Sensitivity of GNSS Blind Interference Detection Algorithm Based on Fourth-order Autocumulants

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Abstract—GNSS signals affected by additive channel noise are approximately Gaussian but often the presence of interference may change that distribution. We analyze the sensitivity of a proposed blind detection method based on the Giannakis-Tsatsanis Gaussianity test to evaluate the presence of narrowband and wideband non-Gaussian interference. Robustness of the proposed method to the blind-spot effect that occurs in pulsed interference signals is also discussed.

Index Terms—Giannakis-Tsatsanis test, Gaussianity tests, interference detection, autocumulants, GNSS receivers, blind-spot effect.

I. INTRODUCTION

Global Navigation Satellite Systems (GNSS) use spread-spectrum modulation schemes but, due to their weak power, the received signals are easily disturbed by the presence of certain types of radio frequency interference (RFI) which can be intentionally provoked (jamming) or due to the proximity of authorized radio sources, such as TV/FM, radar and mobile systems. An example of unintentional interference signal emissions is the malfunctioning of electronic devices (e.g., nonlinear amplifier in TV transmitters) that might generate harmonics or intermodulation products [1]. These interfering systems tend to exhibit narrow bands compared with the GNSS signal bandwidths, whereas jammers typically resort to wideband signals for malicious interference [2], [3]. Thus, we will consider two types of interfering signals: narrowband sinusoidal and wideband FM-modulated chirp signals.

In [4], [5], [6] semi-blind methods for pre-correlation interference detection in GNSS signals were proposed which compare the characteristics of the signal disturbed by RFI with those of an interference-free signal. Although being more sensitive to weak interference than the blind detection schemes, they require that the GNSS signal is sampled at a window, designated as assessment or calibration window, where it is known that the signal is RFI-free. However, often this assumption cannot be guaranteed or it may be desirable to operate with a single time interval where the presence or absence of RFI has to be determined. In those cases, it is convenient to use a blind detection scheme that performs sufficiently well without information from an assessment window.

The complex baseband GNSS received signal contains data from several satellites of different constellations together with Gaussian thermal noise. Due to the weak power of each GNSS received signal (typically −160 dBW at the Earth’s surface) and the large bandwidths (several MHz) the channel noise is preponderant. In fact, typical noise power spectral densities in conventional receivers are about −200 dBW/Hz which means that, before despreading, the power spectral density of each modulated signal is well below the noise level [7]. Besides, the aggregate contribution of the received GNSS signals to the inphase/quadrature components of the baseband complex signal is approximately Gaussian provided that the number of navigation signals impinging the receiver’s antenna at a given band is larger than few percent. For a large number of signals Gaussianity is guaranteed by the Central Limit theorem [8]. In practical scenarios this condition is met most of the time; that is, with the deployment of other systems besides the Global Positioning System (GPS), the number of satellites in view is almost always larger than five [7]. However, this scenario is modified by the existence of a jamming signal that tends to disturb the Gaussian nature of the received signals.

Tests of Gaussianity (or normality) have been used in many different applications to separate (classify) two populations that obey to different statistical distributions when it is known that one of them is Gaussian [9]. In [10], Giannakis and Tsatsanis proposed a test of Gaussianity for real signals that relies on higher-order statistics. Specifically, the algorithm resorts to third or fourth-order autocumulants to measure deviations from Gaussianity and uses the fact that, if a sequence of real samples are Gaussian, then the k-th order autocumulants vanish for $k \geq 3$.

Following previous work carried out by the authors in [11], herein we analyze the sensitivity of the proposed GNSS blind interference detection technique based on fourth-order autocumulants. The analysis will be carried out mainly through the receiver operating characteristic (ROC) plots which are curves of correct interference detection probability versus false alarm probability for a given interference scenario.

The paper is organized as follows. The GNSS incoming signal disturbed by RFI is characterized in Section II. Section III is devoted to the analysis of the proposed technique.
based on the Giannakis-Tsatsanis (G-T) test. In Section IV the simulation results obtained with the new technique are presented. Conclusions are drawn in Section V.

II. SIGNAL CHARACTERIZATION

Assume that the incoming signal of the interference detector, disturbed by RFI (jamming signal) $v(t)$, is [7]

$$r(t) = \sum_{g=1}^{N_g} A_g c_g(t) d_g(t) \cos(2\pi(f_0 + f_g)t + \phi_g) + v(t) + w(t)$$

(1)

where $N_g$ is the number of received GNSS signals in a given RF band and $w(t)$ is AWGN with power spectral density $G_w(f) = N_0/2$. For each received signal, with amplitude $A_g$, the carrier-to-noise power ratio is $(C/N_0)_g = A_g^2/(2N_0)$, $c_g(t) = \pm 1$ is the spreading sequence which may include a digital subcarrier as in the Binary Offset Carrier (BOC) modulations, $d_g(t) = \pm 1$ is the data sequence, $f_0$ and $f_g$ are, respectively, the nominal carrier and the Doppler frequencies, and $\phi_g$ are initial carrier phases. We consider two types of continuous interferers: a sinusoidal waveform of frequency $f_0 + f_j$, with $f_j$ accounting for the jammer’s frequency offset, and a chirp waveform, both described by $v(t) = A_j \cos(\varphi(t))$, where $A_j$ stands for the interference amplitude. The jamming-to-noise power ratio is $J/N_0 = A_j^2/(2N_0)$.

Assume that the receiver’s front-end bandwidth is equal to $2B$. The phase of the sinusoidal (narrowband) interference is given by $\varphi(t) = 2\pi(f_0 + f_j)t + \varphi_0$, with $\varphi_0$ denoting the initial phase. Consider that the RF signal $r(t)$ is heterodyned to baseband using inphase/quadrature carrier replicas and sampled at rate $f_s = 2B$ to yield

$$z_n = s_n + A_j \exp[j(2\pi f_{j} t_n + \varphi_0)] + N_n$$

(2)

at time epoch $t_n = n\Delta$ with $\Delta \equiv f_s^{-1}$ and $|f_{j}\Delta| \ll 1$. Signal

$$s_n = \sum_{g=1}^{N_g} A_g c_g.n d_g.n \exp(j\psi_{g,n}), \quad \psi_{g,n} = 2\pi f_g t_n + \phi_g$$

(3)

is the contribution of $N_g$ visible satellites. Assuming that the phases $\psi_{g,n}$ are uniformly distributed, the pdfs of $A_g c_g.n d_g.n \cos(\psi_{g,n})$ and $A_g c_g.n d_g.n \sin(\psi_{g,n})$ are given by

$$f_Y(y) = 1/(\sqrt{A_g^2 - y^2}), \quad |y| < A_g$$

with variance $A_g^2/2$. The complex noise $N_n = N_{i,n} + jN_{q,n}$ has independent Gaussian zero-mean random components with variance $N_0/\Delta$. GPS C/A receivers typically present $C = -160$ dBW and $N_0 = -200$ dBW/Hz yielding $C/N_0 = 40$ dB-Hz [7]. However, this value may be modify due to changes of signal attenuation (for instance provoked by different satellite elevation angles) and by changes of the antenna effective noise temperature.

For $N_g$ independent GNSS signals, with $N_g \geq 5$ and equal carrier-to-noise power ratios $C/N_0 = A^2/(2N_0)$, the real and imaginary parts of $s_n$ are approximated by zero-mean Gaussian r.v. with variances $A^2 N_g/2$, thanks to the Central Limit Theorem. Therefore, in the absence of an interfering signal and assuming that several satellites are in view, we can consider that the real and imaginary parts of the samples $z_n$ are independent and approximately zero-mean Gaussian with variances equal to $N_0[jf_s + N_g(C/N_0)]$.

Several civil jammers were tested in [2], [3] and the signals found were, typically, of the chirp type with periods of repetition in the order of several dozens of microseconds and frequency sweeps exceeding 20 MHz. Based on the analysis carried out therein we model the wideband interference as a periodic chirp signal with the frequency sweep depicted in Fig. 1, where $W$ stands for the frequency sweep range, and $T_J$ is the sweep period of the jamming signal.

![Fig. 1. Frequency sweep of the chirp jamming signal.](image)

The instantaneous frequency is given by $f_i(t) = f_0 + \gamma t - W/2$ in the interval $0 \leq t < T_J$, with $\gamma = W/T_J$. Let $\tau = \text{mod}(t,T_J)$ be the remaining after division of $t$ by $T_J$; the phase of the interfering signal is given by $\varphi(t) = 2\pi(f_0 - W/2)\tau + \pi\gamma\tau^2 + \varphi_0$. In the interval $0 \leq \tau < T_J$ this signal can be written as $v(t) = R\{u(t) \exp(j2\pi f_0 \tau)\}$, where $u(t)$ is the chirp interference is heterodyned to baseband and sampled at rate $f_s$ to yield

$$z_n = s_n + A_j \exp[j(\pi\gamma n^2 + \pi W \tau_n + \varphi_0)] + N_n$$

(4)

with $\tau_n = \text{mod}(t_n,T_J)$, the signals amplitudes are given by $A_g = \sigma_N \sqrt{2\Delta(C/N_0)_g}$ and $A_j = \sigma_N \sqrt{2\Delta(J/N_0)}$.

As noted in [12], if the chirp frequency sweep exceeds the front-end bandwidth, i.e., $W > 2B$, the interference appears in the receiver as a pulsed chirp signal with duty-cycle $2B/W$. Regardless of the type of interference (sinusoidal or chirp continuous wave), when the RFI signal is sufficient strong, the approximate pdf of the real/imaginary part of samples $z_n$ is given by

$$f_X(x) = \begin{cases} \frac{1}{\pi \sqrt{A_j^2 - x^2}}, & \text{if } |x| < A_j \\ 0, & \text{otherwise.} \end{cases}$$

(5)

III. GAUSSIANITY TEST BASED ON AUTOCUMULANTS

Cumulants of order greater than two are generalizations of the autocorrelation function $c_2(a) \equiv E\{x(i) x(i+a)\}$ for zero-mean processes and can be used to quantify departure from Gaussianity [10]. Assume a zero-mean, real, stationary, sequence $\{x(i), i = 1, \ldots, N\}$, with variance $\sigma_x^2$. The third-order autocumulant is defined by $c_3(a,b) \equiv E\{x(i + a) x(i + b) x(i)\}$ [13], [10]. However, if additionally the samples
$x(i)$ are symmetrically distributed (Gaussian or not) then the autocumulant vanishes regardless of the signal distribution [13]. That is why the application of third-order autocumulant to test Gaussianity may fail. Consider now the fourth-order autocumulant define by [13, 10]

$$c_4(a, b, c) = \frac{E \{x(i + a)x(i + b)x(i + c)x(i)\}}{E \{x^2(i)\}} - 3c_2^2(0)$$

In particular, we have

$$c_4(0, 0, 0) = E \{x^4(i)\} - 3c_2^2(0)$$

and samples with a Gaussian distribution lead to $c_4(0, 0, 0) = c_3(0, 0) = 0$ [14]. The previous result is related to the kurtosis, define as $R = E \{x^4(i)\} / (E \{x^2(i)\})^2$, by

$$c_4(0, 0, 0) = R - 3$$

Let the r.v. $x_i, x_j, x_k, x_l$ are zero mean and jointly Gaussian. The Isserlis’ theorem states that $E \{x_i x_j x_k x_l\} = m_{ij}m_{kl} + m_{ik}m_{jl} + m_{il}m_{jk}$, with $m_{ij} = E \{x_i x_j\}$ denoting the correlation. Using this result it can be shown that, if the samples $x(i)$ are Gaussian, then the autocumulant $c_4(a, b, c)$ vanishes for all lags, regardless of the amount of correlation existing between samples.

The classical or natural estimator of the fourth-order autocumulant $c_4(a, b, c)$ is [10]

$$\hat{c}_4(a, b, c) = \frac{1}{N} \sum_{i=1}^{N-a} x(i + a)x(i + b)x(i + c)x(i) - \hat{c}_2(a)\hat{c}_2(b - c) - \hat{c}_2(b)\hat{c}_2(a - c) - \hat{c}_2(c)\hat{c}_2(a - b)$$

where $\hat{c}_2(a)$ is the estimate of the autocorrelation given by

$$\hat{c}_2(a) = \frac{1}{N} \sum_{i=1}^{N-a} x(i + a)x(i).$$

It was proved in [15] that the estimator defined in (9) is asymptotically unbiased.

The estimates $\hat{c}_4(a, b, c)$, with $0 \leq c \leq b \leq a \leq (L - 1)$ and $L = 1, 2, \ldots$, are collected into a $N_c \times 1$ vector, called $\hat{c}_4$ (vector of estimated autocumulants), where $N_c = L(L + 1)(L + 2)/6$ is the number of lags ($L$ is a selectable value).

The elements of $\hat{c}_4(p)$, with $p = 1, \ldots, N_c$, are ordered according to the following relation between $p$ and the lag indices $a$, $b$, and $c$

$$a = \max_u \left\{ p - 1 \geq \frac{u^2 + 3u^2 + 2u}{6} \right\}$$

$$b = \max_v \left\{ p - \frac{a^3 + 3a^2 + 2a}{6} - 1 \geq \frac{v^2 + v}{2} \right\}$$

$$c = p - \frac{a^3 + 3a^2 + 2a}{6} - \frac{b^2 + b}{2} - 1.$$
This result can be written in terms of the incomplete gamma function $\gamma(a, x)$ as follows

$$P_{fa}(\lambda) = 1 - \frac{\gamma(N_c, \lambda/2)}{(N_c - 1)!}$$  \hspace{1cm} (18)

with [17]

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt.$$  \hspace{1cm} (19)

In general, we assign a given value to $P_{fa}$ and determine the corresponding threshold $\lambda$ by resorting to (18). The probability of correct detection of interference is then given by $P_d = Pr\{V \geq \lambda | H_1\}$. Remarkably, the computation of $\lambda$ depends only on the quantity $N_c$, being independent of the number of samples $N$ and the variance of each sample. Therefore, it can be computed off-line.

IV. Simulation Results

For all the simulations we consider sinusoidal interference with frequency offset $f_d = 70$ kHz and chirp interference with frequency range $W = 40$ MHz, centered at the GNSS signal carrier frequency, with repetition period $T_r = 20 \mu s$. We assume that the baseband bandwidth of the interference detector is equal to 25 MHz. A receiver with typically this amount of bandwidth required to process the new GNSS signals, such as the lower or upper sub-bands of Galileo AltBOC(15,10) modulation [18]. The adopted sampling rate is $f_s = 50$ Msamples/s. Except when indicated the number of complex samples to be processed is constant: $N_s = 5000$. This corresponds to a processing window of 100 $\mu s$. We assume that $N_g = 20$ independent GNSS signals with equal signal-to-carrier ratios $C/N_0 = 40$ dB-Hz and uniformly distributed Doppler frequencies in the interval $-5$ kHz to 5 kHz are received. This number of satellites in view aims to represent a typical multi-constellation scenario.

A. Continuous sinusoidal and chirp interference

Consider the cost function $C(\lambda) = [P_{fa}^2(\lambda) + (1 - P_d(\lambda))^2]^{1/2}$ given by the distance from the ROC curve to the point $(P_{fa} = 0, P_d = 1)$ and assume that the optimum value of $\lambda$ corresponds to minimize $C(\lambda)$. By computing the values of $C_{\text{min}} = \min_{\lambda} C(\lambda)$ we can compare the performance of two tests for a given value of $J/N_0$, the best being the one with the smallest value of $C_{\text{min}}$. Figure 2 displays the curves of $C_{\text{min}}$ versus the ratio $J/N_0$ for sinusoidal and chirp interference using different values of $L$. The curves were obtained with 20000 independent runs. For all the curves $C_{\text{min}} \rightarrow 1/\sqrt{2}$ when $J/N_0 \rightarrow 0$, which corresponds to the ROC curve (straight line) with $P_d = P_{fa}$. In that case, the test is unable to separate hypotheses $H_0$ and $H_1$. On the other side, note that when $C_{\text{min}} \ll 1$, $P_{fa} \approx \sqrt{2}C_{\text{min}}$ and $P_d \approx 1 - \sqrt{2}C_{\text{min}}$.

The curves with $L = 1$ ($N_c = 1$) mean that only autocumulant $c_4(0, 0, 0)$ is used to test Gaussianity, which is equivalent to the kurtosis tests. In this case the detection performance is independent of the considered types of interference (sinusoidal or chirp). The plots show what can be gained by replacing a kurtosis test by the G-T test: as the number of lags increases the latter test is able to distinguish the hypotheses $H_0$ and $H_1$ for smaller amounts of interference (sinusoidal or chirp). With $L = 10$ and $C_{\text{min}} = 0.01$ the gains are approximately 7 dB and 2.5 dB for the sinusoidal and chirp interference, respectively. Note that, for chirp interference, a computationally heavier algorithm ($L$ larger) is required to reach the same sensitivity to the presence of interference.

Figure 2 reveals that the interference detector is able to reliably detect chirp interference with $J/N_0 \geq 73$ dB-Hz and sinusoidal interference with $J/N_0 \geq 68$ dB-Hz (assuming $L = 10$ and $C_{\text{min}} = 0.01$). The sensitivity can be improved by reducing the front-end bandwidth. For instance, by diminishing the baseband bandwidth to 2.5 MHz permits a tenfold decrease in the amount of Gaussian noise that corrupts the $z_n$ samples, thus improving the interference detector sensitivity. The downside of this solution is that, in the case of wideband...
interference, the bandwidth reduction leads to a loss of power of the chirp signal processed by the interference detector. In fact, most of the time RFI is out of band as the effective duty-cycle is reduced to $2.5/20 = 0.125$. As a consequence, the improvement in the detector sensitivity with the bandwidth reduction will be smaller for wideband than for narrowband interference. This conclusion is corroborated by the results of Fig. 3. The curves in the figure show that, with a baseband bandwidth $B = 2.5$ MHz (sampling rate $f_s = 5$ Msamples/s) and $N_s = 25000$ samples, narrowband interference with $J/N_0 \geq 56$ dB-Hz and wideband interference with $J/N_0 \geq 66$ dB-Hz can be reliably detected. Note that this solution only makes sense if the navigation signals to be processed have a significant RF spectral occupancy inside the bandwidth $B_{RF} = 5$ MHz, such as in the case of the GPS C/A signals.

B. Pulsed interference

It is known that the kurtosis tests are sensitive to the duty-cycle $d$ of pulsed sinusoidal RFI (blind-spot effect) [19]. The region of blindness, that is, the region in which the interference will not be detected by the kurtosis for any number of independent samples is given by [19]

$$\psi(R_{max}) \leq d \leq \psi(R_{min})$$

with

$$\psi(R) = \frac{3S^2}{2[(1+S)^2R-3(1+2S)]}$$

where $R_{min} = 3 - \mu$, $R_{max} = 3 + \mu$, and $\mu$ is a small positive quantity that grows as the probability of false alarm decreases. In (21) $S = dA_s^2/(2\sigma^2)$ is the signal-to-noise ratio of the pulsed sinusoid to the noise power. The blind spot concept is illustrated in Fig. 4 for different values of $\mu$. The figure shows that, for a duty-cycle of $1/2$ (50%), the kurtosis tests are unable to detect interference regardless of its power. Besides, as the signal-to-noise ratio decreases the range of values of the duty-cycle which do not permit to detect interference becomes wider.

The blind-spot effect happens when the duty-cycle of the sinusoidal interference is around $1/2$, but simulations have evidenced that it also occurs with chirp interference provided that the bandwidth of the chirp signal is larger than the receiver’s front-end bandwidth. Figure 5 shows the results obtained for $C_{min}$ when the chirp frequency range is $W = 40$ MHz (corresponding to a baseband bandwidth equal to 20 MHz) and the baseband equivalent bandwidth $B$ of the detector input filter varies between 2 and 20 MHz (which corresponds to duty-cycles between 0.1 and 1) for two different values of $J/N_0$. For each sampling rate $f_s = 2B$ the number of processed samples changes but the duration of the processing window is kept constant at 100 $\mu$s. Departure from $C_{min} \approx 0$ in Fig. 5 indicates failure to detect RFI. The figure shows that the blind-spot effect is mitigated when the proposed G-T test (with $L > 1$) replaces the kurtosis test ($L = 1$). Notice for $L = 1$ and $B = 10$ MHz ($d = 1/2$) the significan loss of sensitivity in the interference detector with both values of $J/N_0$. For $L \geq 5$ the G-T test is practically immune to blind-spot degradation. This behavior is an important advantage of the proposed RFI detection technique over the kurtosis-based tests.

Simulations displayed in Fig. 6 reveal that the G-T test can also reduce the blind region in pulsed sinusoidal interference. In contrast with the kurtosis tests which degrade the interference detectability with duty-cycles above $1/3$, the proposed Gaussianity test diminishes the region of blindness to a small interval centered at the duty-cycle of $1/2$. The parameters used in this simulation are: baseband bandwidth $B = 2.5$ MHz (sampling rate $f_s = 5$ Msamples/s), $N_s = 5000$ samples (1 ms window), and constant interference-to-noise power ratio $J/N_0 = d(A_s^2/2)/N_0 = 65$ dB-Hz.

![Fig. 4. Kurtosis blindness to pulsed sinusoidal RFI with $\mu = 0.1$ and 0.2.](image)

![Fig. 5. ROC metrics $C_{min}$ Versus the equivalent baseband bandwidth of the detector input filte for chirp interference with $W = 40$ MHz.](image)
Fig. 6. ROC metrics $C_{\min}$ versus the duty-cycle for pulsed sinusoidal interference with $J/N_0=65$ dB-Hz.

V. CONCLUSION

In this paper we proposed a new technique that is useful to determine the presence of narrowband (sinusoidal) or wideband (chirp) interference in GNSS signals. The rationale behind this technique is the fact that, in the absence of interference, the incoming signal is approximately Gaussian, as both the thermal noise and the ensemble of a large number of navigation signals are, in general, well described by Gaussian distributions. However, in the presence of sufficient strong interference, the distribution of the received signal is often non-Gaussian. The proposed blind detection method is based on the fourth-order autocumulant Giannakis-Tsatsanis Gaussianity test for real signals. With the new technique we can improve the ability to detect weaker interference just by increasing the computational effort, which is a desirable feature not found in other Gaussianity tests. Nevertheless, the trade-off sensitivity to interference versus computational effort is limited as the burden tends to grow faster than the sensitivity. Robustness to the blind-spot effect for wideband interference, and to a lesser degree for narrowband interference, which occurs with duty-cycles around $1/2$, constitutes an additional advantage over the kurtosis-based Gaussianity tests.

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