Performance Evaluation in AltBOC Receivers Affected by Interference

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Abstract—The paper analyzes the joint performance of the PLL and DLL blocks that constitute the conventional receiver for Galileo AltBOC(15,10) signals, in the presence of narrow band (sinusoidal) and wideband (periodic chirp) interference. Full-band and sub-band processing are considered. The simulation results are compared with the expected behavior determined by the computation of the spectral separation coefficient (SSC).

Index Terms—AltBOC signal processing, narrowband and wideband interference, full-band and half-band receivers, spectral separation coefficient.

I. INTRODUCTION

The constant envelope AltBOC(15,10), which is the modulation transmitted in E5 band, centered at frequency 1191.795 MHz, is the most complex signal used in the Galileo system. It consists of four signals in two adjacent sub-bands (E5a and E5b) allowing to carry four separate channels: two data channels (E5a-Q and E5b-Q) and two pilot channels (E5a-I and E5b) [1]. A digital sub-carrier of frequency \( f_s = 15.345 \) MHz, which separates the two sub-bands by approximately 31 MHz, and a high chip rate of 10.23 Mchips/s produce a very large RF bandwidth. The receiver front-end typically uses a bandwidth of at least 51.15 MHz in order to pass the two most important lobes of the signal spectrum [1], [2].

There are mainly two different approaches to demodulate the AltBOC signal. Each sub-band can be processed independently and the resulting RF signal is treated as a complex BPSK(10) modulation with a data and a pilot channel, or the contents of the two sub-bands can be jointly processed by exploiting the full band. In this case, better performance in terms of robustness to multipath, thermal noise and narrow-band interference is expected [3].

Current Global Navigation Satellite Systems (GNSS) use spread-spectrum modulation schemes but, due to the weak power of the received signals at the Earth surface, they can be easily disturbed or even destroyed in the presence of certain types of RFI (radio frequency interference) which are intentionally provoked (jamming) or are due to the proximity of authorized radio sources, such as TV/FM, radar, navigation and mobile systems [4]. In general, non-intentional RFI is provoked by narrowband continuous or pulsed signals whereas jamming is caused by wideband transmitters using frequency-modulated chirp signals. Other papers have been published on narrowband interference detection and mitigation in AltBOC signals [5], [6], [7]. However, the evaluation of the software-defined AltBOC receiver robustness to wideband interference herein presented seems to be novel.

This paper is devoted to the assessment of the AltBOC receiver performance in tracking mode with half-band and full-band processing in the presence of narrowband (continuous-wave sinusoidal) and wideband chirp interference. The receiver under test is based on the architecture presented in [8].

II. RECEIVED SIGNAL

A. AltBOC signal

The RF transmitted signal is defined as

\[
s(t) = \text{Re}\left\{S_{E5}(t)\right\}\cos(\omega_0 t) - \text{Im}\left\{S_{E5}(t)\right\}\sin(\omega_0 t)
\]

where \( S_{E5}(t) \) is the baseband AltBOC signal [1] and \( \omega_0 \) is the nominal carrier frequency. Assuming that the satellite-receiver velocity is \( v \), the Doppler frequency of the received signal is \( \omega_d = -\omega_0 v/c \) and the received signal, using a non-relativistic approximation, is given by

\[
r(t) = \rho \left[ \text{Re}\left\{S_{E5}\left(t\left(1 - \frac{v}{c}\right)\right)\right\}\cos(\omega_0 t + \omega_d t + \phi) + \text{Im}\left\{S_{E5}\left(t\left(1 - \frac{v}{c}\right)\right)\right\}\sin(\omega_0 t + \omega_d t + \phi) + w(t) \right]
\]

where \( \rho \) is an amplitude, \( w(t) \) is additive white Gaussian noise (AWGN) with power spectral density \( G_w(f) = N_0/2 \) and \( c \) is the speed of light. Since \( |v|/c \ll 1 \) and the front-end bandwidth is typically around 50 MHz, we may simplify the expression to
The instantaneous frequency navigation data. The power of the received signal (excluding
\[ = \text{mod} \left( \frac{W}{2} \right) \rho \right) + n(t) \]
with \( A = 2p/(\pi \sqrt{2}) \), \( \omega_s = 2\pi f_s \), \( f_s = 15.345 \) MHz is the frequency of the AltBOC subcarrier, and \( n(t) \) is bandpass noise. In addition we have [1]
\[
\alpha(t) = e^{E_{5a-1}(t)} + e^{E_{5b-1}(t)} \]
\[
\beta(t) = e^{E_{5a-2}(t)} - e^{E_{5b-2}(t)} \]
\[
\gamma(t) = e^{E_{5a-3}(t)} + e^{E_{5b-3}(t)} \]
\[
\mu(t) = e^{E_{5a-4}(t)} - e^{E_{5b-4}(t)} \]
with \( e^{E_{5a-1}(t)} = C_{E_{5a}-1}(t)D_{E_{5a}-1}(t) \), \( e^{E_{5a-2}(t)} = C_{E_{5a}-2}(t)D_{E_{5a}-2}(t) \), \( e^{E_{5a-3}(t)} = C_{E_{5a}-3}(t)D_{E_{5a}-3}(t) \), \( e^{E_{5a-4}(t)} = C_{E_{5a}-4}(t)D_{E_{5a}-4}(t) \), where \( C_X(t) \) are spreading codes and \( D_X(t) \) are navigation data. The power of the received signal (excluding the noise) is equal to \( 2A^2 \). Thus, the carrier-to-noise ratio is
\[
\frac{C}{N_0} = \frac{2A^2}{N_0}. \tag{8}
\]

**B. Interference characterization**

We assume that the received signal \( r(t) \) is disturbed by the additive RFI waveform
\[
s(t) = A_J \cos(\varphi_J(t)) \tag{9}
\]
with the interference-to-noise power being \( (C_J/N_0) = A^2_J/(2N_0) \). Two types of continuous interferers are considered: a sinusoidal waveform of frequency \( f_0 + f_J \), with \( f_J \) accounting for the jammer’s frequency offset regarding the GNSS carrier frequency, and a chirp waveform, with period of repetition \( T_J \), described by
\[
\varphi_J(t) = 2\pi (f_0 - W/2)\tau(t) + \pi \gamma_J r^2(t) + \varphi_0. \tag{10}
\]
\( W \) stands for the frequency sweep range, \( \gamma_J = W/T_J \) and \( \tau(t) = \text{mod}(t, T_J) \) is the remainder after division of \( t \) by \( T_J \). The instantaneous frequency \( (2\pi)^{-1}(d\varphi_J(t)/dt) \), centered at the carrier frequency \( f_0 \), is sketched in Fig. 1.

\[
\text{Fig. 1. Frequency sweep of the chirp jamming signal}
\]

**III. RECEIVER ARCHITECTURE**

The signal \( r(t) \) is heterodyned to baseband to produce the inphase and quadrature components
\[
\begin{bmatrix}
  x_i(t) \\
  x_q(t)
\end{bmatrix}
= \mathcal{A} \mathcal{R}(\omega_d t + \phi) S(t) +
\begin{bmatrix}
  y_i(t) \\
  y_q(t)
\end{bmatrix} +
\begin{bmatrix}
  n_i(t) \\
  n_q(t)
\end{bmatrix} \tag{11}
\]

where
\[
\mathcal{R}(\theta) = \begin{bmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{bmatrix} \tag{12}
\]
is the rotation matrix by the angle \( \theta \),
\[
S(t) = \begin{bmatrix}
  \alpha(t) \cos(\omega_s t) + \beta(t) \sin(\omega_s t) \\
  \gamma(t) \cos(\omega_s t) + \mu(t) \sin(\omega_s t)
\end{bmatrix} \tag{13}
\]
and the vectors \( [y_i(t) \ y_q(t)]^T \) and \( [n_i(t) \ n_q(t)]^T \) consist, respectively, of the inphase and quadrature components of the interference and the Gaussian noise \( n(t) \).

**A. PLL**

The structure of the PLL is presented in Fig. 2.

\[
\text{Fig. 2. Structure of the PLL for AltBOC signals}
\]

The vector \( [x_i(t) \ x_q(t)]^T \) is rotated by the phase \( \tilde{\omega}_d t + \tilde{\phi} \), where \( \tilde{\omega}_d \) and \( \tilde{\phi} \) are, respectively, the estimates of \( \omega_d \) and \( \phi \), to yield \( [I(t) \ Q(t)]^T = \mathcal{R}[-(\tilde{\omega}_d t + \tilde{\phi})][x_i(t) \ x_q(t)]^T \) or
\[
\begin{bmatrix}
  I(t) \\
  Q(t)
\end{bmatrix} = \mathcal{A} \mathcal{R}[-(\tilde{\omega}_d t + \tilde{\phi})] S(t) +
\begin{bmatrix}
  \tilde{y}_i(t) \\
  \tilde{y}_q(t)
\end{bmatrix} +
\begin{bmatrix}
  \tilde{n}_i(t) \\
  \tilde{n}_q(t)
\end{bmatrix} \tag{15}
\]

where \( \omega_e = \tilde{\omega}_d - \omega_d \) and \( \phi_e = \tilde{\phi} - \phi \) are, respectively, the frequency and phase errors. In (15), \( [\tilde{y}_i(t) \ \tilde{y}_q(t)]^T \) and \( [\tilde{n}_i(t) \ \tilde{n}_q(t)]^T \) result from rotating the interference and noise vectors. The operation leading to (15) is the Doppler frequency wipe-off.

We perform next the complex correlation between signals
\[
X(t) = I(t) + jQ(t) \tag{16}
\]
\[
\tilde{X}(t) = \tilde{I}(t) + j\tilde{Q}(t) \tag{17}
\]
where $\tilde{X}(t)$ is the locally generated unfiltered replica of $X(t)$. That is, $\tilde{X}(t)$ is constituted by the parts of vector $[\text{Re}\{S_{E5}(t)\}, \text{Im}\{S_{E5}(t)\}]^T$ that do not carry navigation data, according to

\begin{align}
\tilde{I}(t) &= \beta(t)s_{E5-S}(t - T_s/4) \\
\tilde{Q}(t) &= \gamma(t)s_{E5-S}(t)
\end{align}

(18) (19)

with $T_s = 1/f_s$. The digital sub-carrier $s_{E5-S}(t)$, with period $T_J$, may be expanded as the Fourier series

$$s_{E5-S}(t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right) \times \left[1 + \sqrt{2} \cos\left(\frac{\pi n}{4}\right)\right] \cos(2\pi f_s t)$$

(20)

with $\sin(x) = \sin(\pi x)/(\pi x)$. The prompt cross-correlation of $X(t)$ and $\tilde{X}(t)$ is defined as

$$R_{X\tilde{X}}^{(P)}(\epsilon) = \frac{1}{T} \int_{0}^{T} X(t) \tilde{X}^*(t - \epsilon) dt$$

(21)

where $\epsilon$ is the locally-generated code delay error and $T$ is the correlation interval.

We consider a 4-quadrant arctangent phase discriminator. Assuming that $\omega_c \approx 0$, the discriminator output is given by

$$\tilde{\phi}_c = -\arg\{R_{X\tilde{X}}^{(P)}(\epsilon)\} = -\arctan_2(V(\epsilon), U(\epsilon))$$

(22)

with $R_{X\tilde{X}}^{(P)}(\epsilon) = U(\epsilon) + jV(\epsilon)$ and

$$U(\epsilon) = \frac{1}{T} \int_{0}^{T} [I(t)\tilde{I}(t - \epsilon) + Q(t)\tilde{Q}(t - \epsilon)] dt$$

(23)

$$V(\epsilon) = \frac{1}{T} \int_{0}^{T} [Q(t)\tilde{I}(t - \epsilon) - I(t)\tilde{Q}(t - \epsilon)] dt.$$  

(24)

For the second-order PLL we consider the loop filter described in Fig. 3. The noise equivalent bandwidth (in Hz) of the closed loop can be shown to be

$$B_{n,PLL} = \frac{1}{2\pi T} \int_{0}^{\pi} |H(e^{j\omega})|^2 d\omega.$$  

(25)

The transfer function of the closed loop is

$$H(z) = \frac{c_1 z^{-1} + c_2 z^{-2}}{(c_1 - 2) z^{-1} + (c_2 + 1) z^{-2}}$$

(26)

with $c_1 = (aT/2 + b)T$ and $c_2 = (aT/2 - b)T$. Using the bilinear transformation between the z-domain (time discrete) and the s-domain (time continuous) [10] we obtain the following expressions for the natural frequency $\omega_n$, and the damping factor $\xi$

$$\omega_n = \frac{2}{T} \sqrt{c_1 + c_2}$$(27)

$$\xi = \frac{c_2}{\sqrt{(c_1 + c_2)(c_2 - c_1 + 4)}}$$

(28)

The noise equivalent bandwidth (in Hz) of the closed loop, whose exact value can be numerically evaluated using (25)-(26), is well approximated by

$$B_{n,PLL} \approx \frac{\omega_n}{2} \left(\xi + \frac{1}{4\xi}\right).$$

(29)

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Structure of the PLL filter}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{Structure of the DLL for AltBOC signals}
\end{figure}

B. DLL

The structure of the DLL is sketched in Fig. 4 with $\Delta$ denoting the early-late spacing.

The early and late cross-correlations of $X(t)$ and $\tilde{X}(t)$ are given by

$$\begin{bmatrix}
R_{X\tilde{X}}^{(E)}(\epsilon) \\
R_{X\tilde{X}}^{(L)}(\epsilon)
\end{bmatrix} = \frac{1}{T} \int_{0}^{T} X(t) \tilde{X}^*(t + \Delta/2 - \epsilon) dt.$$  

(30)

We consider a normalized non-coherent early-late power (NELP) discriminator defined as
\[ D_{\text{NELP}}(\epsilon) = \frac{|R_{XX}(E)(\epsilon)|^2 - |R_{XX}(L)(\epsilon)|^2}{|R_{XX}(E)(\epsilon)|^2 + |R_{XX}(L)(\epsilon)|^2} \] 

(31)

with

\[ |R_{XX}(E)(\epsilon)|^2 = U^2(\epsilon - \Delta/2) + V^2(\epsilon - \Delta/2) \] 

(32)

\[ |R_{XX}(L)(\epsilon)|^2 = U^2(\epsilon + \Delta/2) + V^2(\epsilon + \Delta/2). \] 

(33)

The response of the normalized NELP discriminator is displayed in Fig. 5 for different values of the early-late spacing \( \Delta \). The value \( \Delta = 0.2T_c \) will be used in the simulations.

A rate-assisted first-order DLL is used with the estimated code delay being computed as \( \hat{\tau}_{k+1} = \hat{\tau}_k + \gamma_c D_{\text{NELP}}(\epsilon) - (\hat{\omega}_d/\omega_0)T \), where \( \hat{\omega}_d \) is the Doppler frequency estimate assumed by the PLL. The DLL noise bandwidth is \( B_{n,\text{DLL}} \approx \gamma_c g_c/4 \), where \( g_c \) is the normalized NELP discriminator gain and \( \gamma_c \) is a factor that permits to adjust the noise bandwidth. Notice that a small bandwidth is viable because the code loop is aided by the PLL (rate-aided DLL) [11].

IV. SIMULATION RESULTS

Figures 6 and 7 illustrate the mechanism of loss of phase and code tracking due to the introduction of a sinusoidal signal of interference with frequency \( f_I = 1 \) MHz at time \( t = 0.4 \) s. The carrier-to-noise density ratio due to thermal noise is \( (C/N_0) = 50 \) dBHz and the interference-to-noise density ratio is \( (C_I/N_0) = 115 \) dBHz. The correlation interval is \( T = 2 \) milliseconds and the adopted sampling rate is 58.52 Msamples/s which corresponds to 5.72 samples per chip. The PLL filter parameters are \( a = 1348 \) and \( b = 52 \), corresponding to a loop bandwidth \( B_{n,\text{PLL}} \approx 20 \) Hz and damping factor \( \xi \approx 0.71 \). The early-late spacing of the normalized NELP discriminator is made equal to \( 0.2T_c \) yielding a gain \( g_c \approx 3 \times 10^8/\)s. The loop gain \( \gamma_c \) is \( 2 \times 10^{-9} \) producing a DLL noise bandwidth of 0.15 Hz. In all the simulations we considered a Doppler frequency of 5 kHz.

The effective signal carrier-to-noise density ratio [12]

\[ \left( \frac{C}{N_0} \right)_{\text{eff}} = \left( \frac{C}{N_0} \right) \left[ 1 + \left( \frac{C_I}{N_0} \right) k_{\text{ssc}} \right] \] 

(34)

is a useful way to quantify the effect of the interference on the quality of the received signal. In (34),

\[ \eta = \int_{-B}^{B} G_s(f) \, df < 1 \] 

(35)

is the fraction of signal power passed by the precorrelation bandwidth. Parameter \( k_{\text{ssc}} \) is the spectral separation coefficient (SSC) define by

\[ k_{\text{ssc}} = \int_{-B}^{B} G(f)G_s(f) \, df \] 

(36)

where \( 2B \) is the RF bandwidth, \( G_I(f) \) is the normalized interference power spectrum and \( G_s(f) \) is the AltBOC normalized power spectrum, given by [13]
The power spectrum $G_s(f)$ is displayed in Fig. 8.

Equations (34) and (36) show that the reduction of $\frac{(C/N_0)_{eff}}{C/N_0}$ for a given value of $(C/I/N_0)$ depends on the SSC. For narrowband (sinusoidal) interference with frequency offset $f_J$ the interference power spectrum is $G_I(f) = (1/2)[\delta(f + f_J) + \delta(f - f_J)]$ and the SSC is given by $k_{ssc} = G_s(f_J)$. Therefore, the receiver’s resistance to RFI depends on the frequency $f_J$ with maximum robustness being expected when $f_J$ is close to a null of $G_s(f)$ (the first positive minima of $G_s(f)$ occur at 5.115 MHz and 25.57 MHz). This behavior is corroborated by Monte Carlo simulations carried out with full band and upper sub-band signal processing, as shown in Fig. 9. The figure presents the smallest values of $(C/I/N_0)$ (averaged over 25 independent runs) that lead to the loss of lock of the DLL/PLL blocks. We adopted the PLL discriminator output $\tilde{D}$ over 25 independent runs) that lead to the loss of lock of the DLL/PLL blocks. We adopted the PLL discriminator output exceeding 1 radian as the criterion for the loss of lock. Near the spectrum nulls we note an increase of the receiver robustness to interference. On the other hand, the minimum of robustness is obtained at the frequency corresponding to the maximum of the power spectrum which is approximately 15 MHz.

The upper sub-band signal processing still uses the structures of Figs. 2 and 4 but now the reference signals $\tilde{I}(t)$ and $\tilde{Q}(t)$, defined in (18) and (19), are simplified to

\[
\tilde{I}(t) = -e^{E_{\text{SB}}-Q}(t)se^{E_{\text{SB}}-S}(t - T_s/4) \quad (38)
\]

\[
\tilde{Q}(t) = e^{E_{\text{SB}}-Q}(t)se^{E_{\text{SB}}-S}(t) \quad (39)
\]

Since the power contained in the upper sub-band is only half the full-band power, the receiver exhibits a smaller immunity against interference, as shown in the figure. The degradation is around 6 dB.

The simulation results of Fig. 12 were plotted using two different chirp repetition periods: $T_J = 20 \mu s$ and $200 \mu s$. The figure shows that the robustness to interference is maximum in the region with $W < 20$ MHz, which corresponds to the SSC minimum both for full-band and upper sub-band processing.

In Fig. 12 we found a loss of robustness of about 6 dB when half-band processing replaces the full-band processing.
because the power of the incoming signal is decreased by half. Also, there is no clear difference in the receiver's performance between the two chirp repetition periods, $T_J$, for $W < 25$ MHz, but above that value the receiver becomes less robust to the interference with smaller value of $T_J$.

In general, the pattern of robustness for the chirp RFI follows relatively well the evolution of the SSC parameter plotted in Fig. 11. In fact, the maximum of robustness is achieved at low values of $W$ and starts to diminish for $W > 15$ MHz which corresponds to the positive slope of the SSC plot. The minimum of robustness is achieved at $W \approx 35$ MHz which corresponds approximately to the maximum of the SSC curve.

![Fig. 11. Spectral separation coefficient versus the RF chirp signal bandwidth](image1)

Fig. 11. Spectral separation coefficient versus the RF chirp signal bandwidth

**V. CONCLUSION**

In this paper we analyzed the loss of performance of the phase and code locked loops of a conventional AltBOC(15,10) receiver in the presence of two types of radio interference: narrowband continuous wave sinusoid and wideband periodic chirp. Two models of receivers were considered: full band and half band.

Simulations have shown that the performance degradation due to radio frequency interference is well predicted by the spectral separation coefficient (SSC) both for the narrowband and the wideband interference signals under test. In the case of the chirp interference, the SSC value depends both on the power spectral density of the modulated signal and the frequency range of the interfering signal. However, the dependency on the rate of frequency variation in the chirp signal is not clearly identifiable in the SSC parameter. Simulations have shown that, for large chirp bandwidths, the rate of frequency variation is also important with the receiver being more robust to large repetition periods of the chirp interference. Since the computational effort of the Monte Carlo simulation is very significant the results presented so far are only preliminary and more extensive simulations have to be performed in order to determine more accurately the losses of robustness that occur in those cases.

**REFERENCES**


