Available at
www.ElsevierComputerScience.com
POWERED BY SCIENCE DIRECT®

Signal Processing 86 (2006) 1848–1863

SIGNAL PROCESSING

www.elsevier.com/locate/sigpro

Spatial vs. multi-user diversity trade-offs for cross-layer scheduling in limited feedback systems

José López Vicario*,[1], Carles Antón-Haro

CTTC, PMT, AV. Canal Olímpic s/n, 08860 Castelldefells (Spain)

Received 3 January 2005; received in revised form 15 September 2005; accepted 15 September 2005
Available online 21 December 2005

Abstract

In this paper, we explore spatial vs. multi-user diversity trade-offs in a cellular system with or without bandwidth restrictions in the feedback channel. More precisely, we are interested in assessing the impact of both antenna selection and space–time coding strategies in such a multi-user system. The centralized scheduler makes an extensive use of physical layer measures and, hence, this constitutes a cross-layer design. Performance assessment is conducted both analytically and by means of computer simulations, in terms of pdf and CDF distributions, average system capacity and throughput. Closed-form expressions are derived for the SNR densities associated to the different transmission configurations along with an approximate expression for system throughput.

Keywords: Spatial diversity; Multi-user diversity; Antenna selection; Limited feedback

1. Introduction

In a wireless multi-user system, it is well known that the average cell throughput can be increased when in each slot the user with the best channel conditions is scheduled [1–3]. Such an effect is usually referred to as *multi-user diversity* (MUD) and relies on the assumption that different users experience independent fading processes. On the other hand, *spatial diversity* provides an effective means to combat fading and, thus, have a more reliable transmission. In particular, space–time block coding (STBC) is known to provide full diversity order schemes while using low complexity receivers [4,5]. Both schemes aimed at exploiting either spatial or multi-user diversity have been proposed for packet data services in 3G wireless networks. For that reason, much attention has been recently paid to their combined use and the associated trade-offs.

1.1. Previous work

In [6] the inclusion of STBC in multi-user scenarios was analyzed. It was shown that, in such a multi-user context, a single-input single-output (SISO)-based scheme outperforms its STBC counterpart. The reason for that is that STBC schemes are designed to reduce the probability of deep fades at the received SNR but, by averaging over different transmit diversity branches, SNR peaks are

*Corresponding author. CTTC, PMT, Av. Canal Olímpic s/n, Rm. 108, 08860 Barcelona, Spain. Tel.: +34 936 452 918; fax: +34 936 452 901.
E-mail address: jose.vicario@cttc.es (J.L. Vicario).
[1]This work was partially funded by the European Commission under projects IST-2002-507525 NEWCOM, IST-2002-508009 ACE; and the Catalan Government (DURSI) 2005FI 00003.

0165-1684/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2005.09.028

suppressed too. However, in multi-user systems with delayed feedback, the increased robustness of STBC schemes against fading may provide substantial gains with respect to those of SISO approaches [7,8].

Besides, it was proven in [9] that with perfect channel state information (CSI) at the transmitter, spatial diversity can be efficiently exploited in a multi-user context via optimal transmit beamforming. To do so, though, a high-rate return channel is required since in principle all the channel gains (amplitude and phase) must be fedback to the scheduler. Alternatively, a transmit antenna selection scheme where transmit power is concentrated in the antenna with the largest channel gain was also proposed. For this second approach, only a limited amount of CSI must be conveyed to the transmitter and, consequently, a low-rate signalling channel suffices. However, gains resulting from spatial processing decrease as well. Besides, some studies on the combination of STBC with antenna selection (STBC-AS) mechanisms for the *single-user case* can be found in [10]. In particular, antenna selection (AS) strategies were introduced as a means to enhance diversity order at the expense of a very limited increase in terms of feedback load, computational complexity and cost. Indeed, it was proven that the diversity order obtained through AS is identical to that of a situation with all the antennas in use, but requiring a lower number of complete RF-chains.

Selective multi-user diversity (SMUD) concepts were first introduced by Gesbert et al. in [11]. By exclusively letting users report channel state information when channel quality exceeds a pre-defined threshold (i.e. when there is a chance for that user to be eventually scheduled), remarkable benefits can be obtained in terms of reduced load in the feedback channel at the expense of moderate performance losses. Clearly selective MUD is better exploited in systems with a large number of active users. As an example, it was shown in [11] that feedback load can be reduced by a factor of 10 when the number of users is $K \geqslant 20$.

1.2. Contributions

In this paper, we explore the use of STBC schemes in combination *with* transmit antenna selection [10], in a *multi-user scenario* with proportional fair scheduling [11]. In doing so, we also take into consideration signalling issues and, hence, we consider two different scenarios, consisting in systems with either *unlimited* or *limited* feedback channels. By unlimited feedback, we mean a system where *all* users permanently monitor and report a pre-defined channel quality indicator to a centralized scheduler over an error- and delay-free return channel. Conversely, in a limited-feedback case only a subset of the active users is allowed to report the continuously-monitored quality measures. For this second case, we therefore, extend the analysis conducted by Gesbert et al. in [11] on Selective MUD schemes to encompass STBC and antenna selection mechanisms. The rationale behind the combination of STBC, AS and Selective MUD lies in the fact that, as the number of active users increases, their individual contribution to feedback channels should be necessarily decreased. In those conditions, chances are higher for the centralized scheduler in the BS to be forced to make a blind decision (i.e. when no mobile station conveys CSI to the scheduler for a specific time slot). The spatial diversity provided by AS and STBC could then result in an increased robustness against deep fades associated to such random user picks.

As far as performance assessment is concerned, we will not restrict ourselves to mainstream average system capacity measures but will also conduct a throughput analysis. For the throughput curves, we will obtain an approximate lower bound analytically and, to do so, an exact expression for the pdf of the post-scheduling signal-to-noise ratio will be derived first. To the best of authors' knowledge, this has not been done before for a STBC-AS configuration.

Throughout the paper, performance results are compared with those of a baseline case consisting in a SISO scheme with or without antenna selection.

1.3. Organization

This paper is organized as follows: in Section 2, the corresponding signal and system models are presented. A short description of the selected transmission schemes and the associated scheduling algorithms are provided as well. Closed-form expressions for the density functions (pdf and CDF) of the pre-scheduling SNRs are then derived in Section 3. Next, the different transmission schemes are analyzed and compared for both the unlimited and limited feedback scenarios in Sections 4 and 5, respectively. To do that, closed-form expressions for the post-scheduling SNR are obtained first and, then, some simulation results for the measures of interest, average capacity and system throughput, are presented. Along with that, an approximate lower bound is analytically derived for the system throughput. Finally, in Section 6, the conclusions of this work are presented along with some suggestions for future work in this field.

2. Signal and system model

2.1. Signal model

Consider the downlink of a cellular system with one base station (BS) equipped with multiple antennas (N_{BS}), and K single-antenna ($N_{\mathrm{MS}}=1$) Mobile Stations (MS). The received signal at the kth MS is given by

$$r_k = \mathbf{h}_k^T \mathbf{s} + n_k, \tag{1}$$

where the time index has been dropped for the ease of notation, $\mathbf{h}_k \in \mathbb{C}^{N_{\mathrm{BS}}}$ is the channel vector gain between the BS and the kth MS, for which each component is assumed to be independent and identically distributed, circularly symmetric Gaussian random variable with zero mean and unit variance ($\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_{\mathrm{BS}}})$), $\mathbf{s} \in \mathbb{C}^{N_{\mathrm{BS}}}$ is the symbol vector broadcasted from the BS and $n_k \in \mathbb{C}$ denotes additive Gaussian noise (AWGN) with zero mean and variance σ^2. The active users in the system are assumed to undergo independent Rayleigh fading processes and so does the signal being transmitted from different antennas in the BS. Further, we consider quasi-static fading, i.e., the channel response remains constant during one time-slot and it changes to a new independent realization in the subsequent one. Concerning channel state information (CSI), we assume perfect CSI knowledge for *each* user at the receive side, and the availability of a low-rate error-free feedback channel to convey partial CSI to the transmitter. Finally, we denote by γ_k the instantaneous signal-to-noise ratio experienced by user k during time-slot s and by $\bar{\gamma}_k = P_{\mathrm{t}}/\sigma^2$ its average SNR, with P_{t} standing for the total transmitted power, which is constant and evenly distributed among transmit antennas.

2.2. Transmission schemes

As far as the transmission schemes are concerned, we will consider four different cases:

- *SISO* (*single-input*, *single-output*):

This constitutes the first baseline case where the BS uses one single antenna at all the times. In this situation, $N_{\mathrm{TX}} = N_{\mathrm{BS}} = 1$ where N_{TX} stands for the number of *active* antennas at the BS. Consequently, the received SNR for user k becomes

$$\gamma_{k,\mathrm{SISO}} = \bar{\gamma}_k |h_{1,k}|^2. \tag{2}$$

- *SISO-AS* (*SISO with antenna selection*):

In this scheme, the *best* antenna ($N_{\mathrm{TX}}=1$) out of the $N_{\mathrm{BS}} > N_{\mathrm{TX}}$ antennas available in the BS will be selected for data transmission, more precisely, the one that maximizes the received SNR for user k

$$\gamma_{k,\mathrm{SISO\text{-}AS}} = \bar{\gamma}_k \max_{1 \leqslant i \leqslant N_{\mathrm{BS}}} \{|h_{i,k}|^2\}. \tag{3}$$

This approach requires the antenna index to be fedback along with SNR information. For that purpose, $\log_2 N_{\mathrm{BS}}$ additional signalling bits are needed.

- *STBC* (*space–time block coding*),

Where a two-antenna Alamouti scheme is adopted for transmission ($N_{\mathrm{TX}} = N_{\mathrm{BS}} = 2$). In this case, the received SNR for user k turns out to be

$$\gamma_{k,\mathrm{STBC}} = \frac{\bar{\gamma}_k}{2}(|h_{1,k}|^2 + |h_{2,k}|^2) = \frac{\bar{\gamma}_k}{2}\,|h_k|^2 \tag{4}$$

since power is evenly allocated to transmit antennas.

- *STBC-AS* (*STBC with antenna selection*):

Now, the antenna subset with two out of the N_{BS} antennas available in the BS that maximizes the received SNR will be chosen and thus we have

$$\gamma_{k,\mathrm{STBC-AS}} = \frac{\bar{\gamma}_k}{2} \max_{1 \leqslant v \leqslant V} \{|h_k^{(v)}|^2\}, \tag{5}$$

where superscript v is an index to antenna subsets. As happened in the *SISO-AS* case, a number of additional signalling bits equal to $\log_2 V = \log_2 \binom{N_{\mathrm{BS}}}{N_{\mathrm{TX}}}$ are needed in order to convey such antenna subset index over the feedback channel (with $N_{\mathrm{TX}} = 2$).

2.3. Centralized scheduler

At the base station, the scheduling process is organized in a slot-by-slot basis following a proportional fair scheduling (PFS) rule [12]. Throughout this work, we will assume identical average SNRs for all the active users ($\bar{\gamma} = \bar{\gamma}_k$) and a large enough scheduling time scale, this turning the

PFS scheme into a *max-SNR* (*greedy*) scheduler. That is, at time-slot s the scheduler picks the active user $k^*(s)$ satisfying

$$k^*(s) = \arg\max_k\{\gamma_1(s), \ldots, \gamma_k(s), \ldots, \gamma_K(s)\}.$$

Note that such scheduler requires all the active users to continuously monitor and report over the feedback channel the measured SNR and/or the optimal transmit antenna subset.

In order to reduce bandwidth requirements in the feedback channel, a *selective* multi-user diversity (SMUD) approach can be adopted instead [11]. In this situation, only users experiencing SNRs above a pre-defined threshold (γ_{th}) will report their channel state information to the BS. Thus, the max-SNR scheduler will conduct the search over such a subset of the active users only, that is,

$$k^*(s) = \arg\max_k\{\gamma_k(s) \text{ s.t. } \gamma_k > \gamma_{\text{th}}\}.$$

Conversely, when all the users remain silent (i.e. in the case of a *scheduling outage*) the scheduling rule amounts to:

$$k^*(s) = \text{rand}\{1, \ldots, k, \ldots, K\}$$

that is, a random user is selected for transmission. In the sequel, subscript s will be dropped for the ease of notation.

3. Pre-scheduling SNR statistics

In this section, we are interested in deriving analytical expressions for the statistics (pdf and CDF) of the pre-scheduling SNRs. We define *pre-scheduling* signal-to-noise-ratio *for user* k as the SNR that is measured and reported by such user to the base station (the SNR expressions for the different transmission schemes can be found in Eqs. (2)–(5) above).[1]

Certainly, the probability density function of the pre-scheduling SNR, $f_\gamma(\gamma)$, will strongly depend on the transmission scheme but, conversely, the cumulative density function unequivocally relates to the pdf through

$$F_{\gamma'}(\gamma) = \text{Prob } (\gamma' \leqslant \gamma) = \int_0^\gamma f_{\gamma'}(\gamma')\,\mathrm{d}\gamma'. \tag{6}$$

[1]On the contrary, we define *post-scheduling* SNR as the signal-to-noise ratio experienced by the scheduled user.

A case-by-case analysis follows (subscript k has been dropped for the ease of notation):

- *SISO*:

Since a Rayleigh fading case is assumed, the received SNR for user k is a chi-square random variable with two degrees of freedom, χ_2^2. Hence, the pdf and CDF take the following expressions:

$$f_{\gamma_{\text{SISO}}}(\gamma) = \frac{1}{\bar{\gamma}}\,\mathrm{e}^{-\gamma/\bar{\gamma}}, \quad F_{\gamma_{\text{SISO}}}(\gamma) = 1 - \mathrm{e}^{-\gamma/\bar{\gamma}}. \tag{7}$$

- *SISO-AS*:

Bearing in mind that channel coefficients corresponding to different BS antennas fade independently from each other, it becomes apparent that

$$\begin{aligned} F_{\gamma_{SISO\text{-}AS}}(\gamma) &= (F_{\gamma_{SISO}}(\gamma))^{N_{\text{BS}}} = (1 - \mathrm{e}^{-\gamma/\bar{\gamma}})^{N_{\text{BS}}}, \\ f_{\gamma_{SISO\text{-}AS}}(\gamma) &= \partial F_{\gamma_{\text{SISO-AS}}}(\gamma)/\partial\gamma \\ &= \frac{N_{\text{BS}}}{\bar{\gamma}}\mathrm{e}^{-\gamma/\bar{\gamma}}(1 - \mathrm{e}^{-\gamma/\bar{\gamma}})^{N_{\text{BS}}-1} \end{aligned} \tag{8}$$

- *STBC*:

Now, the signal-to-noise ratio becomes a chi-square random variable with $2N_{\text{TX}}$ degrees of freedom, $\chi^2_{2N_{\text{TX}}}$. For the two-antenna Alamouti scheme, we have

$$f_{\gamma_{\text{STBC}}}(\gamma) = \frac{4\gamma}{\bar{\gamma}^2}\,\mathrm{e}^{-2\gamma/\bar{\gamma}}, \tag{9}$$

$$F_{\gamma_{\text{STBC}}}(\gamma) = 1 - \mathrm{e}^{-2\gamma/\bar{\gamma}}\left(\frac{2\gamma}{\bar{\gamma}} + 1\right). \tag{10}$$

- *STBC-AS*:

In the proposed scheme, the best two out of N_{BS} BS antennas will be selected for data transmission. In the sequel, we will restrict ourselves to the case $N_{\text{BS}} = 3$ in order to keep analytical derivations tractable (in particular for expressions derived in Section 5). Since the search over the available antenna subsets is aimed at maximizing the received SNR, the resulting problem can be readily solved by means of the so-called *order statistics* [13]. By arranging the (squared) channel gains for each antenna subset in an increasing order ($X_1 \leqslant X_2 \leqslant X_3$), it is clear that the resulting SNR is maximized when the highest and second-highest elements are selected, that is

$$F_{\gamma_{\text{STBC-AS}}}(\gamma) = \text{Prob}\left(\max_{1\leqslant i,j\neq i\leqslant N_{\text{BS}}}\left\{\frac{\bar{\gamma}}{2}(X_i+X_j)\right\}\leqslant\gamma\right)$$
$$\underset{N_{\text{BS}}=3}{=} \text{Prob}\left((X_2+X_3)\leqslant\frac{2\gamma}{\bar{\gamma}}\right). \quad (11)$$

The joint density function of the (individually χ_2^2-distributed) *ordered* random variables X_2 and X_3 is given by

$$f_{X_3X_2}(x_3,x_2) = \begin{cases} 3!(1-\mathrm{e}^{-x_2})\mathrm{e}^{-(x_2+x_3)} & \text{for } x_2\leqslant x_3, \\ 0 & \text{otherwise} \end{cases}$$

and from that we can obtain the corresponding closed-form expression for the CDF

$$F_{\gamma_{\text{STBC-AS}}}(\gamma) = \int_{x_3=0}^{\gamma/\bar{\gamma}}\int_{x_2=0}^{x_3} f_{X_3X_2}(x_3,x_2)\,\mathrm{d}x_3\,\mathrm{d}x_2 + \int_{x_3=\gamma/\bar{\gamma}}^{2\gamma/\bar{\gamma}}\int_{x_2=0}^{2\gamma/\bar{\gamma}-x_3} f_{X_3X_2}(x_3,x_2)\,\mathrm{d}x_3\,\mathrm{d}x_2$$
$$= 1-\mathrm{e}^{-2\gamma/\bar{\gamma}}\left(\frac{6\gamma}{\bar{\gamma}}+4\mathrm{e}^{-\gamma/\bar{\gamma}}-3\right). \quad (12)$$

In Fig. 1, we depict both the pdf and CDF of the pre-scheduling SNR for the different transmission schemes. When comparing the curves for the SISO and STBC schemes, one can observe that STBC substantially *stabilizes* the SNR distribution. This means that the probability that a deep fade occurs decreases but, unfortunately, the number of SNR peaks (those that multi-user diversity can exploit) is reduced, as well. However, this undesirable effect can be compensated by introducing antenna selection mechanisms (STBC-AS) which partly restore the *missing* SNR peaks (for $\gamma\geqslant 14\,\text{dB}$) and, simultaneously, further suppress deep fades. As a result, the effective SNR experienced by users in the system increases. Besides, one can also observe that a SISO-AS configuration with $N_{\text{BS}}=2$ (i.e. curve labelled with SISO-AS2) performs better than the STBC-AS scheme in terms of SNR peaks generation. However, its capability of mitigating fades is far more limited (see crossing of CDF curves around $\gamma=10\,\text{dB}$), which can be improved by increasing the number of transmit antennas to $N_{\text{BS}}=3$ (SISO-AS3 configuration).

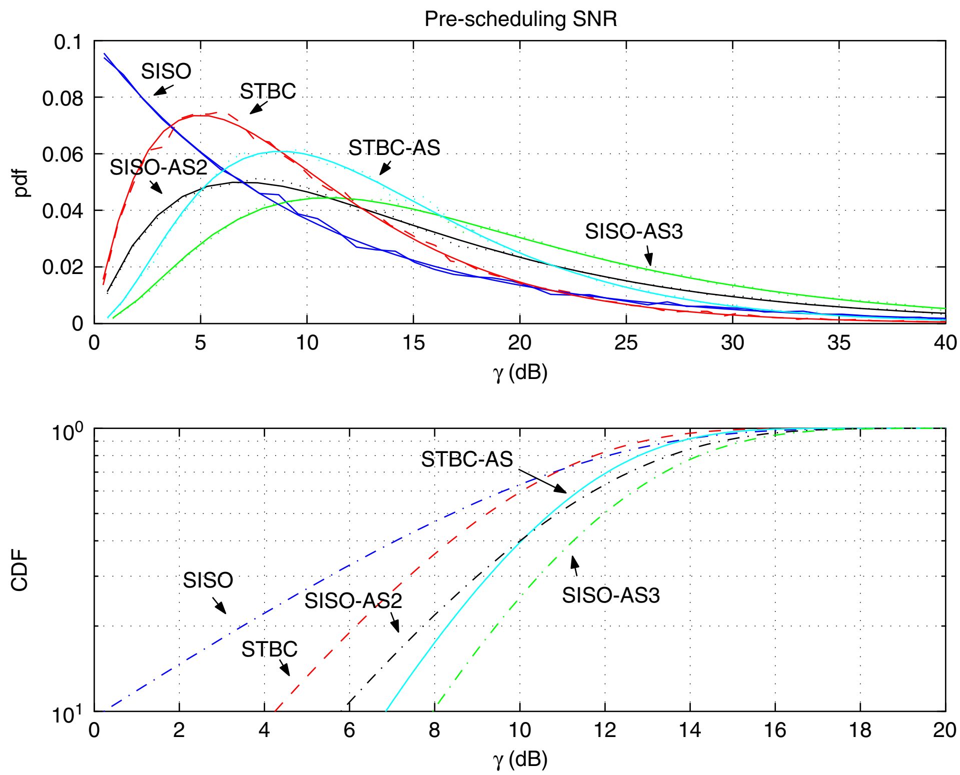

Fig. 1. Top: Analytical and simulated curves for the probability density functions of the pre-scheduling SNRs. Bottom: Cumulative density functions in log-scale. ($\bar{\gamma}=10\,\text{dB}$).

In summary, antenna selection strategies provide an effective means to shape the pre-scheduling SNR statistics exhibited by the STBC and SISO schemes in terms of fade occurrence and SNR peak generation. However, one should bear in mind that it measures like average system capacity or throughput (and not SNR distributions) that provide a more accurate view on the actual performance. Besides, we have not yet come to the point of analyzing robustness issues resulting from bandwidth constraints in the feedback channels. The following sections are devoted to conduct such an analysis.

4. Unlimited feedback scenario

In this section, we analyze and compare performance for the different transmission schemes in a multi-user scenario with unlimited feedback. As explained above, by unlimited feedback scenario we mean a system where *all* users permanently monitor and report a pre-defined channel quality indicator, such as the *pre-scheduling* SNR, to a centralized scheduler over a (error- and delay-free) return channel. Ultimately, we aim at conducting a capacity and throughput analysis but, to do so, we first have to obtain the closed-form expressions of the *post-scheduling* SNRs.

4.1. Post-scheduling SNR

Assuming that all users experience i.i.d Rayleigh fading, the CDF of the *post-scheduling* SNR, i.e. the SNR experienced by the scheduled user, $F_{\gamma^*}(\gamma)$, can be readily expressed in terms of the *pre-scheduling* CDFs, $F_\gamma(\gamma)$, as

$$\begin{aligned} F_{\gamma^*}(\gamma) &= \mathrm{Prob}(\gamma^* \leqslant \gamma) = \mathrm{Prob}(\max\{\gamma_1, \ldots, \gamma_K\} \leqslant \gamma) \\ &= \mathrm{Prob}(\gamma_1 \leqslant \gamma, \ldots, \gamma_K \leqslant \gamma) = (F_\gamma(\gamma))^K, \end{aligned} \tag{13}$$

where index s was dropped for brevity. By differentiating with respect to γ, the corresponding pdf expressions can be found

$$f_{\gamma^*}(\gamma) = K(F_\gamma(\gamma))^{K-1} f_\gamma(\gamma),$$

where $F_\gamma(\gamma)$ in the above equations for the different transmission schemes can be found in Eqs. (7)–(12). In summary, the closed-form expressions for the pdf of the post-scheduling SNRs are

$$f_{\gamma^*_{\mathrm{SISO}}}(\gamma) = K\,\frac{\mathrm{e}^{-\gamma/\bar{\gamma}}}{\bar{\gamma}}(1-\mathrm{e}^{-\gamma/\bar{\gamma}})^{K-1}, \tag{14}$$

$$f_{\gamma^*_{\mathrm{SISO\text{-}AS^*}}}(\gamma) = KN_{\mathrm{BS}}\,\frac{\mathrm{e}^{-\gamma/\bar{\gamma}}}{\bar{\gamma}}(1-\mathrm{e}^{-\gamma/\bar{\gamma}})^{KN_{\mathrm{BS}}-1}, \tag{15}$$

$$f_{\gamma^*_{\mathrm{STBC}}}(\gamma) = K\,\frac{4\gamma}{\bar{\gamma}^2}\mathrm{e}^{-2\gamma/\bar{\gamma}}\left(1-\mathrm{e}^{-2\gamma/\bar{\gamma}}\left(\frac{2\gamma}{\bar{\gamma}}+1\right)\right)^{K-1}, \tag{16}$$

$$\begin{aligned} f_{\gamma^*_{\mathrm{STBC\text{-}AS^*}}}(\gamma) = {} & K\,\frac{12}{\bar{\gamma}}\,\mathrm{e}^{-2\gamma/\bar{\gamma}}\left(\mathrm{e}^{-\gamma/\bar{\gamma}}+\frac{\gamma}{\bar{\gamma}}-1\right) \\ & \times\left(1-\mathrm{e}^{-2\gamma/\bar{\gamma}}\left(\frac{6\gamma}{\bar{\gamma}}+4\mathrm{e}^{-\gamma/\bar{\gamma}}-3\right)\right)^{K-1}. \end{aligned} \tag{17}$$

Fig. 2 shows the CDF function of the post-scheduling SNR in a system with $K=1$, 5 and 50 users. Note that the case with $K=1$ users is essentially equivalent to that of *round-robin* scheduling, i.e., letting K users take turns in utilizing the channel, regardless of their SNR (this featuring a SNR density function identical to that of the pre-scheduling case). From the relative ordering in each group of curves, it becomes clear that in a STBC-AS scheme the additional *spatial* diversity provided by the antenna selection mechanism is better exploited in a scenario with a low to moderate number of users ($K=1,5$). The approximate gain with respect to the SISO configuration turns out to be 4 and 2 dB, respectively. Conversely, as the number of users increases ($K=50$) the beneficial effect of multi-user diversity on a SISO scheme exceeds spatial diversity gains.

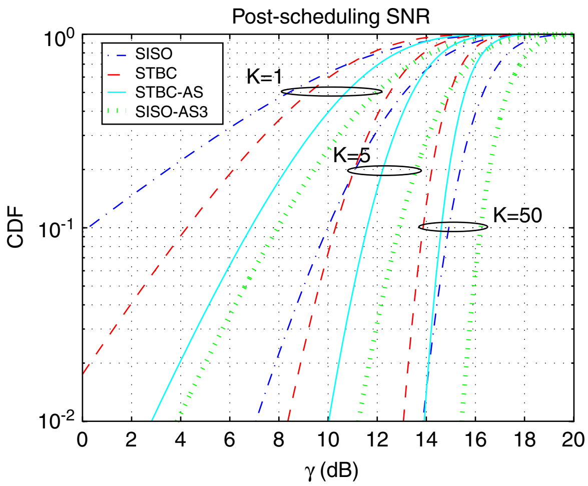

Fig. 2. CDF of the received SNR for the different transmission schemes, for K = 1,5,50 users (left to right, $\bar{\gamma} = 10$ dB).

Apart from that, one can also observe that the SISO-AS3 scheme is far more successful than its STBC-AS counterpart in shifting the SNR curves towards higher values. In other words, the joint exploitation of MU diversity and the spatial diversity component provided by the antenna selection mechanisms is more effectively done when no space–time block coding scheme is used. Actually, the introduction of antenna selection mechanisms is often interpreted as an increase in the number of active users since, in the end, SNRs are measured for a higher number of antenna pairs. In the STBC-AS case, though, the SNRs for the different antenna subsets are correlated due to the fact that some antenna subsets have one or more elements in common. As a result the effective increase in the number of users is less than V-fold (to recall, $V = \binom{N_{\mathrm{BS}}}{N_{\mathrm{TX}}}$ is the number of antenna subsets).

4.2. Capacity and throughput analysis

So far, the different transmission schemes have been compared in terms of post-scheduling SNR densities only. For that reason a complementary study addressing system capacity and throughput aspects becomes necessary. The Shannon capacity achievable by the scheduled user k^* over the equivalent single-input single-output channel is given by

$$C^* = \log_2(1+\gamma^*)$$

and, consequently, the *average* system capacity achievable with a max-SNR scheduling policy can be expressed as

$$\overline{C} = \mathrm{E}[C^*] = \int_0^\infty \log_2(1+\gamma) f_{\gamma^*}(\gamma)\,\mathrm{d}\gamma.$$

By plugging expressions (14)–(17) into the above equation, one can obtain the corresponding capacities for the SISO, SISO-AS, STBC and STBC-AS schemes. This can be easily done for the SISO case with or without antenna selection (see for instance [11]) but in the remaining cases, resorting to numerical integration is mandatory.

Nonetheless, capacity measures simply provide a rough idea on how spectrally efficient the system can be when an *infinite* number of modulation and coding schemes are available. In practical cases, though, the number of AMC schemes can be rather limited (ex: two modulation schemes in HSDPA, QPSK and 16-QAM). In that scenario, it is also worth investigating to what extent can MU diversity and spatial diversity be jointly exploited. In particular, we will restrict ourselves to consider a *single* modulation scheme in the PHY layer and, then, assess the associated average system throughput. In this context, the (post-scheduling) average throughput can be expressed as

$$\eta^* = b\cdot(1-\mathrm{PER}(\gamma^*)) = b\cdot(1-\mathrm{SER}(\gamma^*))^L,$$

$$\bar{\eta} = \mathrm{E}[\eta^*] = b\int_0^\infty (1-\mathrm{SER}(\gamma))^L f_{\gamma^*}(\gamma)\,\mathrm{d}\gamma, \tag{18}$$

where L stands for the number of symbols in the transmitted burst and b is the number of bits per symbol. With the accustomed bound for the SER [14], the expression above is barely integrable. Alternatively, we will adopt the approximation for M-QAM modulation schemes presented in [15]

$$\mathrm{SER}(\gamma) \approx b0.2\mathrm{e}^{-1.6\gamma/2^b-1} = \alpha\mathrm{e}^{-\beta\gamma}. \tag{19}$$

By substituting Eqs. (14)–(17) and (19) into (18), using the binomial expansion, integrating by parts, and equation [16], Eq. 3.351.3 ($\int_0^\infty \mathrm{e}^{-\mu t}t^n\,\mathrm{d}t = n!\mu^{-n-1}$) one can derive the following closed-form expressions:

$$\bar{\eta}_{\mathrm{SISO}}(K) \approx Kb\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\sum_{l=0}^{L}\binom{L}{l}\frac{(-\alpha)^l}{\bar{\gamma}\beta l+1} \tag{20}$$

$$\bar{\eta}_{\mathrm{SISO\text{-}AS}}(K) \approx \bar{\eta}_{\mathrm{SISO}}(KN_{\mathrm{BS}}) \tag{21}$$

$$\bar{\eta}_{\mathrm{STBC}}(K) \approx 4Kb\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\sum_{l=0}^{L}\binom{L}{l}(-\alpha)^l \times\sum_{n=0}^{k}\binom{k}{n}\frac{2^n(n+1)!}{(\bar{\gamma}\beta l+2k+2)^{n+2}} \tag{22}$$

$$\begin{aligned}\bar{\eta}_{\mathrm{STBC\text{-}AS}}(K) \approx{}& 12Kb\sum_{k=0}^{K-1}\binom{K-1}{k}3^k\sum_{l=0}^{L}\binom{L}{l}(-\alpha)^l \\ &\times\sum_{n=0}^{k}\binom{k}{n}\left(-\frac{1}{3}\right)^n\sum_{s=0}^{n}\frac{6^{n-s}4^s n!}{s!} \\ &\times\left[\frac{1}{(\bar{\gamma}\beta l+2k+s+3)^{n-s+1}}+\frac{1}{(\bar{\gamma}\beta l+2k+s+2)^{n-s+1}}\right. \\ &\left.\times\left(\frac{n-s+1}{\bar{\gamma}\beta l+2k+s+2}-1\right)\right]\end{aligned} \tag{23}$$

namely, the average system throughput for the different transmission schemes.

4.3. Computer simulation results and discussion

Throughout this section, we will consider a system with a number of active users in the range $K = 1\ldots30$ transmitting data packets with $L = 50$ QPSK symbols in each. The average SNR is set to $\bar{\gamma} = 10\,\mathrm{dB}$.

In Fig. 3, we plot the average system *capacity* as a function of the number of active users. One can observe that STBC systems without antenna selection mechanisms exclusively outperform SISO schemes in the $K = 1$ case, that is, when multi-user diversity cannot be exploited at all. Conversely, in the presence of MU diversity ($K \geqslant 2$) the spatial diversity component associated to the STBC scheme penalizes (rather than boosts) system performance. However, the system exhibits a different behavior when an additional diversity component is introduced via antenna selection mechanisms. On the one hand, the STBC-AS scheme efficiently combines antenna diversity and multi-user diversity and, thus, performs better than the SISO approach in the $K = 1\ldots8$ users range. This can be very beneficial, for instance, in the early phases of deployment of packet data systems or, perhaps, in scarcely populated areas where the number of users per cell is potentially low. On the other hand, a remarkable shift can be observed between STBC-AS and STBC curves for the whole range of users. Due to correlation effects among antenna subsets, the effective increase in the number of users is a factor of 2 (e.g. 5 vs. 10 users @ $C = 4.5\,\mathrm{bits/s/Hz}$) whereas the number of subsets is actually $V = 3$. As for the comparison between the SISO-AS and STBC-AS schemes in terms of channel capacity, the former outperforms the latter for the whole range of users (as pointed out in the SNR analysis above).However, this analysis has been conducted in a rather idealized scenario: with an infinite number of AMC levels and for an error- and delay-free infinite-bandwidth feedback channel. The throughput analysis to follow will bring things closer to the real world.

The average system *throughput* curves depicted in Fig. 4 show that, for a given modulation scheme (QPSK), MU diversity and spatial diversity can still be effectively combined in the STBC-AS scheme, in particular for a low to moderate number of users. As opposed to channel capacity, throughput curves for SISO-AS3 exhibit slightly better performance with respect to the STBC-AS approach. On the other hand, STBC outperforms SISO for the whole range of users, the reason for that is that diversity gains associated to STBC (in terms of reduced BER due to an increased diversity order) clearly exceeds those of multi-user diversity (lower BER resulting from instantaneous SNR peaks). In summary, for a given modulation scheme (and, possibly, for a reduced set of them where bit cap and granularity effects play a dominant role [17]), the trade-offs among transmission schemes, MU diversity and antenna selection mechanisms must be carefully established.

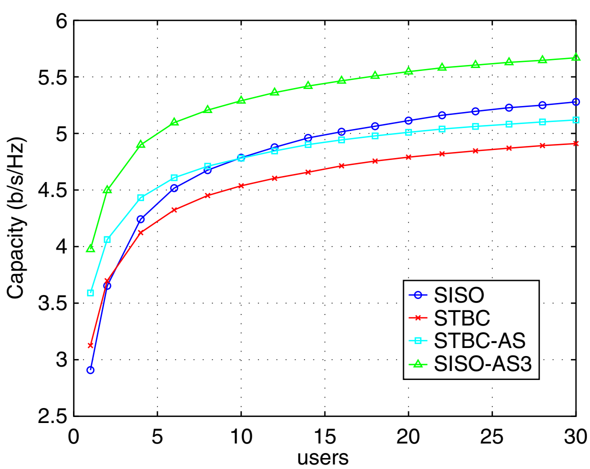

Fig. 3. Channel capacity vs. the number of active users for the different transmission schemes in an unlimited feedback scenario ($\bar{\gamma} = 10\,\mathrm{dB}$).

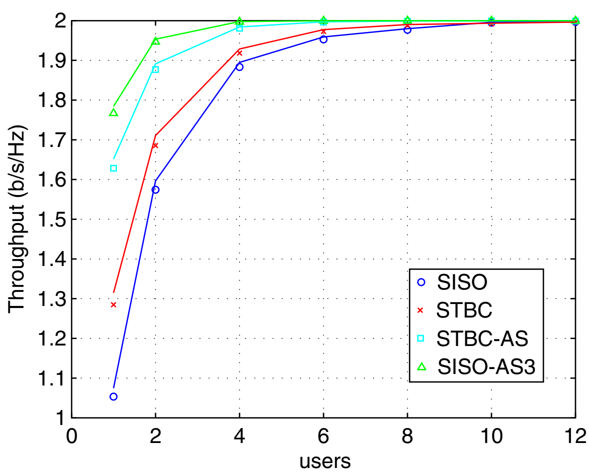

Fig. 4. Link throughput vs. the number of users for the different transmission schemes in an unlimited feedback scenario ($\bar{\gamma} = 10\,\mathrm{dB}, L = 50$). Solid line: simulation, symbols: analytical expression.

5. Limited-feedback scenario

As a natural extension to the analysis conducted in the previous section, we will now consider a scenario where only a subset of the active users (i.e. those with channel quality measures exceeding a pre-defined threshold) are allowed to report their pre-scheduling SNRs to the scheduler. By doing so, we aim at analyzing the sensitivity of the different transmission schemes (SISO, SISO-AS, STBC and STBC-AS) to imperfections in the feedback channel, in particular, those resulting from bandwidth constraints leading to scheduling outages and random user scheduling. In this situation, the spatial diversity provided by AS and STBC mechanisms could well make the system more robust against the deep fades encountered when users are randomly scheduled.

5.1. Post-scheduling SNR

In a limited-feedback scenario, the analysis of the *post-scheduling* SNR, γ^*, must be conducted in two different SNR regions: $\gamma \leqslant \gamma_{\text{th}}$ (i.e. all users remain silent), and $\gamma > \gamma_{\text{th}}$ (at least one user reports its CSI to the BS). For the $\gamma \leqslant \gamma_{\text{th}}$ case and by recalling that all users experience i.i.d fading, the CDF can be expressed as

$$\begin{aligned} F_{\gamma^*}(\gamma) &= \text{Prob}(\gamma^* \leqslant \gamma, \gamma_k \leqslant \gamma_{\text{th}} \text{ for all } k = 1 \dots K) \\ &= \text{Prob}(\gamma^* \leqslant \gamma)(F_\gamma(\gamma_{\text{th}}))^{K-1}. \end{aligned} \tag{24}$$

On the other hand, for $\gamma > \gamma_{\text{th}}$ where at least one user reports its CSI, the CDF function is related to the maximum SNR in the cell

$$\begin{aligned} F_{\gamma^*}(\gamma) &= \text{Prob}(\gamma_k \leqslant \gamma, \text{ for all } k = 1 \cdots K) \\ &= \text{Prob}(\max\{\gamma_1, \dots, \gamma_K\} \leqslant \gamma). \end{aligned} \tag{25}$$

Therefore, the expressions derived in Section 4.1 are still valid for this second region. Next, the pdf expressions are obtained for the different transmission schemes:

- *SISO:*

For the $\gamma \leqslant \gamma_{\text{th}}$ case, we have

$$F_{\gamma^*_{\text{SISO}}}(\gamma) = (F_{\gamma_{\text{SISO}}}(\gamma_{\text{th}}))^{K-1} F_{\gamma_{\text{SISO}}}(\gamma)$$

and hence

$$f_{\gamma^*_{\text{SISO}}}(\gamma) = \frac{\delta F_{\gamma^*_{\text{SISO}}}}{\delta \gamma} = \frac{\mathrm{e}^{-\gamma/\bar{\gamma}}}{\bar{\gamma}} \left(1 - \mathrm{e}^{-\gamma_{\text{th}}/\bar{\gamma}}\right)^{K-1}, \tag{26}$$

whereas for $\gamma > \gamma_{\text{th}}$, the CDF/pdf functions are given by

$$\begin{aligned} F_{\gamma^*_{\text{SISO}}}(\gamma) &= (F_{\gamma_{\text{SISO}}}(\gamma))^K, \\ f_{\gamma^*_{\text{SISO}}}(\gamma) &= K \frac{\mathrm{e}^{-\gamma/\bar{\gamma}}}{\bar{\gamma}} (1 - \mathrm{e}^{-\gamma/\bar{\gamma}})^{K-1}. \end{aligned} \tag{27}$$

- *SISO-AS:*

As done in Section 4.1, the *post-scheduling* pdf and CDF for the SISO-AS case can be readily obtained by simply replacing the actual number of users K by KN_{BS} in the SISO expressions. This holds true for both SNR regions, $\gamma \leqslant \gamma_{\text{th}}$ and $\gamma > \gamma_{\text{th}}$ since, in the former case, both the scheduled user *and* the BS transmit antenna are randomly selected.

- *STBC:*

It can be readily shown that, for the Alamouti scheme the pdf of the *post-scheduling* SNR can be expressed as

$$\begin{aligned} f_{\gamma^*_{\text{STBC}}}(\gamma) &= \frac{4\gamma}{\bar{\gamma}^2} \mathrm{e}^{-2\gamma/\bar{\gamma}} \left(1 - \mathrm{e}^{-2\gamma_{\text{th}}/\bar{\gamma}} \left(\frac{2\gamma_{\text{th}}}{\bar{\gamma}} + 1\right)\right)^{K-1} \quad \gamma \leqslant \gamma_{\text{th}}, \\ f_{\gamma^*_{\text{STBC}}}(\gamma) &= K \frac{4\gamma}{\bar{\gamma}^2} \mathrm{e}^{-2\gamma/\bar{\gamma}} \left(1 - \mathrm{e}^{-2\gamma/\bar{\gamma}} \left(\frac{2\gamma}{\bar{\gamma}} + 1\right)\right)^{K-1} \gamma > \gamma_{\text{th}}. \end{aligned} \tag{28}$$

- *STBC-AS:*

Since transmit antenna subsets are no longer statistically independent, the *post-scheduling* SNR analysis (for the $\gamma \leqslant \gamma_{\text{th}}$ region) is somewhat involved and, thus, will be omitted (see Appendix A for details). The resulting expressions for the pdf function in both regions are

$$\begin{aligned} f_{\gamma^*_{\text{STBC-AS}}}(\gamma) &= \frac{4}{\bar{\gamma}} \mathrm{e}^{-2\gamma_{\text{th}}/\bar{\gamma}} \left(\mathrm{e}^{-\gamma/\bar{\gamma}} + \frac{\gamma}{\bar{\gamma}} \mathrm{e}^{-2\gamma + \gamma_{\text{th}}/\bar{\gamma}} - 1\right) \\ &\times \left(1 - \mathrm{e}^{-2\gamma_{\text{th}}/\bar{\gamma}} \left(\frac{6\gamma_{\text{th}}}{\bar{\gamma}} + 4\mathrm{e}^{-\gamma_{\text{th}}/\bar{\gamma}} - 3\right)\right)^{K-1} \gamma \leqslant \gamma_{\text{th}} \end{aligned}$$

$$f_{\gamma^*_{\text{STBC-AS}}}(\gamma) = K\,\frac{12}{\bar\gamma}\,\mathrm{e}^{-2\gamma/\bar\gamma}\left(\mathrm{e}^{-\gamma/\bar\gamma}+\frac{\gamma}{\bar\gamma}-1\right) \times\left(1-\mathrm{e}^{-2\gamma/\bar\gamma}\left(\frac{6\gamma}{\bar\gamma}+4\mathrm{e}^{-\gamma/\bar\gamma}-3\right)\right)^{K-1} \quad \gamma>\gamma_{\text{th}}. \tag{29}$$

In Fig. 5, we depict the pdf of the post-scheduling SNR for the different transmission schemes. Apart from a close matching between analytical and simulated curves, we can appreciate in the pdf behavior a well-defined transition region between random ($\gamma<\gamma_{\text{th}}$) and max-SNR scheduling. In the $\gamma<\gamma_{\text{th}}$ region, one would emphasize the enhanced fading mitigation capabilities exhibited by the STBC-based schemes (reduced probability at low SNR values). In the max-SNR scheduling region, instead, it is worth noting the improvement in terms of SNR peak generation (increased probability at high-SNR values) obtained with the SISO-based approaches.

5.2. Normalized average feedback load

In [11], the authors define *normalized average feedback load*, $\bar F$, as the usage ratio per time slot averaged over the total number of active users. This measure can also be interpreted as the probability for a given user to effectively signal its CSI over the feedback channel. Thus, in the case that all users experience i.i.d. fading channels, $\bar F$ can be readily expressed as $\bar F = 1 - F_\gamma(\gamma_{\text{th}})$.

For a given feedback load, the associated SNR thresholds, that will ultimately depend on the selected transmission scheme, can then be obtained from (7), (8), (10) and (12) in Section 3. This is straightforward in the SISO case ($\gamma_{\text{th}} = -\bar\gamma\ln(\bar F)$) but for the other cases one should resort to numerical methods.

As shown in Fig. 6(a), the stabilizing effect of STBC with respect to a SISO configuration has a clear impact on the determination of the normalized threshold. In the high feedback region ($\bar F \geqslant 0.28$) the threshold is higher in the STBC case because of the reduction in the number of deep fades (e.g. the effective SNR is higher and, for a given feedback load, so is the threshold). Conversely, in the low feedback region where only the highest SNR peaks can be exploited by the selective MU diversity schemes, the threshold associated to STBC must be lower (w.r.t. that of SISO) since the number of such peaks is substantially lower too. By introducing AS mechanisms (STBC-AS) the normalized thresholds can be increased again due to the associated SNR peak restoration (for $\bar F \geqslant 0.08$) and fade suppression effects. Finally, one can also observe how the fade mitigation capability of the SISO scheme can be improved with the introduction of antenna selection mechanisms. As the number of available antennas for selection increases (SISO-AS2, SISO-AS3) so do the associated SNR thresholds.

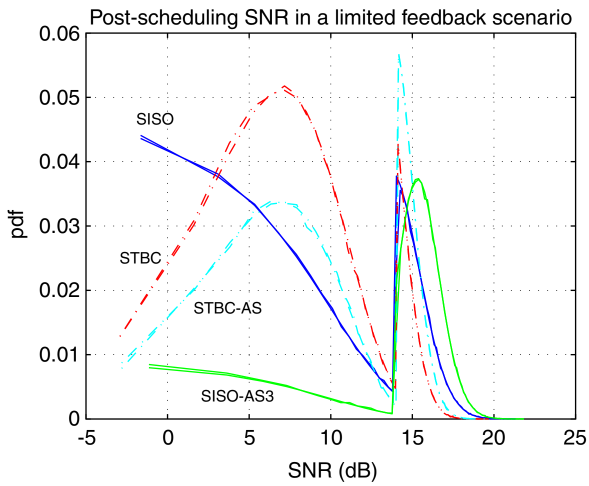

Fig. 5. Analytical and simulated pdf of the post-scheduling SNR for the different transmission schemes in a limited feedback scenario ($\bar\gamma = 10\,\text{dB}, \gamma_{\text{th}} = 14\,\text{dB}$, 10 users).

5.3. Capacity and throughput analysis

For the sake of brevity, the reader is referred to the analysis conducted in Section 4.2 where the capacity and throughput expressions were derived for the unlimited feedback case. Instead, the pdf functions obtained in Section 5.1 should now be used.

We will skip the derivation of the average channel capacity formulas since that was already done in [11] (SISO and SISO-AS cases) or, on the contrary, one should resort to numerical integration methods (STBC and STBC-AS). As for throughput expressions, by substituting Eqs. (26)–(29) and (19) into (18), considering the equality Eq. 3.351.1 in [16] ($\Gamma(n,x) = \int_x^\infty \mathrm{e}^{-t}t^{n-1}\,\mathrm{d}t$), using the binomial expansion and integrating by parts,

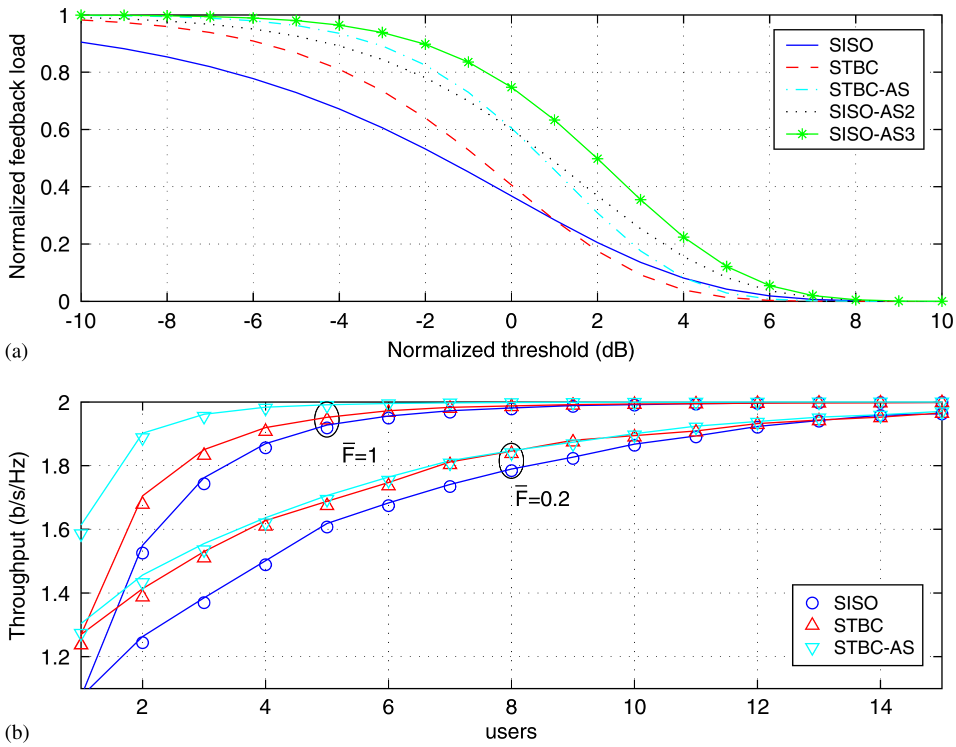

Fig. 6. (a) Normalized feedback load vs. normalized threshold $\gamma_{\mathrm{th}}/\bar{\gamma}$ for the different transmission schemes. (b) Analytical (symbols) vs. simulation (solid) results for the different transmission schemes.

we have

$$\bar{\eta}_{\mathrm{SISO}}(K) \approx \sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\sum_{l=0}^{L}\binom{L}{l}(-\alpha)^l \left[\frac{\mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}k}}{\bar{\gamma}\beta l+1}+\left(\frac{K}{\bar{\gamma}\beta l+k+1}-\frac{1}{\bar{\gamma}\beta l+1}\right) \times \mathrm{e}^{-\gamma_{\mathrm{th}}/\bar{\gamma}(\bar{\gamma}\beta l+k+1)}\right] \tag{30}$$

$$\bar{\eta}_{\text{SISO-AS}}(K) \approx \bar{\eta}_{\mathrm{SISO}}(KN_{\mathrm{BS}}) \tag{31}$$

$$\begin{aligned}\bar{\eta}_{\mathrm{STBC}}(K) &\approx 4\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\sum_{l=0}^{L}\binom{L}{l}(-\alpha)^l \\ &\times\left[\frac{(2(\gamma_{\mathrm{th}}/\bar{\gamma})+1)^k\mathrm{e}^{-(2k\gamma_{\mathrm{th}}/\bar{\gamma})}}{(\bar{\gamma}\beta l+2)^2}\right. \\ &\times\left(1-\mathrm{e}^{-(\gamma_{\mathrm{th}}/\bar{\gamma})(\bar{\gamma}\beta l+2)}\left(\frac{\gamma_{\mathrm{th}}(\bar{\gamma}\beta l+2)}{\bar{\gamma}}+1\right)\right) \\ &+K\sum_{n=0}^{k} \\ &\times\left.\binom{k}{n}\frac{2^n\Gamma(n+2,(\gamma_{\mathrm{th}}/\bar{\gamma})(\bar{\gamma}\beta l+2k+2))}{(\bar{\gamma}\beta l+2k+2)^{n+2}}\right]\end{aligned} \tag{32}$$

$$\begin{aligned}\bar{\eta}_{\text{STBC-AS}}(K) &\approx -4\mathrm{e}^{-(2\gamma_{\mathrm{th}}/\bar{\gamma})} \\ &\times\left(1-\mathrm{e}^{-(2\gamma_{\mathrm{th}}/\bar{\gamma})}\left(\frac{6\gamma_{\mathrm{th}}}{\bar{\gamma}}+4\mathrm{e}^{-(\gamma_{\mathrm{th}}/\bar{\gamma})}-3\right)\right)^{K-1} \\ &\times\left[\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}+\sum_{l=1}^{L}\binom{L}{l}\frac{(-\alpha)^l(1-\mathrm{e}^{-\gamma_{\mathrm{th}}\beta l})}{\bar{\gamma}\beta l}-\sum_{l=0}^{L}\binom{L}{l}\right. \\ &\times(-\alpha)^l\left(\frac{(1-\mathrm{e}^{-(\gamma_{\mathrm{th}}/\bar{\gamma})(\bar{\gamma}\beta l+1)})}{\bar{\gamma}\beta l+1}+\frac{\mathrm{e}^{(2\gamma_{\mathrm{th}}/\bar{\gamma})}}{(\bar{\gamma}\beta l+2)^2}\right. \\ &\times\left.\left.\left(1-\mathrm{e}^{-(\gamma_{\mathrm{th}}/\bar{\gamma})(\bar{\gamma}\beta l+2)}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}(\bar{\gamma}\beta l+2)+1\right)\right)\right)\right] \\ &+12K\sum_{k=0}^{K-1}\binom{K-1}{k}3^k\sum_{l=0}^{L}\binom{L}{l}(-\alpha)^l\end{aligned}$$

$$\times \sum_{n=0}^{k} \binom{k}{n} \left(-\frac{1}{3}\right)^n \sum_{s=0}^{n} \binom{n}{s} 6^{n-s} 4^s$$
$$\times \left[\frac{\Gamma(n-s+1, (\gamma_{\text{th}}/\bar{\gamma})(\bar{\gamma}\beta l + 2k + s + 3))}{(\bar{\gamma}\beta l + 2k + s + 3)^{n-s+1}} \right.$$
$$+ \frac{1}{(\bar{\gamma}\beta l + 2k + s + 2)^{n-s+1}}$$
$$\times \left(\frac{\Gamma(n-s+2, (\gamma_{\text{th}/\bar{\gamma}})(\bar{\gamma}\beta l + 2k + s + 2))}{\bar{\gamma}\beta l + 2k + s + 2} \right.$$
$$\left.\left. - \Gamma(n-s+1, (\gamma_{\text{th}}/\bar{\gamma})(\bar{\gamma}\beta l + 2k + s + 2)) \right) \right]. \quad (33)$$

In Fig. 6(b), it can be observed that the approximations are very tight with respect to the simulated curves.

5.3.1. Asymptotic analysis

In order to gain some insight for the throughput expressions above, we will force the feedback channel bandwidth to be arbitrarily low (i.e. $\gamma_{\text{th}} \to \infty$) and conduct the corresponding asymptotic analysis. Bearing in mind that $\lim_{x\to\infty} \Gamma(n,x) = 0$ we can easily prove that

$$\lim_{\gamma_{\text{th}}\to\infty} \bar{\eta}_{\text{SISO-AS}} = \lim_{\gamma_{\text{th}}\to\infty} \bar{\eta}_{\text{SISO}} = \sum_{l=0}^{L} \binom{L}{l} \frac{(-\alpha)^l}{\bar{\gamma}\beta l + 1}, \quad (34)$$

$$\lim_{\gamma_{\text{th}}\to\infty} \bar{\eta}_{\text{STBC-AS}} = \lim_{\gamma_{\text{th}}\to\infty} \bar{\eta}_{\text{STBC}} = 4 \sum_{l=0}^{L} \binom{L}{l} \frac{(-\alpha)^l}{(\bar{\gamma}\beta l + 2)^2}. \quad (35)$$

Two main conclusions can be drawn from the expressions above. First, the inclusion of AS makes no difference in the asymptotic regime since, clearly, bandwidth constraints in the feedback channel will prevent the scheduler from being informed on the optimal antenna subset. Besides, MU diversity can be barely exploited in this context (the system will often resort to random scheduling) and, consistently, there is no dependence in Eqs. (34), (35) on the number of active users in the system.

Going one step further, we will assume a large enough average SNR, $\bar{\gamma}$, and will approximate the summations in Eqs. (34) and (35) by their first two elements:

$$\lim_{\gamma_{\text{th}}\to\infty} \bar{\eta}_{\text{SISO}} \approx 1 - \frac{\alpha L}{\bar{\gamma}\beta + 1},$$

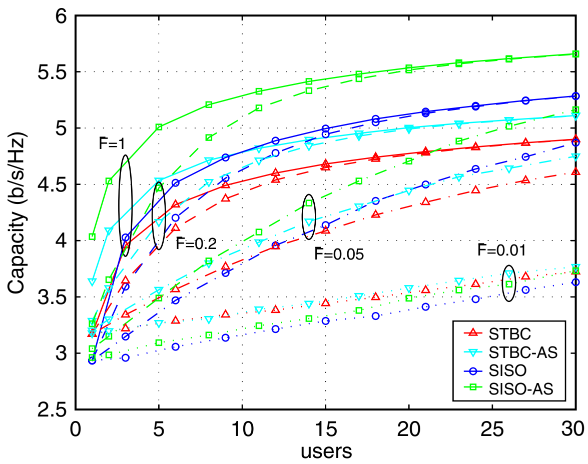

Fig. 7. Channel capacity vs. number of users for the different transmission schemes and feedback loads $\bar{F} = 1$ (solid), 0.2 (dashed), 0.05 (dash–dotted), 0.01 (dotted). $\bar{\gamma} = 10\,\text{dB}$.

$$\lim_{\gamma_{\text{th}}\to\infty} \bar{\eta}_{\text{STBC}}(K) \approx 1 - \frac{4\alpha L}{(\bar{\gamma}\beta + 2)^2}.$$

Finally, for positive values of the $\beta^2\bar{\gamma}^2$ product (as in our case), one can easily verify after some manipulations that

$$\frac{4}{(\bar{\gamma}\beta + 2)^2} < \frac{1}{\bar{\gamma}\beta + 1} \quad (36)$$

i.e., in terms of average throughput STBC-based schemes outperform SISO-based ones in the asymptotic case. This conclusion is not so surprising since a multi-user scenario with very low feedback channel bandwidth and high SNR is equivalent to a single-user case where, ultimately, it is the spatial diversity gain provided by space–time block coding (BER curves' slope in the high SNR regime) that makes the difference. Simulation results shown in Fig. 8 are aligned with and, thus, confirm the asymptotic results obtained in this section.

5.4. Computer simulation results and discussion

In Fig. 7, we depict the average system capacity as a function of the number of active users and different feedback loads[2] ($\bar{F} = 1 \ldots 0.01$). As in [11], performance loss in limited-feedback system with $\bar{F} = 0.2$ and a moderate-to-high number of

[2]For a given feedback load, the SNR threshold, γ_{th}, for each transmission scheme must be accordingly adjusted.

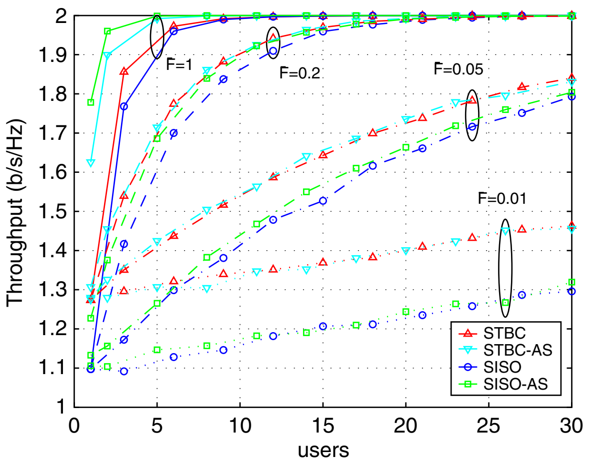

Fig. 8. Link layer throughput vs. number of users for the different transmission schemes and feedback loads $\bar{F} = 1$ (solid), 0.2 (dashed), 0.05 (dash–dotted), 0.01 (dotted). $\bar{\gamma} = 10\,\mathrm{dB}$. QPSK.

users ($K \geqslant 15$) can be neglected for all transmission schemes. When the average feedback load per user is further reduced ($\bar{F} = 0.05$), one can observe that the degradation experienced by the SISO-based schemes is larger than that exhibited by the STBC ones (approximately 24% vs. 16%, respectively, for a $K = 10$ case). Furthermore, for $\bar{F} = 0.01$, STBC approaches outperform *both* SISO schemes. As a conclusion, when the scheduling outage probability increases (due to bandwidth limitations in the feedback channel), STBC approaches provide additional robustness against deep fades associated to random user selection. In all cases, additional gains in terms of spatial diversity can be obtained from the introduction of AS mechanisms (albeit very moderate when feedback load is low).

As for the average system throughput, curves in Fig. 8 reveal that, for limited-feedback schemes ($\bar{F} < 1$),, spatial diversity gains associated to STBC clearly exceed those of multi-user diversity. This is true even for cases with a moderate scheduling outage probability (e.g. $\bar{F} = 0.2$). Diversity order can be further increased by introducing AS mechanisms but the improvement in terms of system throughput is rather limited (as opposed to capacity enhancements). Finally, throughput curves reveal that the gap between the SISO- and STBC-based approaches becomes wider when feedback load is reduced. In other words, a STBC transmission scheme is far less sensitive to restrictions and constraints in the feedback channel.

6. Conclusions

In this paper, a number of transmission schemes (SISO, SISO-AS, STBC and STBC-AS) aimed at jointly exploiting both multi-user and spatial diversity arising from antenna selection strategies and space–time block coding were presented. The scenario consisted of a cellular system featuring MISO links (with up to $N_{\mathrm{BS}} = 3$ antennas in the base station) where a centralized radio resource manager in the base station conducted scheduling in the downlink following a proportional fair scheduling rule. To do so, an extensive use of PHY-layer information (i.e. measured SNRs) is done at the link layer and, hence, this constitutes a cross-layer design.

In terms of pre-scheduling SNR, we observed that the stabilizing effect (i.e. reduced number of deep fades and SNR peaks) associated to STBC can be modified by the introduction of antenna selection mechanisms that partly restore the missing SNR peaks.

Overall system performance, though, was assessed in terms of average system capacity and throughput, corresponding to the limiting cases of having infinite and single AMC levels available, respectively. According to the number of active users being allowed to report quality measures, two different cases, namely unlimited and limited feedback (also referred to as Selective Multi-user diversity) were studied. The latter case was aimed at analyzing the sensitivity of the different transmission schemes to imperfections in the feedback channel and, in particular, those resulting from bandwidth constraints leading to scheduling outages and random user scheduling. For both feedback cases, differences in terms of performance resulted from differences in the associated post-scheduling SNR, for which closed-form expressions were analytically derived for all transmission schemes. By combining space–time block-coding with transmit antenna selection (STBC-AS), it was shown that, for a low to moderate number of active users ($K = 1, 8$), the suppression of the SNR peaks due to the SNR-stabilizing effect associated to STBC can be partly compensated for. Apart from that, the number of antenna subsets resulting from the introduction of AS can be interpreted as an increase in the number of active users in the system, this causing a shift in the post-scheduling SNRs towards higher values (its impact being lower when combined with STBC due to inter-subset correlations). However, for a higher number of users the beneficial effect of multi-user diversity on a SISO scheme exceeds by far that of spatial diversity. In the limited-feedback case, two different behaviors were observed in the pdf and CDF functions in regions above or below the SNR thresholds, these ultimately depending on the selected transmission scheme.

In the unlimited-feedback case, conclusions drawn from capacity curves were, as expected, totally aligned with the results obtained from SNR densities. In terms of capacity, the STBC-AS scheme combines antenna diversity and multi-user diversity and performs better than the MU-exploiting SISO approach in the $K = 1\ldots8$ users range. However, the SISO-AS configuration performs better for the whole range of users. Due to correlation effects, STBC-AS is equivalent to a twofold increase in the number of active users with respect to the STBC scheme without antenna selection, in terms of system capacity. In terms of throughput, part or all of the advantage exhibited by the SISO and SISO-AS configurations is lost in favor of the STBC and STBC-AS configurations respectively. For instance, STBC outperforms SISO for the whole range of users as opposed to channel capacity where that only happened for the case with $K = 1$ active users.

In the limited-feedback case, when the average feedback load per user is reduced one can observe that the degradation, both in terms of capacity and throughput, experienced by the SISO-based schemes is larger than that exhibited by the STBC ones. Additional capacity gains can be obtained from the introduction of AS mechanisms, albeit very moderate when feedback load is low. In terms of throughput, the effect of spatial diversity associated to STBC (in terms of reduced BER due to an increased diversity order) clearly exceeds that of multi-user diversity on SISO, even for cases with a moderate scheduling outage probability of 20. From the asymptotic analysis (and also the simulated throughput curves), it follows that the inclusion of AS makes no difference when feedback channel bandwidth is made arbitrarily low and, in general, gains are far more limited than in the capacity case. Furthermore, in the high-SNR region, STBC-based schemes outperform SISO-based ones.

Future work in this field may encompass the study of the proposed transmission schemes in more realistic environments. For instance, scenarios where users' fading statistics are not assumed identical, the correlation between transmit antennas is not neglected, or where effects such as errors or delay are introduced in the feedback channel.

Appendix A. Proof of (29)

Since in the $\gamma \leqslant \gamma_{th}$ region, all users remain silent, both the scheduled user and the transmit antenna subset in the BS are randomly selected. Therefore, the probability $\text{Prob}(\gamma^* \leqslant \gamma)$ in (24) can be expressed as

$$\begin{aligned}
&\text{Prob}(\gamma^* \leqslant \gamma) \\
&\quad = \text{Prob}(\gamma^* \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th} | \gamma^* = \gamma_{\min})\text{Prob}(\gamma^* = \gamma_{\min}) \\
&\qquad + \text{Prob}(\gamma^* \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th} | \gamma^* = \gamma_{\text{med}})\text{Prob}(\gamma^* = \gamma_{\text{med}}) \\
&\qquad + \text{Prob}(\gamma^* \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th} | \gamma^* = \gamma_{\max})\text{Prob}(\gamma^* = \gamma_{\max}) \\
&\quad = \text{Prob}(\gamma_{\min} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th})\text{Prob}(\gamma^* = \gamma_{\min}) \\
&\qquad + \text{Prob}(\gamma_{\text{med}} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th})\text{Prob}(\gamma^* = \gamma_{\text{med}}) \\
&\qquad + \text{Prob}(\gamma_{\max} \leqslant \gamma)\text{Prob}(\gamma^* = \gamma_{\max}), \qquad (37)
\end{aligned}$$

where $\gamma_{\min}$, γ_{med} and $\gamma_{\max}$ stand for the received SNR associated with the worst, the second worst and the best antenna subsets and clearly

$$\begin{aligned}
\text{Prob}(\gamma^* = \gamma_{\min}) &= \text{Prob}(\gamma^* = \gamma_{\text{med}}) \\
&= \text{Prob}(\gamma^* = \gamma_{\max}) = \tfrac{1}{3} \qquad (38)
\end{aligned}$$

due to the random choice of antenna subsets. In Eq. (37) above, the probability that the SNR associated to the best antenna subset is below γ_{th}, can be readily computed from the pre-scheduling SNR distribution (see Eq. (12)), that is

$$\text{Prob}(\gamma_{\max} \leqslant \gamma) = F_{\gamma_{\text{STBC-AS}}}(\gamma). \qquad (39)$$

However, a more elaborate analysis is needed to calculate the joint probabilities in Eq. (37) which cannot be factorized due to the fact that the antenna subsets are not statistically independent, i.e.

$$\begin{aligned}
&\text{Prob}(\gamma_{\min} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th}) \\
&\quad \neq \text{Prob}(\gamma_{\min} \leqslant \gamma)\text{Prob}(\gamma_{\max} \leqslant \gamma_{th}),
\end{aligned}$$

$$\begin{aligned}
&\text{Prob}(\gamma_{\text{med}} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{th}) \\
&\quad \neq \text{Prob}(\gamma_{\text{med}} \leqslant \gamma)\text{Prob}(\gamma_{\max} \leqslant \gamma_{th}).
\end{aligned}$$

Again, one can resort to order statistics to solve this problem. The joint distribution of the squared channel gains (X_1, X_2, X_3) can be written as

$$f_{X_3X_2X_1}(x_3, x_2, x_1) = \begin{cases} 3!\mathrm{e}^{-(x_1+x_2+x_3)} & \text{for } x_1 \leqslant x_2 \leqslant x_3, \\ 0 & \text{otherwise.} \end{cases}$$

For the $0 \leqslant \gamma \leqslant \gamma_{\rm th}/2$ interval , we have

$$\begin{aligned}
&\mathrm{Prob}(\gamma_{\min} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{\rm th}) \\
&= \mathrm{Prob}\left(X_1 + X_2 \leqslant \frac{2\gamma}{\bar{\gamma}} = t, X_2 + X_3 \leqslant \frac{2\gamma th}{\bar{\gamma}} = m\right) \\
&= \int_{x_3=0}^{t/2}\int_{x_2=0}^{x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{t}\int_{x_2=0}^{t/2}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{t}\int_{x_2=t/2}^{x_3}\int_{x_1=0}^{t-x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \\
&+ \int_{x_3=t}^{m-t}\int_{x_2=0}^{t/2}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t}^{m-t}\int_{x_2=t/2}^{t}\int_{x_1=0}^{t-x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=m-t}^{m-t/2}\int_{x_2=0}^{t/2}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=m-t}^{m-t/2}\int_{x_2=t/2}^{m-x_3}\int_{x_1=0}^{t-x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3 \\
&+ \int_{x_3=m-t/2}^{m}\int_{x_2=0}^{m-x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,dx_2\,\mathrm{d}x_3
\end{aligned} \tag{40}$$

$$\begin{aligned}
&\mathrm{Prob}(\gamma_{\rm med} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{\rm th}) \\
&= \mathrm{Prob}\left(X_1 + X_3 \leqslant \frac{2\gamma}{\bar{\gamma}} = t, X_2 + X_3 \leqslant \frac{2\gamma_{\rm th}}{\bar{\gamma}} = m\right) \\
&= \int_{x_3=0}^{t/2}\int_{x_2=0}^{x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{t}\int_{x_2=0}^{t-x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{t}\int_{x_2=t-x_3}^{x_3}\int_{x_1=0}^{t-x_3} f_{X_3X_2X_1}(x_3,x_2,x_1)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3
\end{aligned} \tag{41}$$

whereas for the $\frac{\gamma_{\rm th}}{2} \leqslant \gamma \leqslant \gamma_{\rm th}$ interval

$$\begin{aligned}
&\mathrm{Prob}(\gamma_{\min} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{\rm th} \\
&= \mathrm{Prob}\left(X_1 + X_2 \leqslant \frac{2\gamma}{\bar{\gamma}} = t, X_2 + X_3 \leqslant \frac{2\gamma_{\rm th}}{\bar{\gamma}} = m\right) \\
&= \int_{x_3=0}^{t/2}\int_{x_2=0}^{x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{m/2}\int_{x_2=0}^{t/2}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{m/2}\int_{x_2=t/2}^{x_3}\int_{x_1=0}^{t-x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=m/2}^{m-t/2}\int_{x_2=0}^{t/2}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=m/2}^{m-t/2}\int_{x_2=t/2}^{m-x_3}\int_{x_1=0}^{t-x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=m-t/2}^{m}\int_{x_2=0}^{m-x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3
\end{aligned} \tag{42}$$

$$\begin{aligned}
&\mathrm{Prob}(\gamma_{\rm med} \leqslant \gamma, \gamma_{\max} \leqslant \gamma_{\rm th}) \\
&= \mathrm{Prob}(X_1 + X_3 \leqslant \frac{2\gamma}{\bar{\gamma}} = t, X_2 + X_3 \leqslant \frac{2\gamma_{\rm th}}{\bar{\gamma}} = m) \\
&= \int_{x_3=0}^{t/2}\int_{x_2=0}^{x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{m/2}\int_{x_2=0}^{t-x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=t/2}^{m/2}\int_{x_2=t-x_3}^{x_3}\int_{x_1=0}^{t-x_3} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=\frac{m}{2}}^{t}\int_{x_2=0}^{t-x_3}\int_{x_1=0}^{x_2} f_{X_3X_2X_1}(x_3,x_2,x_1)\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3 \\
&+ \int_{x_3=\frac{m}{2}}^{t}\int_{x_2=t-x_3}^{m-x_3}\int_{x_1=0}^{t-x_3} f_{X_3X_2X_1}(x_3,x_2,x_1)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3
\end{aligned} \tag{43}$$

By solving the integrals in Eqs. (40)–(43) above and plugging the results (along with Eqs. (38) and (39)) into Eq. (37), we have

$$\begin{aligned}
\mathrm{Prob}(\gamma^* \leqslant \gamma) = &-\frac{\mathrm{e}^{-2\gamma_{\rm th}/\bar{\gamma}}}{\bar{\gamma}}\Big(4\bar{\gamma}\mathrm{e}^{-\gamma/\bar{\gamma}} \\
&+2\gamma\mathrm{e}^{-2\gamma-\gamma_{\rm th}/\bar{\gamma}} + \bar{\gamma}\mathrm{e}^{-2\gamma-\gamma_{\rm th}/\bar{\gamma}} - \bar{\gamma}\mathrm{e}^{2/\bar{\gamma}}\gamma_{\rm th} + 4\gamma - 4\bar{\gamma}\Big)
\end{aligned}$$

Finally, by substituting this last expression into (24) and taking its derivative, (29) follows.

References

[1] R. Knopp, P. Humblet, Information capacity and power control in single-cell multiuser communications, in: Proc. IEEE ICC, 1995.

[2] D. Tse, Optimal power allocation over parallel gaussian broadcast channels, in: Proc. IEEE ISIT, 1997.

[3] P. Viswanath, D. Tse, R. Laroia, Opportunistic beamforming using dumb antennas, IEEE Trans. Inform. Theory 48 (6) (2002) 1277–1294.

[4] V. Tarokh, H. Jafarkhani, A.R. Calderbank, Space–time block codes from orthogonal designs, IEEE Trans. Inform. Theory, 1999.

[5] S. Alamouti, A simple transmit diversity technique for wireless communications, IEEE J. Selected Areas Communication, 1998.

[6] R. Gozali, R. Buehrer, B.D. Woerner, The impact of multiuser diversity on space–time block coding, IEEE Comm. Lett., 2003.

[7] L.T. Berger, et al., Interaction of transmit diversity and proportional fair scheduling, in: Proc. IEEE VTC Spring, 2003.

[8] M. Kobayashi, G. Caire, D. Gesbert, Antenna diversity versus multiuser diversity: quantifying the tradeoffs, in: Proc. WWRF 11, 2004.

[9] E.G. Larsson, On the combination of spatial diversity and multiuser diversity, IEEE Comm. Lett., 2004.
[10] D.A. Gore, A.J. Paulraj, MIMO antenna subset selection with space–time coding, IEEE Trans. Signal Process., October 2002.
[11] D. Gesbert, M. Alouini, How much feedback is multi-user diversity really worth? in: Proc. IEEE ICC, 2004.
[12] J. Holtzman, CDMA forward link waterfilling power control, in: Proc. IEEE VTC Spring, 2000.
[13] H. David, Order Statistics, Wiley, New York, 1981.
[14] J. Proakis, Digital Communications, McGraw Hill, New York, 2001.
[15] S. Chung, A. Goldsmith, Degrees of freedom in adaptive modulation: A unified view, IEEE Trans. Comm., 2001.
[16] I. Gradsthteyn, I. Ryzhik, Tables of Integrals: Series and Products, Academic, New York, 1965.
[17] T. Starr, J. Cioffi, P. Silverman, Understanding Digital Subscriber Line Technology, Prentice-Hall, Upper Saddle River, NJ, 1999.