Adaptive GNSS Carrier Tracking Under Ionospheric Scintillation: Estimation vs. Mitigation

Jordi Vilà-Valls, Pau Closas, Carles Fernández-Prades, Jose A. López-Salcedo, and Gonzalo Seco-Granados

Abstract—This letter deals with carrier synchronization in Global Navigation Satellite Systems. The main goals are to design robust methods and to obtain accurate phase estimates under ionospheric scintillation conditions, being of paramount importance in safety critical applications and advanced receivers. Within this framework, the estimation versus mitigation paradigm is discussed together with a new adaptive Kalman filter-based carrier phase synchronization architecture that copes with signals corrupted by ionospheric scintillation. A key point is to model the time-varying correlated scintillation phase as an AR(p) process, which can be embedded into the filter formulation, avoiding possible loss of lock due to scintillation. Simulation results are provided to show the enhanced robustness and improved accuracy with respect to state-of-the-art techniques.

Index Terms—Carrier phase synchronization, robust/adaptive tracking, GNSS, ionospheric scintillation, Kalman filter.

I. INTRODUCTION

THE vast deployment of Global Navigation Satellite Systems (GNSS) receivers in personal electronic devices and new scientific/industrial applications are pushing the limits of traditional receiver architectures, which were initially designed to operate in clear sky, benign propagation conditions. In harsh propagation scenarios, signals may be affected by severe multipath, high dynamics or ionospheric scintillation. From a signal processing standpoint, the latter is the most challenging degradation effect due to the combination of deep fadings and rapid phase changes in a simultaneous and random manner, where the largest amplitude fades are usually associated with half-cycle phase jumps –the so-called canonical fades [1]. Estimation and mitigation of those undesired effects is of paramount importance for safety-critical applications, ionosphere characterization and advanced integrity receivers.

Conventional carrier synchronization architectures rely on traditional well established phase-locked loops (PLLs), which have been shown to deliver poor performances or even fail under severe propagation conditions [2] because of the noise

reduction versus dynamic trade-off (i.e., a small bandwidth is needed to filter out as much noise as possible, whereas a large bandwidth is needed to track high signal dynamics). Several improved PLL-based architectures have been proposed in the literature, but their limitations and performances have been clearly overcome by Kalman filter (KF)-based tracking techniques [2]–[4]. In general, their broad flexibility and improved accuracy is the reason why KFs are in the core of all the advanced carrier phase tracking techniques [5].

Considering the scintillation mitigation problem, the main drawback of KFs in their standard form is basically on the choice of the dynamic model, only taking into account the phase dynamics due to the relative movement between the satellite and the receiver. This leads to the *estimation versus mitigation paradigm* appearing in the current carrier tracking techniques: if a filter is well designed to cope with time-varying dynamic phase changes, it will not be able to mitigate undesired propagation effects such as scintillation phase variations. A possible way to resolve such a trade–off is to include the statistical knowledge about the propagation disturbances into the system model, therefore being able to decouple both contributions and mitigating such undesired effects. Moreover, standard KFs assume fully known system noise parameters (i.e., Gaussian noise covariances), what is not of practical interest in real-life applications. Consequently, adaptive approaches must be considered [6], [7].

In this contribution, *i*) a comprehensive scintillation phase modeling using a p$^{\text{th}}$ order autoregressive model (AR(p)) is given, *ii*) the estimation versus mitigation paradigm is discussed, and *iii*) a new adaptive KF is proposed for carrier synchronization and scintillation mitigation. Simulation results are provided to support the discussion and to show the improved accuracy. Note that this contribution generalizes the results presented in [8], [9], where a standard KF and an AR(1) model was used considering an almost static user scenario and fully known KF characterization, being of limited applicability for the advanced GNSS receivers of interest here.

II. IONOSPHERIC SCINTILLATION AR(P) MODELING

The purpose of this section is to fully analyze the AR(p) modeling of the scintillation phase time-series. The ionospheric scintillation, which is the disturbance caused by the propagation path through the ionosphere affecting the GNSS signals with amplitude fades and phase variations, can be modeled as a multiplicative

$$x(t) = \xi_s(t)s(t) + n(t); \quad \xi_s(t) = \rho_s(t)e^{j\theta_s(t)} \qquad (1)$$

where $s(t)$ and $x(t)$ are the complex-valued baseband equivalent of the transmitted and received signal, $n(t)$ is the noise term, $\xi_s(t)$ is the stochastic process representing the presence of

scintillation with the corresponding envelope and phase components, $\rho_s(t)$ and $\theta_s(t)$, respectively. The strength of amplitude scintillation is described by the so-called scintillation index S_4, which is usually considered within three main regions [10]: weak ($S_4 \le 0.3$), moderate ($0.3 < S_4 \le 0.6$) and strong/severe ($0.6 < S_4$) scintillation [10]. Recent experimental results in [10] show that a Ricean distribution can be used to model the envelope of this amplitude scintillation, while preserving a close fit with empirical data. Using this idea, a method to synthesize realistic scintillation time-series called Cornell Scintillation Model (CSM)[1] was introduced in [10], [11]. The CSM is very convenient for simulation purposes, and it is considered hereafter for realistic scintillation time-series generation where the two parameters to be specified are the scintillation intensity ($0 < S_4 \le 1$) and correlation ($0.1 \le \tau_0 < 2$). As a rule, higher S_4 and lower τ_0 lead to more severe scintillation.

The fact that the correlated scintillation phase can be fairly modeled as an AR model was introduced in [8], but the modeling/fitting was not analyzed and considered only the AR(1) case. From further testing, it was noticed that the model order p which best fits the scintillation time-series depends on the scintillation intensity S_4 and the intrinsic correlation τ_0. This is quite intuitive when going deep into the analysis of such scintillation time-series, because the lower the scintillation intensity, the more correlated and less nervous is the scintillation phase evolution, a behavior that can not be characterized by an AR(1) process. It is important to emphasize that the CSM does not intrinsically use an AR model to obtain the scintillation time-series.

An example of a scintillation phase AR(p) model approximation is given in Fig. 1 with $T_s = 10$ ms, where for the sake of completeness the three main scintillation intensity categories were considered: low (bottom figure - $\{S_4 = 0.3, \tau_0 = 1.2\}$), moderate (middle figure—$\{S_4 = 0.5, \tau_0 = 0.5\}$) and severe (top figure—$\{S_4 = 0.8, \tau_0 = 0.1\}$) scintillation. For each case, the scintillation phase empirical power spectral density (psd) is plotted together with the psd frequency response of the fitted AR(1), AR(2), and AR(3) models. It is clear that the severe scintillation case is well approximated by the AR(1), as already stated in [8], but the moderate and low scintillation scenarios are best approximated by an AR(2) and AR(3) models, respectively. This result is also valid for $T_s = 1$ and 4 ms.

Note that the general AR(p) model for a sequence z_k is formulated as $z_k = \sum_{i=1}^{p} \beta_i z_{k-i} + \eta_k$, where η_k is a white Gaussian noise sequence with variance σ_η^2. The set of p parameters β_i and the driving noise variance σ_η^2 can be easily computed from the Yule-Walker equations, using the autocorrelation function of the simulated CSM time-histories.

III. GNSS SIGNAL MODEL & NEW STATE-SPACE FORMULATION

Taking into account the problem at hand (i.e., carrier phase tracking under scintillation conditions), a simplified signal model with a perfect timing synchronization can be considered, at the input of the carrier tracking stage is given by

$$y_k = \alpha_k e^{j\theta_k} + n_k, \qquad (2)$$

[1]The CSM has been embedded in the so-called Cornell Scintillation Simulation Matlab toolkit, which is available at http://gps.ece.cornell.edu/tools.php.
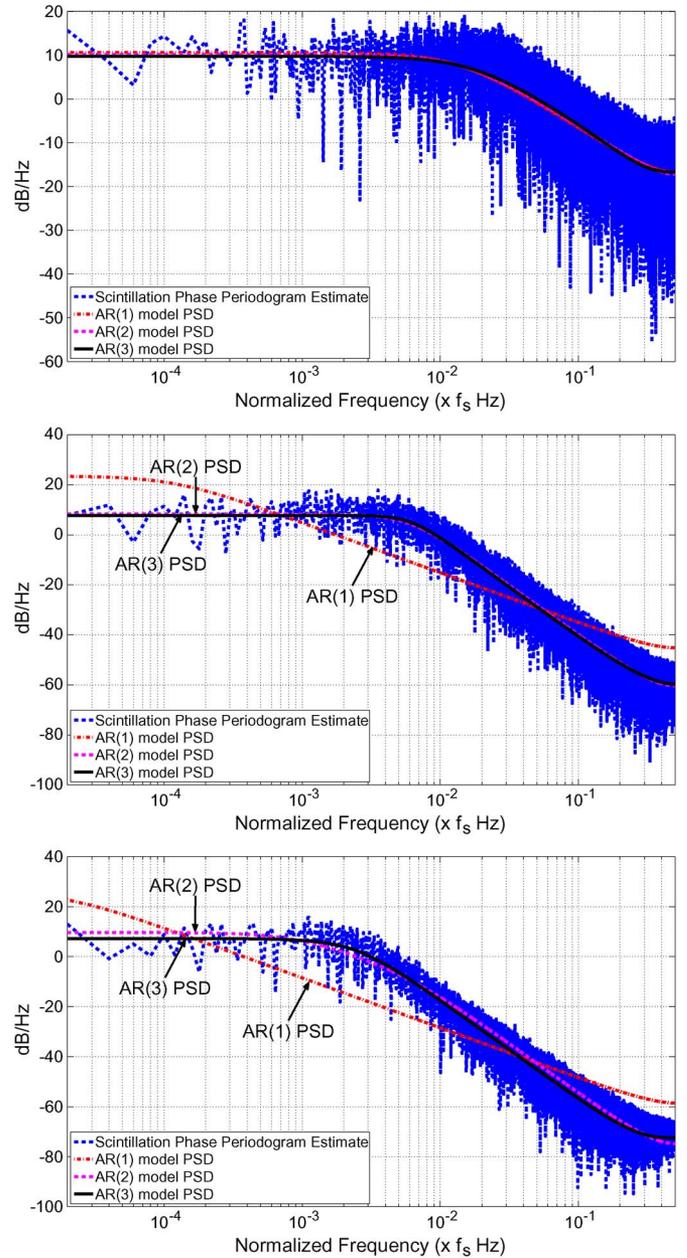
Fig. 1. Scintillation phase CSM time-series AR(p) approximation example. Severe (top) $\{S_4 = 0.8, \tau_0 = 0.1\}$, moderate (middle) $\{S_4 = 0.5, \tau_0 = 0.5\}$ and low (bottom) $\{S_4 = 0.3, \tau_0 = 1.2\}$ scintillation phase cases.

where k stands for the discrete time $t_k = kT_s$, A_k is the signal amplitude at the output of the correlators after accumulation over T_s, the amplitude α_k may include the scintillation amplitude effects, $\alpha_k = A_k \rho_{s,k}$; the carrier phase includes both the phase variations due to the receiver's dynamics, $\theta_{d,k}$, and the scintillation phase variation, $\theta_{s,k}$, $\theta_k = \theta_{d,k} + \theta_{s,k}$; and the Gaussian measurement noise is $n_k \sim \mathcal{N}(0, \sigma_{n,k}^2)$. Notice that both phase contributions, $\theta_{d,k}$ and $\theta_{s,k}$, are independent, which allows to build the following state-space model.

In standard KF architectures, the carrier phase is usually modeled using a Taylor approximation of the time-varying evolution caused by the receiver dynamics (i.e., $\theta_k = \theta_{d,k}$), where the order depends on the expected dynamics. Modern mass-market receivers implement a 3rd order PLL, which implicitly assume a 3rd order Taylor approximation of the phase

evolution, thus to obtain a fair comparison this is the case considered to construct the KF state-space model,

$$\theta_k = \theta_0 + 2\pi \left(f_{d,k} k T_s + \frac{1}{2} f_{r,k} k^2 T_s^2 \right), \tag{3}$$

where θ_0 (rad) is a random constant phase value, $f_{d,k}$ (Hz) the carrier Doppler frequency shift and $f_{r,k}$ (Hz/s) the Doppler frequency rate (i.e., the Doppler dynamics). This formulation can be used together with (2) to construct a state-space formulation of the carrier tracking problem, where the state to be tracked would be $\mathbf{x}_k^{(1)} \doteq [\theta_k \; f_{d,k} \; f_{r,k}]^T$. In this contribution, a key point is to take advantage of the AR(p) model approximation of the scintillation phase introduced in Section II, which can be written as $\theta_{s,k} = \sum_{i=1}^{p} \beta_i \theta_{s,k-i} + \eta_k$, with $\eta_k \sim \mathcal{N}(0, \sigma_\eta^2)$, and therefore can also be introduced into the state-space formulation as $\mathbf{x}_k^{(2)} \doteq [\theta_{d,k} \; f_{d,k} \; f_{r,k} \; \theta_{s,k} \; \cdots \theta_{s,k-p+1}]^T$. As already stated, this state-space formulation allows the filter to be aware of both phase contributions, being much more powerful than its standard version only taking into account the dynamics, $\theta_{d,k}$. The state evolution can be modeled as

$$\mathbf{x}_k^{(2)} = \underbrace{\begin{pmatrix} \mathbf{F}_d & \mathbf{0}_{3\times p} \\ \mathbf{0}_{p\times 3} & \mathbf{F}_s \end{pmatrix}}_{\mathbf{F}_k} \mathbf{x}_{k-1}^{(2)} + \mathbf{v}_k, \;\; \mathbf{v}k \sim \mathcal{N}(0, \mathbf{Q}k), \tag{4}$$

where the process noise $\mathbf{v}_k = [v_{1,k} \; v_{2,k} \; v_{3,k} \; \eta_k \; 0 \; 0]^\top$ stands for possible uncertainties or mismatches on the dynamic model, $\mathbf{Q}_k = \mathrm{diag}\left(\mathbf{Q}_{d,k}, \sigma_\eta^2, 0, \ldots, 0\right)$, and $\mathbf{Q}_{d,k}$ is usually *a priori* fixed according to the expected dynamics. The dynamics and scintillation transition matrices are

$$\mathbf{F}_d = \begin{pmatrix} 1 & 2\pi T_s & \pi T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{pmatrix}; \;\; \mathbf{F}_s = \begin{pmatrix} \beta_1 & \beta_2 & \ldots & \beta p \\ 1 & & & \\ & \ddots & & \\ & & 1 & \end{pmatrix}$$

Equations (2) and (4) define the new state-space formulation.

IV. NEW ADAPTIVE SCINTILLATION MITIGATION AR(P)-KF

From the state-space formulation given in the previous section it is almost straightforward to construct a standard KF for carrier tracking (i.e., using $\mathbf{x}_k^{(1)}$ in its standard form). Note that the observation equation is nonlinear, therefore a slight modification must be considered to apply the linear KF formulation. Emulating the traditional PLL architectures, a discriminator is used to obtain phase measurements and apply the linear KF equations. The noise variance of the linear phase model at the output of the discriminator, which is needed in the filter formulation, is no longer σ_n^2. An approximation of the phase noise variance for the ATAN discriminator is given by $\sigma_{n_\theta}^2 = \frac{1}{2C/N_0 T_s} \left(1 + \frac{1}{2C/N_0 T_s}\right)$ [rad^2].

The new architecture is derived from the standard KF. The state to be tracked is given by $\mathbf{x}_k^{(2)}$, using the augmented state-space formulation presented in the previous section. As already stated, the measurement noise variance must be perfectly characterized for the KF to be optimal formulation. In practice, this parameter is unknown to a certain extend and must be
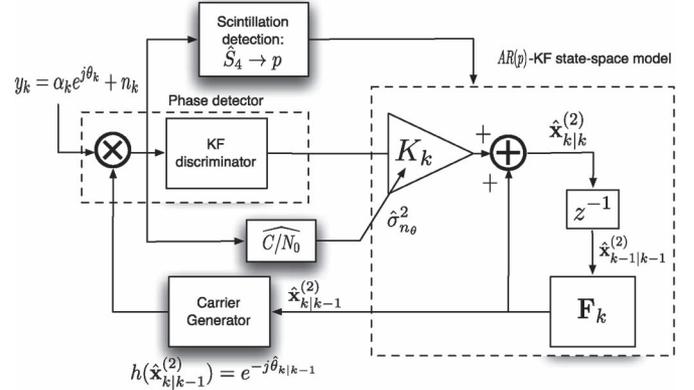
Fig. 2. Block diagram of the new adaptive AR(p)-KF approach.

somehow adjusted. In GNSS receivers, a C/N_0 estimator is always available and thus it is easy to obtain a sequential phase variance estimate at the output of the discriminator [6], [7], being straightforward to construct an adaptive KF (AKF). Moreover, from the incoming signal it is possible to estimate the scintillation intensity S_4 [12]. This scintillation detection (i.e., distinguishing between the three main scintillation regions) may be used to fix the AR order p, which directly affects the state-space formulation. The block diagram of the new architecture is sketched in Fig. 2.

V. COMPUTER SIMULATIONS AND DISCUSSION

In the interest of providing illustrative numerical results, the performance of the proposed method was analyzed with respect to current state-of-the-art techniques: a 3rd order PLL, a standard KF (tracking $\mathbf{x}_k^{(1)}$) and an AKF adjusting the measurement noise variance from the C/N_0 estimate. Two scenarios were simulated, the first one considering a signal corrupted by severe scintillation ($S_4 = 0.8$, $\tau_0 = 0.1$) and tracked with the AR(1)-KF, and a second one considering the moderate scintillation case ($S_4 = 0.5$, $\tau_0 = 0.5$) and using the AR(2)-KF. To show the method' robustness, both scenarios considered a very low $C/N_0 = 30$ dB-Hz , and a rapidly varying Doppler profile: $T_s = 10$ ms, initial Doppler $f_{d,0} = 50$ Hz and constant rate $f_{r,0} = 100$ Hz/s , which corresponds to an aeronautical user (acceleration = 20 m/s^2).

To support the discussion on the estimation versus mitigation paradigm, Fig. 3 plots for the first scenario (severe scintillation) one realization of the frequency estimation. The standard KF is tuned to be able to track the desired signal dynamics while being flexible enough to adapt to dynamic changes. Therefore, when the signal is corrupted by severe scintillation, the filter is not able to decouple both phase contributions and correctly tracks the complete phase. These scintillation fast phase variations are seen by the filter as frequency changes, as can be observed in Fig. 3. On the other hand, the AR(p)-KF is designed to decouple both phase evolutions, therefore the frequency state variable $f_{d,k}$ in $\mathbf{x}_k^{(2)}$ correctly tracks the signal frequency without being corrupted by the scintillation variations, which are gathered in $\theta_{s,k}$. Therefore, using standard techniques it is difficult to track the desired phase and frequency, and to mitigate undesired scintillation effects to avoid possible receivers' loss of lock, a problem which is solved by the new AR(p)-KF.
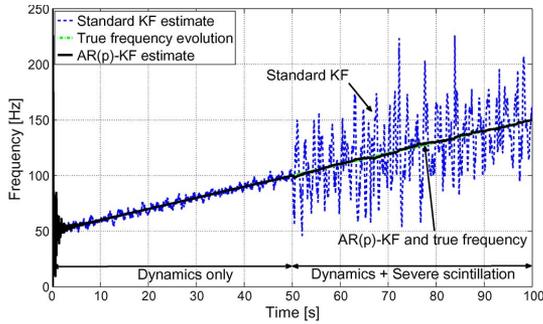
Fig. 3. One realization of the frequency estimation (severe scintillation scenario) given by the standard KF and the AR(p)-KF.
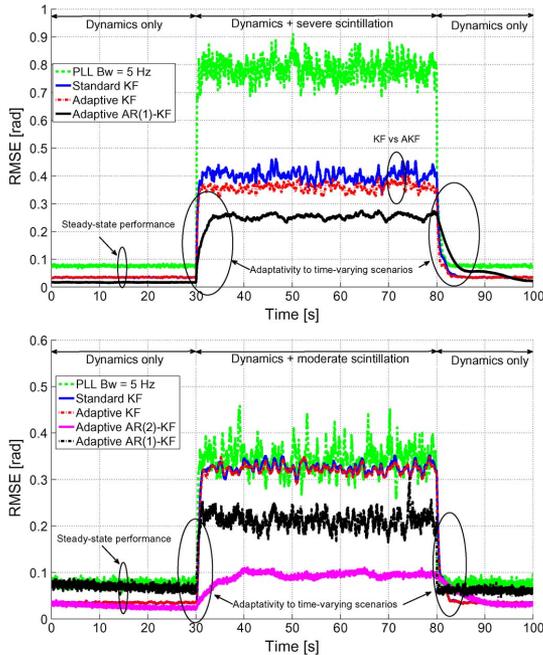
Fig. 4. RMSE on θ_d obtained for the first (top) and second (bottom) scenarios with: 3rd order PLL, standard KF, AKF and adaptive AR(p)-KF.

In pursuance of obtaining statistically significant results, the root mean square error (RMSE) on θ_d was used as a measure of performance and obtained from 500 Monte Carlo runs. The results obtained for the first scenario (severe scintillation) are given in Fig. 4 (top). The improved performance obtained with the new adaptive AR(p)-KF seems clear in both steady-state regime and under severe scintillation. While the steady-state performances (no scintillation) are given for all the techniques within the standard lock values, the main advantage of the proposed approach relies on its capability to deal with scintillation conditions. The loss-of-lock rule of thumb for the standard deviation is usually fixed to $\sigma = 0.52$ radians (i.e., $3\sigma = 90$ degrees). Therefore, whereas the AR(p)-KF is far from losing lock, the PLL is out of lock and the standard KF is on the limit of the comfort region. The only disadvantage of the AR(p)-KF is a slightly slower convergence in the transitory regions, which is not critical at all because the scintillation events are much longer and this is only a limit case (i.e., in realistic environments the scintillation intensity varies over time and does not reach its maximum suddenly). The AKF is slightly better than the standard KF, a result which proves that the good per-

formance of the AR(p)-KF is because of its formulation and not for the adaptive estimation of the measurement noise variance.

The results for the second scenario are shown in Fig. 4 (bottom). Again, the AR(p)-KF clearly outperforms the standard techniques, as shown in the previous severe scintillation scenario. For the sake of completeness, the results obtained with a wrong estimation of the AR model order ($p = 1$, AR(1)-KF) are shown together with the correct ones ($p = 2$, AR(2)-KF). Notice that under moderate scintillation the AR(2) modeling gives better performances than the AR(1), a result that supports the scintillation phase modeling introduced in Section II.

VI. CONCLUSION

This letter introduced a new approach for scintillation mitigation and robust carrier phase tracking in advanced GNSS receivers. The scintillation phase has been shown to be correctly modeled with an AR(p) process, thus being embedded into the KF formulation. This allows the filter to be aware of the different phase contributions, what leads to an effective and powerful scintillation mitigation solution, far beyond the performance obtained with standard techniques and being a promising approach for both commercial and scientific applications. The AR model order selection p is obtained from a scintillation intensity detection method, and the measurement noise variance is adaptively adjusted from a C/N_0 estimate. The performance of this approach was shown in scenarios with a rapidly varying Doppler profile and very low nominal C/N_0.

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