorrelated terms. This is the case of the Power-based Capon (PBC) [13] and the Hybrid (HYB) beamformer [14].

As briefly introduced before, these multipath-effective techniques need both spatial and temporal information about the LOSS, normally relying on external aid to provide said parameters. The expression to find the optimal coefficients for the PBC is mathematically expressed in Eq. 4, where  $\hat{\Gamma} = (\mathbf{r}_{\mathbf{x}s_r} - \sigma_s^2 \mathbf{a}(\theta_0)) \mathbf{a}^H(\theta_0) - \mathbf{a}(\theta_0) (\mathbf{r}_{\mathbf{x}s_r} - \sigma_s^2 \mathbf{a}(\theta_0))^H$  is the estimated term that removes the contributions of the cross-correlation between the LOSS and multipath.

$$\mathbf{w}_{\text{PBC}} = \frac{(\mathbf{R}_{\mathbf{x}} - \hat{\Gamma})^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0) (\mathbf{R}_{\mathbf{x}} - \hat{\Gamma})^{-1} \mathbf{a}(\theta_0)} \quad (4)$$

Note from the previous equation (Eq. 4) that a temporal reference of the LOSS is needed,  $s_r$ , so that the receiver can calculate  $\mathbf{r}_{\mathbf{x}s_r}$ . Furthermore, the power of the LOSS,  $\sigma_s^2$ , must also be available at the receiver for the technique to work properly.

Similarly, the HYB beamformer, presented in Eq. 5, also removes the undesired terms in  $\mathbf{R}_{\mathbf{x}}$  that hinder the performance of the CAP algorithm through a previously estimated matrix,  $\hat{\mathbf{W}}$ , which takes the form  $\hat{\mathbf{W}} = \mathbf{r}_{\mathbf{x}s_r} \mathbf{r}_{\mathbf{x}s_r}^H / \sigma_{s_r}^2$ .

$$\mathbf{w}_{\text{HYB}} = \frac{(\mathbf{R}_{\mathbf{x}} - \hat{\mathbf{W}})^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0) (\mathbf{R}_{\mathbf{x}} - \hat{\mathbf{W}})^{-1} \mathbf{a}(\theta_0)} \quad (5)$$

The description of both techniques clearly shows their dependence on the accurate knowledge of the DoA and the temporal reference of the LOSS, meaning that their efficacy will be reduced when said parameters are inexact.

## III. BLIND APPROACH TO COUNTERACT MULTIPATH: S-LMS

In scenarios that are constantly changing, information such as the DoA of the LOSS can easily become outdated if the system relies on external assistance. A solution to this is to find an approach capable of mitigating reflections without requiring any prior knowledge, i.e., a blind technique, which is the focus of this section. The proposed method is based on the combination of the SAGE algorithm [16], able to perform DoA and amplitude estimation, with the well-known LMS [17], which will compute an up-to-date set of weights with the information provided by the SAGE.

### A. SAGE

The SAGE algorithm finds the DoA and amplitudes of the different signals through their maximum-likelihood (ML) estimates, following an iterative two-step process where each iteration computes the expectation and subsequently estimates the values that maximise it [18]. To do so, SAGE decomposes the received signal into its different contributions, assuming the general model shown in Eq. 6, so that each of the components DoA,  $\theta_i$ , and amplitude,  $\alpha_i$ , can be estimated independently [19].

$$\mathbf{g}_i[n] = \mathbf{a}(\theta_i)s[n; \tau_i] + \mathbf{n}_i[n] \quad (6)$$

The first step of the algorithm is to compute the expected value of each contribution,  $\hat{\mathbf{g}}_i[n]$ , once the observation data  $\mathbf{x}[n]$  is available. This conditional expectation results in the expression in Eq. 7, where  $\mathbf{A}(\boldsymbol{\theta})$  and  $\boldsymbol{\alpha}$  are an  $L \times (M + 1)$  matrix and an  $(M + 1) \times 1$  vector containing the spatial signature and the amplitude of the different contributions, built with the estimates from the previous iteration, and  $\boldsymbol{\eta}_i = [\theta_i \ \alpha_i]$  is a  $1 \times 2$  vector referring to the DoA and amplitude values of the  $i$ -th component that must be estimated.

$$\hat{\mathbf{g}}_i[n; \boldsymbol{\eta}_i] = \mathbf{a}(\theta_i)s[n; \tau_i] + \mathbf{x}[n] - \mathbf{A}(\boldsymbol{\theta})\boldsymbol{\alpha} \quad (7)$$

With the expression for the expectation computed, the DoA that maximises it is the one presented in Eq. 8, which locates the direction of maximum power of the  $i$ -th contribution in the  $j$ -th iteration of the algorithm [20].

$$\hat{\theta}_i^{(j)} = \max_{\theta} \mathbf{a}^H(\theta)\mathbf{R}_{\hat{\mathbf{g}}_i}(\boldsymbol{\eta}_i^j)\mathbf{a}(\theta) \quad (8)$$

Once  $\hat{\theta}_i^{(j)}$  is available, the receiver can use it to find the value of the amplitude that maximises Eq. 7,  $\hat{\alpha}_i^{(j)}$ , as described in Eq. 9.

$$\hat{\alpha}_i^{(j)} = \frac{1}{L} \mathbf{a}^H(\hat{\theta}_i^{(j)})\hat{\mathbf{g}}_i[n; \boldsymbol{\eta}_i^{(j)}] \quad (9)$$

Since SAGE is an iterative algorithm, the previous computations are repeated until convergence, using  $\hat{\theta}_i^{(j)}$  and  $\hat{\alpha}_i^{(j)}$  to build the model in Eq. 7 for iteration  $j+1$ . When the algorithm has converged, i.e.,  $\hat{\theta}_i \approx \theta_i$  and  $\hat{\alpha}_i \approx \alpha_i$ , these parameters can be used by the LMS to find an optimal solution for the beamforming technique.

### B. S-LMS

The LMS algorithm finds the coefficients of the beamformer minimising the the mean square error between its output,  $y[n]$ , and a reference signal that must be available at the receiver. This problem formulation is presented in Eq. 10.

$$\mathbf{w}_{\text{LMS}} = \min_{\mathbf{w}} \mathbb{E} \left\{ |\mathbf{w}^H \mathbf{x}[n] - s_r[n]|^2 \right\} \quad (10)$$

When addressing GNSS signals, the structure of the LOSS is known by the receiver, but not its amplitude or power, so an accurate reference cannot be built. The use of the LMS together with SAGE, i.e., S-LMS, overcomes this limitation,

since the amplitude estimate found by the latter can be employed by the LMS to build a new local replica of the LOSS as  $\hat{s}_r[n] = \hat{\alpha}_0 s_r[n]$ . Therefore, the solution for the coefficients of the S-LMS beamformer is shown in Eq. 11, obtained after substituting  $s_r[n]$  in Eq. 10 for  $\hat{s}_r[n] = \hat{\alpha}_0 s_r[n]$ .

$$\mathbf{w}_{\text{S-LMS}} = \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{r}_{\mathbf{x}\hat{s}_r}[n] \quad (11)$$

## IV. SIMULATION RESULTS

To complete the previous discussion, this section offers a comparative of the simulation results obtained with the proposed S-LMS approach with those of the techniques previously discussed.

To carry out the analysis, a static receiver equipped with an 8-element uniform linear array (ULA) was used. The received signal consisted of a LOS component with a DoA of  $\theta_0 = 0^\circ$ , along with a single multipath reflection with a DoA of  $\theta_m = 15^\circ$ , assuming a 3 dB difference in power between the reflection and the LOS component.

The performance of the techniques has been measured through two metrics, the spatial cancellation and the remaining code delay error at the output of the technique. The spatial cancellation, defined as the ratio between the array response at DoA  $\theta_m$  and  $\theta_0$ , allows to acquire the mitigation capacity of each of the algorithms. However, in a GNSS receiver the ultimate objective is to obtain a reduced code delay error, since it has direct impact on the final positioning solution, so the inaccuracies in delay estimation induced by the remaining multipath were also studied.

Lastly, for the sake of acquiring a broad view of the effect, a sweep over several multipath delays  $\tau_m$  was conducted, starting from the case where the reflection is completely aligned with the LOSS,  $\tau_m = 0$ , to a time delay of two chips,  $\tau_m = 2T_C$ .

The first case under analysis assumes a perfect knowledge of the DoA of the LOSS signal for those techniques that rely on this information, i.e., HYB, PBC and CAP, and compares their performance with that achieved with the S-LMS approach. In Fig. 1 these results are depicted in terms of spatial cancellation.

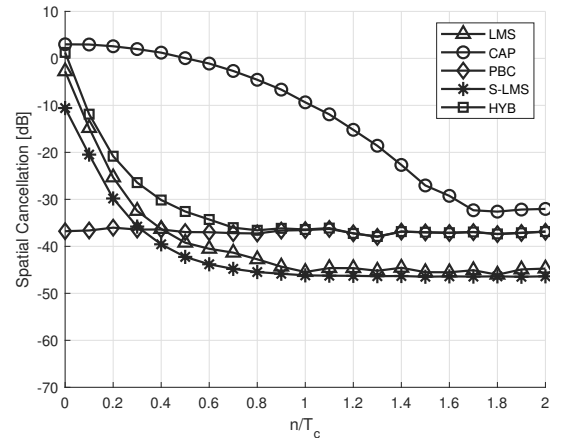


Fig. 1. Spatial cancellation.

The poor performance of the CAP beamformer is clearly manifested in Fig. 1, since the cross-correlation terms between LOSS and replica cause a positive spatial cancellation, increasing the multipath contribution while cancelling out the desired component. Accordingly, this behaviour has a negative effect in the code delay error, depicted in Fig. 2, where errors over 10m are obtained for closely spaced reflections,  $< 1T_C$ .

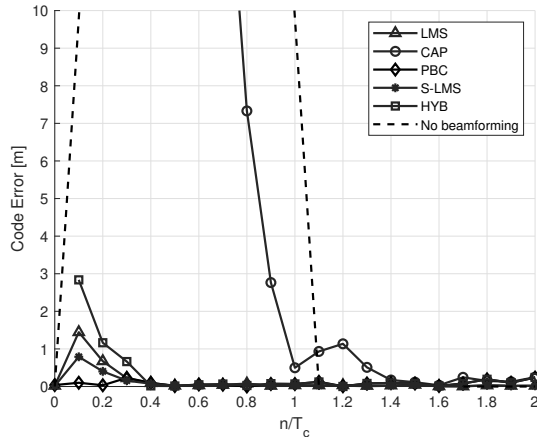


Fig. 2. Code delay error.

The spatial cancellation of the proposed technique, the S-LMS, exhibits very promising results, only outperformed by the PBC for extremely small delays,  $< 0.3T_C$ . As the time spacing of the replica increases, so does the efficacy of the S-LMS, yet the results observed for smaller delays are quite positive, introducing a 20 dB attenuation for a  $0.1T_C$  delay. This behaviour is seconded by that displayed in Fig. 2, where one can see that the code error after applying the S-LMS lies below 1m for all multipath delays, again only being surpassed by the PBC.

Up to this point, the S-LMS is only outperformed by the PBC, a DoA-dependent technique. This proves the promising capabilities of the proposed approach, since no prior information must be used to achieve large mitigation capacities. However, it is fair to compare the behaviour of the techniques when a mismatch between the real DoA and the reference at the receiver exists, since this could easily be the situation in a dynamic scenario.

In Fig. 3, the spatial cancellation is shown when a  $2^\circ$  difference between the available DoA and the real one arises. The performance of the PBC beamformer, as well as that of all the DoA-dependent techniques, is greatly affected by the inaccuracy in the reference, while the S-LMS is completely unaffected by this error. This advantage is provided by the SAGE up-to-date amplitude estimate, which allows the beamformer to be solidly resilient against reference mismatches.

Once again, the previous performance is backed by the code delay error of this simulation. The results for this metric are presented in Fig. 4, where the PBC shows a reduced accuracy with respect to the accurate reference case displayed in Fig. 2.

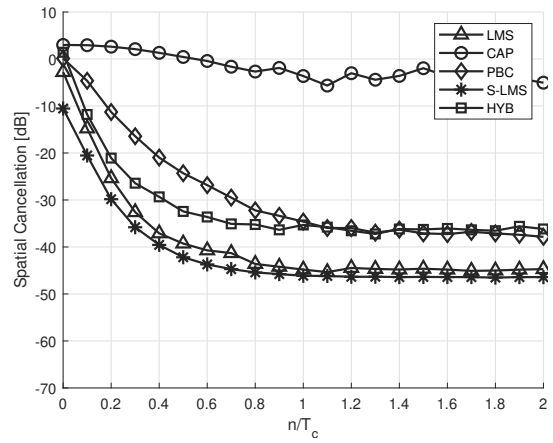


Fig. 3. Spatial cancellation with a  $2^\circ$  mismatch in LOSS DoA.

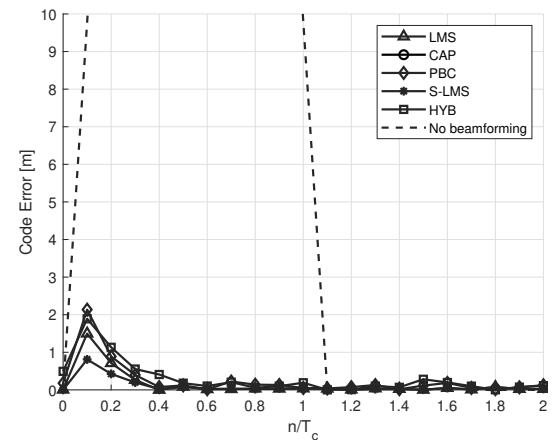


Fig. 4. Code delay error with a  $2^\circ$  mismatch in LOSS DoA.

It is the S-LMS beamformer that remains unaffected by the DoA error, offering the same accuracy it exhibited in the case of perfect DoA knowledge. Furthermore, the performance of the DoA-dependent algorithms will further decline as the mismatch increases, as it will also be more noticeable as the number of antennas at the receiver grows. As proved by the presented results, these problems can be solved using the S-LMS, where a favourable multipath mitigation capacity is achieved without relying in external or out-dated information.

## V. CONCLUSIONS

A blind approach for multipath mitigation in GNSS receivers has been proposed, exploiting the benefits of two well-known algorithms, SAGE and LMS. Supported by the results presented in this work, the proposed joint algorithm shows a promising performance, able to achieve great mitigation capacities that lead to very small errors in the code delay estimation. This approach clearly outperforms the HYB and PBC techniques in the presence of DoA mismatches, proving its suitability for GNSS receivers operating in dynamic environments, such as urban scenarios, where sudden changes in the DoA might cause the malfunction of DoA-dependent techniques.

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