Improving Detection of Incorrect GPS Carrier Phase Integer Ambiguity Solutions with the Ambiguity Filter

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\textbf{Abstract—}The Global Positioning System (GPS) can be used to estimate the attitude of a vehicle. To do this, multiple receivers should be used with antennas in known fixed positions in the vehicle body frame. Since the estimation of the attitude angles needs high precision, the carrier phase measurements (with millimeter precision) are used in combination with the pseudorange measurements. Nevertheless, the phase measurements have the disadvantage of having an unknown integer number of cycles, known as phase ambiguities. When low-cost receivers with single-frequency measurements (L1 band) are used, the method called Ambiguity Filter must be applied to stabilize the solution given by the LAMBDA method. This filter makes use of the prior knowledge of the distance between the antennas. If the Ambiguity Filter fixes an incorrect set of ambiguities, the solution for the baseline (and attitude angles) starts to degrade, which may not be immediately visible.

This paper proposes a new implementation within the Ambiguity Filter in order to improve the detection of false locks for the solution of the phase ambiguities. This implementation also decreases the Ambiguity Filter sensitivity to the value of the baseline length that is given as input to this method. The results presented in this paper were based on field tests performed in static conditions with a single-baseline setup in the university campus, allowing the estimation of heading and pitch.

\textbf{Index Terms—}GPS, attitude determination, phase integer ambiguity, Ambiguity Filter, LAMBDA method.

\section{Introduction}

The estimation of the attitude of a vehicle based on GPS can be accomplished by using multiple receivers with the antennas in known fixed positions in the body frame. The vectors between the antennas (baselines) must be estimated with precision, which can be done using interferometric techniques. Since the carrier phase measurements have millimeter precision, they are used in combination with the pseudorange measurements to attain the desired precision. However, the carrier phase measurements have an unknown number of cycles (phase ambiguities) that needs to be solved,\cite{1,2}.

To find these phase ambiguities, the search method LAMBDA,\cite{3}, is employed. The LAMBDA is known as the most efficient to solve the ambiguities,\cite{4}–\cite{6}, however, this only happens with dual-frequency measurements (L1 and L2 bands). When low cost receivers are used, with single-frequency measurements (L1 band), another method, known as Ambiguity Filter, must be applied to stabilize the solution given by the LAMBDA method,\cite{7}–\cite{10}. As soon as the phase ambiguities are found, the baseline vector can be estimated and applied to determine the attitude angles.

When single-frequency measurements are used, the Ambiguity Filter makes use of the prior knowledge of the distance between the antennas and, with the baseline configuration used for this paper, it is possible to achieve a precision of $10^{-2}$ degrees for the attitude angles, as shown in previous work,\cite{11}. If the baseline length is not known with precision, the Ambiguity Filter could fix an incorrect set of ambiguities as fixed solution and, in this case, the solution for the baseline and attitude angles starts to degrade.

This work proposes the implementation of a new verification step within the Ambiguity Filter in order to improve the detection of false locks. The comparison between previous work,\cite{12}, and the new performance of the filter allows to verify that the Ambiguity Filter sensitivity to the input value of the baseline length decreases with the new implementation.

In Section II, it is defined the system that represents the problem in consideration. In Section III, it is explained how the Ambiguity Filter finds the solution for the phase ambiguities. In Section IV, it is explained how to estimate the attitude angles. In Section V, it is described the field tests that were conducted in order to obtain the results presented in Section VI. The final conclusions are summarized in Section VII.

\section{System Definition}

The majority of the error sources, like clock offsets and ionosphere and troposphere delays, present at the carrier phase (phase) and pseudorange (code) measurements, are eliminated by creating double differences (DD or $\nabla \Delta$) of these measurements.

The carrier phase measurement taken by a receiver $k$ to a satellite $p$ is given, in meters, by

$$\phi_k^p = \rho_k^p + \lambda N_k^p + c(t_k^p + t_p) + T_k^p - I_k^p + \epsilon_k^p$$  \hfill (1)$$

where

- $\rho_k^p$ is the distance between the satellite $p$ and the receiver $k$ (meters),
- $\lambda$ is the carrier wavelength (meters),
- $N_k^p$ is the carrier phase ambiguity (cycles),
- $t_k^p$ is the receiver clock offset (msec),
- $t_p$ is the satellite clock offset (msec),
- $c$ is the speed of light (m/sec),
- $T_k^p$ is the carrier clock bias (meters),
- $I_k^p$ is the integer ambiguity (cycles),
- $\epsilon_k^p$ is the phase noise (cycles).

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where the equations (3) and (4), is given by measurements.

Using the phase measurement from a different receiver, a single difference (SD) can be created by subtracting both measurements ($\phi_p$ and $\phi_m$):

$$\Delta \phi_{km} = \phi_p - \phi_m = \Delta \rho_{km}^p + \lambda \Delta N_{km}^p + c \Delta t_{km} + \Delta e_{km}^p$$

(2)

where the satellite clock offset and the atmospheric delays (for a baseline with less than 50 km, [1]) are eliminated.

Within the same logic, the receiver clock offset is canceled by using a single difference from another satellite, $q$, ($\Delta \phi_{km}^q$), as shown below

$$\nabla \Delta \phi_{km}^q = \nabla \Delta \phi_{km}^p - \nabla \Delta \phi_{km}^q = \nabla \Delta \rho_{km}^p + \lambda \nabla \Delta N_{km}^p + \nabla \Delta e_{km}^p.$$  

(3)

This new measurement, given by equation (3), is the double difference (SD) that was desired.

Applying the same process to pseudorange measurements, leads to a code double difference:

$$\nabla \Delta P_{km}^{pq} = \nabla \Delta \rho_{km}^{pq} + \nabla \Delta \xi_{km}^{pq}.$$  

(4)

where $\xi$ is the noise due to other sources, in meters, for code measurements.

The double difference of the distance ($\nabla \Delta \rho_{km}^{pq}$) present in the equations (3) and (4), is given by

$$\nabla \Delta \rho_{km}^{pq} = \mathbf{b} \cdot (e^p - e^q) = \mathbf{b} \cdot e^{pq}$$

(5)

where $\mathbf{b}$ is the baseline vector, and $e^p$ and $e^q$ are the direction cosines of satellites $p$ and $q$, respectively. Since the baseline is several orders of magnitude smaller than the distance to the satellite, [1], the direction cosines are considered approximately equal for both receivers ($e_k^p \approx e_m^p \approx e^p$ and $e_k^q \approx e_m^q \approx e^q$).

The system that defines the problem in study can be created by using equations (3) and (4) and by combining them with equation (5). When single-frequency measurements are used and the GPS constellation has $n$ satellites, the linear system has $(2(n-1))$ equations, $((n-1)+3)$ states and assumes the following form:

$$y_{[2(n-1) \times 1]} = B_{[2(n-1) \times 3]} b_{[3 \times 1]} + A_{[2(n-1) \times (n-1)]} a_{[(n-1) \times 1]} + e_{[2(n-1) \times 1]}$$

(6)

where

- $y$ is the vector with the phase and code double differences,
- $B$ is the matrix with the differenced direction cosines,
- $b$ is the baseline vector,
- $A$ is the matrix with the carrier wavelength,
- $a$ is the vector with the phase ambiguities,
- $e$ is the vector with unmodeled noise.

When dual-frequency measurements are used the new linear system has $(4(n-1))$ equations and $(2(n-1)+3)$ states.

III. AMBIGUITY FILTER

A Weighted Least Squares (WLS) algorithm can be used to find the floating point solution for the phase ambiguities by minimizing the residual error:

$$\min_{a,b} \| y - A a - B b \|_W^2$$

(7)

where $W$ is the weight matrix and $\| \cdot \|_W = (\cdot)^T W (\cdot)$, [13].

Nonetheless, the float solution does not correspond to the most accurate solution for the baseline (and attitude angles) because the phase ambiguities are integers. This float solution is then used as initial guess in the search method called LAMBDA, [3]. With dual-frequency measurements, the LAMBDA is known as the most efficient to find the ambiguities, [4]–[6]. However, when using only single-frequency measurements, the LAMBDA solution has a low success rate, making it necessary to apply the Ambiguity Filter after the LAMBDA, [7]–[10].

The Ambiguity Filter selects the best solution for the ambiguities by processing the output given by the LAMBDA method, which corresponds to an ordered list of candidates, $\hat{a}$, (set of ambiguities) and the corresponding baseline vector, $\hat{b}$. This filter had originally three steps: validation, selection and stabilization. In this paper, a new verification step is added to the Ambiguity Filter in order to improve the detection of false locks for the solution of the ambiguities.

A. Validation Step

In this step, all of candidates, that do not comply with a threshold of 10 cm, [9], [10], defined for the baseline length error ($\delta_b$), are excluded:

$$\delta_b = |\hat{b} - l| \leq 0.1 \text{ m}$$

(8)

where $\hat{b}$ is the baseline length solution correspondent to the candidate given by the LAMBDA method ($\hat{a}$) that needs to be tested and $l$ is the known baseline length.

B. Selection Step

In this step, the remaining candidates, given by the LAMBDA method, are sorted in ascending order, according to three available tests: Baseline Length Constraint, Residual Ratio and Up Coordinate Constraint, [9], [10]. Then, two of these tests are combined to create a metric that allows the selection of the best candidate for the ambiguities in the current epoch.

1) Baseline Length Constraint

The baseline length error is calculated by using the same equation (8) from the constraint in the Validation Step:

$$\delta_b = |\hat{b} - l| .$$

(9)

2) Residual Ratio

The residual ratio is calculated as shown below

$$\|V\|^2 = V^T (\text{cov}(\nabla \Delta \phi))^{-1} V$$

(10)

where $V$ is the phase residual vector and $\text{cov}(\nabla \Delta \phi)$ is the covariance matrix for the phase double differences.
The phase residual vector is computed by subtracting the estimated phase DDs (for each candidate given by LAMBDA) and the phase DDs ($\nabla \Delta \phi$):

$$V = B\dot{b} + A\dot{\alpha} - \nabla \Delta \phi.$$  \hspace{1cm} (11)

3) **Up Coordinate Constraint**

The Up coordinate error can be calculated when the vehicle is stable during the initialization of the Ambiguity Filter and is given by

$$\delta_u = ||\dot{u}(\hat{u}) - u_{real}||$$  \hspace{1cm} (12)

where $\dot{u}(\hat{u})$ is the estimated baseline Up coordinate (for each candidate given by LAMBDA) and $u_{real}$ is the real baseline Up coordinate which is calculated by making the difference between the altitude of the two antennas.

4) **Candidate Selection**

For each one of the tests defined, the merit, [9], [10], is assigned to the candidates according to its sorted order, $i$, as shown below:

$$M_i = 1/i.$$  \hspace{1cm} (13)

With the previous definition in mind, it is possible to sum the merit given to a candidate from two tests and create a new metric. The candidate with higher merit is then selected as solution for the current epoch. The results presented in Section VI were obtained for two metrics, the Baseline Length Constraint plus the Residual Ratio (Metric 1), and the Baseline Length Constraint plus the Up Coordinate Constraint (Metric 2).

**C. Stabilization Step**

Despite the improvement over the LAMBDA solution for each epoch, the previous steps are still not enough to solve the ambiguities. So, to stabilize the solution, the solutions given by the Ambiguity Filter are recorded for each time epoch, and then the candidate that is first selected in 50 different epochs is chosen as the fixed solution, [9], [10].

D. **The Proposed Verification Step**

When using carrier phase measurements, it is possible to achieve results for the baseline vector with millimeter precision, however, this only happens when the solution for the ambiguities is correctly determined. If the Ambiguity Filter fixes an incorrect set of ambiguities, the solution for the baseline (and attitude angles) starts to degrade, which may not be immediately visible. An example of these two situations is shown in Figure 1.

This new step is the main contribution of this paper and was added to the Ambiguity Filter in order to improve the detection of incorrect solutions for the phase integer ambiguities. As previously explained, the Ambiguity Filter is dependent on the prior knowledge of the baseline length which will lead to wrong solutions when this value is not precisely known. Having this verification step, it is possible to decrease the sensitivity of the Ambiguity Filter to the baseline length by detecting wrong solutions for the ambiguities.

All of the measurements from the available satellites, for which the ambiguities are known, are used to estimate the baseline vector. The verification step explores the redundant measurements, above the required minimum, that are used to estimate the baseline.

Over several tests, it was found that, when a satellite was lost at a given time and the set of ambiguities was incorrectly fixed, it was more likely the occurrence of a large jump in the baseline length solution, coincident with the moment when the satellite was lost. This does not happen when the set of ambiguities is correctly fixed. These two cases can be seen in Figure 1, where the satellite was lost around epoch 1300 seconds.

![Baseline Length Solution with Correct and Wrong Ambiguities Fixed](image)

**Fig. 1. Baseline Length Solution with Correct and Wrong Ambiguities Fixed**

Given this behavior, the additional verification step was created for the Ambiguity Filter. Whenever the solution for the ambiguities is considered fixed (stabilization step), it is tested if the baseline length obtained (with $n$ satellites) suffers any sudden change by removing one or two satellites (solution obtained with $(n-1)$ or $(n-2)$ satellites), if enough redundant data is available. If it is verified the existence of a jump in the baseline length solution, typically above 5 millimeters, the correspondent solution for the phase ambiguities ceases to be considered fixed and the respective count for the stabilization step returns to 1.

If redundant data is available this detection is immediate and thus the degradation that usually follows a false lock is not experienced.

**IV. Attitude Estimation**

To determine the attitude of a vehicle with two GPS receivers, the antennas are usually placed longitudinally in the vehicle in order to coincide with the direction of the movement. In this situation, it is possible to estimate the heading ($\psi$) and pitch ($\theta$) by resorting to the definition of azimuth and elevation, respectively, and by making use of the baseline local coordinates ($n,e,d$) (NED - North, East, Down), [14]:

$$\psi = \tan^{-1}\left(\frac{e}{n}\right)$$  \hspace{1cm} (14)

$$\theta = -\tan^{-1}\left(\frac{d}{\sqrt{e^2+n^2}}\right)$$  \hspace{1cm} (15)

where $\psi \in [-\pi, \pi]$ and $\theta \in [-\pi/2, \pi/2]$. To place $\psi$ in the right quadrant the signs of $e$ and $n$ must be taken in consideration.
V. Solution Implementation

Experimental tests were conducted at IT GNSS Monitoring Station, using two receivers from Ashtech (ZXW-Sensor and ProFlex 500). Both antennas were complemented with choke rings, to minimize the multipath effects, and were placed at the North Tower rooftop at Instituto Superior Técnico campus, at the University of Lisbon.

The baseline vector was pointing North and had a length of 12.47 meters ($l$). The expected values for heading is $-4$ degrees and for pitch is zero degrees, since the antennas were placed at the same height.

VI. Results

Figure 2 shows the results for heading and pitch obtained with single-frequency measurements (L1) and with dual-frequency measurements (L1+L2). Table I presents the performance for these results, the mean value ($\mu$) and the standard deviation ($\sigma$). The Ambiguity Filter was used to obtain the results with single-frequency measurements, using the value of 12.47 meters as the baseline length input for this method. The LAMBDA method was complemented with a validation step to ensure the correctness of the solution obtained with dual-frequency measurements.

![Figure 2. Final Solution](image)

Comparing the results, it can be verified that it is possible to obtain the same level of precision with single-frequency and dual-frequency measurements. The results obtained for heading and pitch have a precision with an order of magnitude of $10^{-3}$ degrees and $10^{-2}$ degrees, respectively. Additionally, the mean values of the results for heading and pitch are close to the expected values. It was possible to obtain these results with single-frequency measurements since the Ambiguity Filter makes use of the initial knowledge of the baseline length, which causes the filter to become dependent on this value. When the value of the baseline length is not accurately given to the Ambiguity Filter, it may cause the filter to fix the wrong solution for the set of ambiguities and, thereby, impair the accuracy of the solution obtained for the baseline vector.

In Figure 1 were shown the results obtained for the baseline vector with the implementation of the new verification step in the Ambiguity Filter, twelve data sets were analyzed which were collected under the same conditions in different moments (September-November, 2015). For each one of these data sets, the Ambiguity Filter (using single-frequency measurements) was tested with different inputs for the baseline length, starting at its true value (12.47 meters). The range within which the solution (given by the Ambiguity Filter) remained correct was registered (in centimeters). The results obtained with the original Ambiguity Filter (without the implementation of the verification step), shown in Table II, were analyzed in previous work, [12]. Now, the new results of the Ambiguity Filter with the implementation of the proposed verification step are presented in Table III. This allows to directly compare the results and to verify the improvements achieved with the new implementation.

In both Tables (II and III), the worst results ($\pm 1 - 3$ cm) are in bold and the best results for each data set (comparing the two metrics) are highlighted. In Table III, the results, that were not improved with the implementation of the verification step in the Ambiguity Filter, are marked with an asterisk.

Comparing Table II and Table III, it can be seen that the results obtained without and with the verification step are not the same.

For some data sets, the bounds have good values (like data set 10), however, some other results are not as good (bounds of $\pm 1 - 3$ cm) (emphasized in bold). Looking at the worst results in Table II, there are six worst results for metric 1 and five for metric 2. After the implementation of the verification step (Table II), the number of worst results was reduced to three for both metrics.

For each data set, the results were compared between both metrics and the best ones were highlighted (lower and upper bounds). When a data set does not have two highlighted values (for two bounds), it means that the results obtained were the same for both metrics. In Table II, metric 2 shows an advantage

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<td>Pitch</td>
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For some data sets, the bounds have good values (like data set 10), however, some other results are not as good (bounds of $\pm 1 - 3$ cm) (emphasized in bold). Looking at the worst results in Table II, there are six worst results for metric 1 and five for metric 2. After the implementation of the verification step (Table II), the number of worst results was reduced to three for both metrics.

For each data set, the results were compared between both metrics and the best ones were highlighted (lower and upper bounds). When a data set does not have two highlighted values (for two bounds), it means that the results obtained were the same for both metrics. In Table II, metric 2 shows an advantage
with this new step. Besides this, it was not observed any result made worse not always happens for the same combination of data set and

improvement with the implementation of the verification step, (7 highlights) over metric 1 (5 highlights). On the other hand, after the implementation of the new verification step (Table III), metric 1 shows an advantage (7 highlights) over metric 2 (4 highlights). Although the results show a greater improvement with the implementation of the verification step, it is not possible to conclude that there is a clear advantage of one metric over the other. Further testing should be carried out to confirm if any of the metrics is better than the other.

Finally, when comparing the results from Table II and Table III, it can be seen that the implementation of the verification step in the Ambiguity Filter decreases the filter sensitivity to the precision to which the baseline length has to be known. From a total of 24 results (for each metric), 10 results were not improved (marked with an asterisk), however, this does not always happen for the same combination of data set and bound. Besides this, it was not observed any result made worse with this new step.

The results above show that the new verification step, implemented in the Ambiguity Filter, improves the detection of false locks for the ambiguities, contributing also to decrease the sensitivity of the Ambiguity Filter to the input of the baseline length.

Within this verification step, the ambiguous measurements from a satellite (or two if possible) are eliminated but not necessarily from the satellite with lower elevation. Besides this, the value used to verify whether there is a jump in the baseline length solution was 0.005 meters.

**VII. CONCLUSIONS**

In this article, it was shown that it is possible to obtain the level of precision achieved by LAMBDA method with dual-frequency measurements, by using the Ambiguity Filter with single-frequency measurements. Nevertheless, if the Ambiguity Filter fixes an incorrect set of ambiguities, the solution obtained for the baseline vector (and attitude angles) begins to degrade which may not be immediately visible. In
order to improve the detection of false locks provided by the Ambiguity Filter, it was implemented a new verification step within this filter. Since the Ambiguity Filter uses the baseline length (given as input) in order to improve the final solution, when only single-frequency measurements are used, the filter becomes highly dependent on this input. The new verification step also allows to decrease the sensitivity of the Ambiguity Filter to the baseline length previously known.

The verification step was developed from a behavior that was observed in different tests: when a satellite is lost, there is a higher variation in the baseline length solution if the ambiguities are fixed incorrectly. To test this new implementation, twelve data sets, acquired during the period of three months, were analyzed using different input values for the baseline length, without and with the new verification step within the Ambiguity Filter. Within this verification step, the ambiguous measurements from a satellite (or two if possible) are eliminated but not necessarily from the satellite with lower elevation and, the value used to verify whether there is a jump in the baseline length solution was 0.005 meters.

After reviewing the results obtained, several conclusions can be drawn. The number of worst results (bounds of $±1 - 3$ cm) is reduced after the implementation of the verification step in the Ambiguity Filter. The metric 1 obtains better results (7 out 24 possible) than the metric 2 (4 out 24 possible) after the implementation of the verification step, but this is not enough to show that metric 1 is better since the opposite happens without the new step. Comparing the results obtained with and without the implementation of the new verification step in the Ambiguity Filter, it is possible to verify that from a total of 24 results (for each metric) 14 results were improved with the new implementation. Additionally, it was not observed any result made worse with this new step.

The results obtained show that the new verification step, implemented in the Ambiguity Filter, improves the detection of false locks for the ambiguities, contributing also to decrease the sensitivity of the Ambiguity Filter to the input of the baseline length.

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