Robust Vector Tracking for GNSS carrier phase signals

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Abstract—A new formulation of a robust vector phase-locked loop (VPLL) for joint tracking of carrier phase from the navigation satellites is presented. This partitioned tracking method leads to an increased tolerance to unmodeled or unexpected disturbances in the carrier phase tracking, providing a trade off between the pure VPLL and the classic scalar tracking loops. It also provides a solution for smoother transitions between scalar tracking and vector tracking operation modes, which can be of importance in more difficult environments, where the bias estimation may not cope with the fast changing dynamic, leading to harsh transitions and possibility of phase lock loss or inaccurate results.

I. INTRODUCTION

Vector tracking loops have been used in GPS over the last two decades, especially for code and frequency tracking (see [12], [7], [5], [11]). By using the information of all the satellites’ tracking errors to maintain the lock, it is possible to track the code and frequency of satellites with low carrier-to-noise ratio, and to reduce the code/frequency error variance. Phase tracking using a vector approach has been a topic where some research has been done, where these benefits are also present. However, the errors due to disturbances (ionosphere, troposphere, for example) significantly lower than the code length, are considerable when comparing with the wavelength of the carrier signal. And the dynamics of these errors, which are typically slow, can be considerable, especially in scenarios of ionospheric scintillation.

In the article from Zhodzishsky [13] the Co-Op tracking was presented, which introduced a major breakthrough, allowing for vector tracking of the receiver’s dynamics (position and clock), while individually tracking the satellites own errors and effects such as ionosphere and troposphere delays. This was based on a frequency region definition for the individual tracking errors, where the assumption is that lower dynamics which are not related to the position and clock drift can be tracked by suitably choosing a small frequency spectrum for the individual filters. Periodic reinitialization was also required, as referred by Brewer [1].

Later, Henkel [3] introduced a multifrequency/multisatellite VPLL, where the vector mapping included the ionosphere and troposphere errors, modelled with suitable time constants, which was then filtered and back-transformed to phase error estimates. The operation mode would switch from scalar to vector tracking in an on-off setting, and could cope with ionospheric scintillation intensity changes. However, it did not consider a phase stress that can also appear in ionospheric scintillations, and the transition from scalar to vector tracking could be a challenge. Giger [2] complemented the work with the introduction of a Kalman filter to replace that vector mapping and the back transformation.

Soloviev [10] added inertial aiding to the loops, leading to acquisition and continuous tracking of weak signals, when short periods of degraded signal conditions are present. Sarnadas [9] made a comparison between three different methods for robust carrier phase tracking: open loop scalar tracking, closed loop scalar tracking and vector tracking. The vector tracking algorithms based on the VPLL showed poor performance when compared to a VDFLL (frequency and code vector tracking) scheme that aids the scalar carrier tracking. This is due to the magnitude of the errors when compared to the carrier wavelength, that can easily lead to loss of lock in the case of vector tracking.

More recently, Brewer [1] allowed for differential corrections to be introduced in the vector phase tracking loops. This requires a base station which has errors common to the rover receiver, thus making the difference in phases small enough for the vector tracking architecture to succeed, and avoiding a complicated solution where, besides the vector phase tracking, individual channels are also tracked and numerous states have to be considered.

In this work we introduce the partitioned phase tracking of the GNSS signals, where a weighted balance of scalar tracking and vector tracking is used, which can be adapted to the environment and individual satellite errors, that would otherwise be a problem in the pure VPLL. The apparent loss of optimality with respect to noise variance reduction, which is valid when we considered perfectly modeled disturbances, is replaced by an increased robustness against individual satellite errors, showing an improvement in performance, while keeping the receiver’s architecture simple, as this method is simply a weighted combination of vector and scalar tracking.
II. Problem Formulation

In the classical approach (scalar phase tracking), for each PLL there is the objective of reducing the individual error to zero. The output of the loop filter is the estimate of the phase error that is then processed by a navigation filter to obtain the position solution. The problem can then be stated as:

For each individual PLL

\[
\minimize ||\Delta \theta||
\]

taking into consideration the noisy measurements present. In Figure 1 a scalar tracking architecture receiver is shown.

While this classical approach is very intuitive, the fact that there is no coupling between the channels leads to the minimization of the individual error, and any degradation in a single channel can lead to the loss of lock and the use of fewer satellites.

In the vector tracking approach, the mapping from the phase error vector to the state vector \([\Delta x, \Delta y, \Delta z, \Delta c\delta t]\) (position and clock error states in this case, but it can be extended to include other error states) is considered and assumed as true, and the objective can then be stated as:

\[
\minimize ||H\Delta x|| = \minimize ||H(H^T H)^{-1}H^T \Delta \theta||
\]

where the matrix H has rows \(h_i\) given by \([h_{i,1:3} 1]\), being \(h_{i,1:3}\) the line of sight unit vector from the user to the satellite \(i\). The phase error signal obtained by the vector mapping and used in the loop filter, to control the numerically controlled oscillator (NCO) is given by

\[
\Delta \hat{\theta} = H\Delta x = H(H^T H)^{-1}H^T \Delta \theta
\]

When referring to the partitioned carrier phase tracking, the problem can be stated as

\[
\minimize ||\alpha \Delta \theta + (1 - \alpha)H\Delta x||
\]

In this approach, \(\alpha\) sets the trust that is put on the vector solution when compared to the scalar tracking solution. If the mapping from the phase error to the position and time error is very accurate, vector tracking is the optimal solution. However, errors that affect in a different manner the satellites tend to distort that vector mapping. Henkel [3] chose to model the slowly varying biases directly in the mapping and use a pure vector tracking architecture. Zhodzishky [13] chose an architecture where the vector tracking is aided by a scalar tracking with different bandwidths, assuming different frequency spectra for the signals of interest in the vector tracking and scalar tracking loops.

In figure 2 the typical architecture of this partitioned vector phase tracking scheme is depicted. In the classic receiver architecture, \(\alpha = 1\). In pure vector tracking receivers, \(\alpha = 0\). Here the weighed combination of both solutions is considered, and that is then filtered to generate a signal that steers the NCO and maintains the phase lock.

This gives a degree of freedom, which can account for environmental disturbances in an adaptive way, allowing for smoother transitions between operating modes and being robust to unmodeled errors, as well as providing the benefits of reduced error variance that are found in the vector tracking methods, as long as the number of satellites used in the solution is greater than four.

III. Partitioned Phase Tracking

As referred in the previous section, the partitioned carrier tracking problem can be stated as in equation (1).

The study of the noise variance in the vector tracking methods, either code or frequency tracking, presents a rather simple approach, as the errors in the code/frequency tracking are small compared to the code/frequency range. In the phase tracking case, that assumption is only true as long as the error in the angle tracking is small. Assuming that the variance of
the discriminator output is the same, an approximation of the variance of the phase corrections, is given by:

$$\text{cov}(\Delta \theta) = \sigma^2_v (H^T H)^{-1} H$$

(2)

with $\Delta \theta$ being the carrier phase error and $\sigma^2_v$ the variance of the carrier phase measurements.

In this article second-order loop filters are considered. For each channel, the output of the loop filter satisfies

$$\dot{\theta}_i + 2\xi_0 \omega_0 \theta_i + \omega_0^2 \theta_i = K \{ \alpha (D(\theta_i, \theta_i, \text{meas})) + (1 - \alpha) [h_i (H^T \Sigma^{-1} H)^{-1} H \Sigma^{-1} \theta_{\text{meas}}] \}$$

(3)

where $H$ is the geometry matrix, $\Sigma$ is the covariance matrix of the phase measurements, $h_i$ is given by $[h_{i,1:3}]^T$, being $h_{i,1:3}$ the line of sight vector from the user to the satellite $i$, $D(\theta_i, \theta_i, \text{meas})$ is the discriminator output, with respect to the true phase and the measured phase. Here the formulation of the problem includes already a weight matrix $\Sigma$ that takes into account the different noise statistics for the discriminator output.

It is interesting to notice that with this formulation, there may be, in the equilibrium states, solutions where the phase error is nonzero. In fact, this differential equation, when at rest, with phase error equal to zero, leads to the equality

$$\alpha \{ D(\theta_i, \theta_i, \text{meas}) \} = \alpha - 1 \{ h_i (H^T \Sigma^{-1} H)^{-1} H \Sigma^{-1} \theta_{\text{meas}} \}$$

As seen, there may exist equilibrium states where a bias in the discriminator output is verified. This can be a problem if the receiver is operating in a vector tracking mode and changes suddenly to a scalar operating mode, and vice versa.

IV. SIMULATION RESULTS

In order to test the performance of the proposed algorithm, a simulation with a static user was done. In the simulation, the initial position of the receiver is $40^\circ N 9^\circ W$ with an altitude over the WGS84 ellipsoid of 20m, and 11 satellites were in view. The loop filters of the PLLs have a bandwidth of 15Hz, while the loop filters of the DLLs have a bandwidth of 0.5 Hz.

A Hatch filter followed by an extended Kalman Filter was used to estimate the 3D position and velocity, as well as the clock bias and drift. The sampling rate of the discriminators is equal to 100Hz, as well as the sampling rate at which the EKF steps are executed.

The user dynamics is estimated by a PV model (six states), while the clock dynamics was generated from a model considering the Allan variance parameters, with two states. That dynamic is estimated by the EKF, where the elements $x_{7,k}$ and $x_{8,k}$ of the state vector concern the receiver’s clock. The discrete-time dynamics model is $X_{k+1} = \Phi X_k + U_k$, where $\Phi = \text{diag}(M, M, M, M)$ with

$$M = \begin{bmatrix} 1 & \Delta \\ 0 & 1 \end{bmatrix}$$

(4)

and vector $U_k$ collects the dynamics model noises. The corresponding covariance matrix is $Q_k = E\{U_k U_k^T\}$ with entries

$$q_{ii} = q_v \Delta^2 / 3, \quad q_{i,i+1} = q_v \Delta^2 / 2 \quad q_{i+1,i+1} = q_v \Delta$$

(5)

$$q_{77} = \frac{q_f \Delta^3}{3}, \quad q_{78} = \frac{q_f \Delta^2 c^2}{2} \quad q_{88} = q_f c^2 \Delta$$

(6)

The measurements used in the EKF are the outputs of the Hatch filter, that is, estimated pseudorange errors. Assuming that the $N$ scalar observations are affected by independent errors with equal variances, the observations noise vector $[\epsilon_{1,k} \ldots \epsilon_{N,k}]^T$ is characterized by the diagonal covariance matrix $R_k = \sigma^2_{\text{ERK}} I$. The partitioned vector tracking structure was tested in different scenarios

1) Constant carrier to noise ratio, with a linearly changing parameter $\alpha$ over a period of time
2) Periodic perturbation in the tracked phase of one satellite
3) Transition between different operating modes, with the parameter $\alpha$ changing suddenly.

A. Dependence of the variance on $\alpha$

In order to study the gain of the vector tracking with respect to the scalar case, a scenario where the receiver starts in pure scalar tracking mode, and then changes suddenly ($t = 20$s, or sample 2000) to pure vector tracking is simulated. Then, in $t \in [20, 120]$ the parameter $\alpha$ changes linearly, until reaching pure scalar tracking mode again ($\alpha = 1$), switching again to pure vector tracking for 10s more and finishing with $\alpha = 0.33$ until the end of the simulation.

In figures 3 and 4 the output of the loop filter is shown for four different satellites, for different scenarios, one with all carrier-to-noise ratios equal to 43 dBHz and 27 dBHz. It is clear that vector tracking, under these conditions of pure gaussian noise has a better performance than the scalar case, and the parameter $\alpha$ close to 0 leads to a very good approximation of pure vector tracking performance.

In figure 5, the discriminator output is shown for $S/N = 27$ dBHz. As observed, especially due to the first satellite, there is a loss of lock at about $\alpha = 0.6$. Vector tracking is advantageous under these conditions.

B. Perturbations in two satellites

In this scenario, a sinusoidal perturbation on the pseudorange affecting the first satellite is introduced, with amplitude 0.1m and a period of 10s, while another sinusoidal perturbation affects satellite 3, with an amplitude of 0.05m and a period of 2s. This accounts for unmodeled scenarios, that could be due to atmospheric effects. The carrier to noise ratio present is equal to 32 dBHz.

In figures 6 and 7, the output of the loop filters and the discriminators are shown. As seen, even though the loop filter provides an estimate of the angle with less variance, that is not enough to track the satellites in a pure vector setting, leading to a noisier output in the discriminators.

When changing from the scalar to the vector tracking, as the
Fig. 3. Output of the loop filter for satellites 1, 2 and (3, 4), for a carrier to noise ratio of 43 dBHz for all the satellites. The receiver starts in pure scalar tracking operation mode, switches to pure vector tracking (sample=2000) and then linearly goes back to scalar tracking at the sample 12000. It then finishes with a partitioned vector tracking with $\alpha = 0.33$. It is interesting to observe that the improvements on the carrier phase noise error depend on the geometry, and can be considerable for some satellites, while for others they are small. This can be seen when comparing the improvements in satellites 3 and 4.

Fig. 4. Output of the loop filter for satellites 1, 2 and (3, 4), for a carrier to noise ratio of 27 dBHz for all the satellites. The receiver starts in pure scalar tracking operation mode, switches to pure vector tracking (sample=2000) and then linearly goes back to scalar tracking at the sample 12000. It then finishes with a partitioned vector tracking with $\alpha = 0.33$.

Fig. 5. Output of the discriminators for all satellites 1, 2 and (3, 4), for a carrier to noise ratio of 27 dBHz for all the satellites. The receiver starts in pure scalar tracking operation mode, switches to pure vector tracking (sample=2000) and then linearly goes back to scalar tracking at the sample 12000. It then finishes with a partitioned vector tracking with $\alpha = 0.33$.

Phase error biases in that instant are not too large, the transient is smaller. Nevertheless, as $\alpha$ increases, the perturbations become less noticeable. That works for values of $\alpha$ as small as $0.1 - 0.2$.

In figures 8 and 9, the evaluation of the performance of the standard deviation of the discriminator outputs is done, for different values of $\alpha$, under the scenarios of no individual perturbations, and a perturbation in satellite 1. Vector tracking tends to reduce the variance of the phase errors, as long as the vector mapping between the phase error and the state error is valid. That is shown in figure 8. However, the presence if an individual perturbation, as seen in figure 9, leads to a bad performance of the vector tracking, which can lead to divergence. But for small values of $\alpha$ that error is very limited, approaching the optimal performance of the vector tracking for a non perturbed scenario.

C. Sudden transitions in operating modes

Finally, in this last scenario, a transition from scalar tracking to partitioned vector tracking, is considered. The same model of perturbations used in the last subsection is included, with perturbations in 2 satellites, but in this case the period of the perturbation in satellite 3 is equal to 1s. The carrier to noise ratio of the incoming signals used in the simulations is equal to 32 dBHz.

The transition from scalar tracking to partitioned vector tracking occurs at $t = 120s$ at sample 12000.

Figures 10, 11 and 12 show the intervals when the transition occurs for $\alpha = 0, 0.1$ and 0.2. As observed, in figure 10, the perturbations lead to values of the phase discriminator that cause loss of lock. Unmodeled high dynamics are a sensitive issue in vector phase lock loops. When the parameter $\alpha$ increases, those perturbations are attenuated, and especially
at $\alpha = 0.2$, there is an improvement with respect to the scalar tracking, while keeping the conditions to sustain the lock condition.

In fact, using a partitioned vector phase tracking loop can be a good option, for small values of $\alpha$, as a control loop exists for individual perturbations, but a higher weight is put on the vector tracking.

V. CONCLUSION

A partitioned vector phase tracking architecture is presented here, which tries to combine the ability of the scalar tracking loops of tracking individual satellite errors, with a vector mapping structure, that leads to aided possibilities of all satellites to an individual phase tracking loop.

If the perturbations on the satellite tracking errors were...
just gaussian or caused by a joint effort (clock dynamics, 3D movement), vector phase tracking would be an optimal solution. However, as the perturbations can have non-gaussian contributions, or individual satellite tracking errors, closing the loop with a contribution of both tracking schemes reveals a better solution, and a more conservative choice of the weights can lead to solutions that have no need to be reinitialized periodically. Future work includes the study of optimality of these solutions in different real case scenarios.

ACKNOWLEDGMENT

The authors would like to thank Fundação para a Ciência e Tecnologia (FCT) for the support by the project UID/EEA/50008/2013.

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