Alternative Implementations of the GNSS Power-Based Capon Beamformer

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***Abstract*—Power-Based Capon beamforming has recently been proposed to avoid the well-known cancellation effects between direct signal and correlated multipaths of the Capon beamformer. This novel technique exploits the fact that in some applications the power of the direct signal can be known at the receiver, in addition to the more usual assumptions of known spatial and temporal signatures. At the implementation stage, however, it involves solving a given two-dimensional rank minimization problem, which must be approximated to avoid inherent numerical limitations. In this work, we present an equivalent minimization problem that overcomes such limitations, and therefore leads to alternative implementations that can provide more precise and reliable results. The new approach is justified mathematically, and also supported by several simulations results.**

***Index Terms*—Arrays, beamforming, Capon, coherent, correlated, GNSS, multipath.**

I. Introduction

GLOBAL Navigation Satellite Systems (GNSS) enable the calculation of a user position by using the signals transmitted by a constellation of specific satellites. For GNSS, only the received Line-Of-Sight Signal (LOSS) is useful to obtain information about the user position. Multipath reflections usually bias the apparent distance between the user and each available satellite, and they also hamper the ambiguity resolution process needed for carrier-phase ranging [1]. For this reason, significant research has been devoted to the mitigation of multipath effects, and many techniques have been proposed so far. On one hand, there are single-antenna techniques, that attempt to discriminate the received signals by exploiting temporal diversity [2]–[8]. On the other hand, there are multiple-antenna techniques, that attempt to discriminate the received signals by exploiting spatial diversity [9]–[22].

The most practical multiple-antenna solutions are based on data-dependent beamforming, where specifically designed weights are calculated that depend on the statistics of the incoming data [23]. These weights are used to spatially filter the incoming signals by the antenna array, and thus attenuate the interference and noise present. Since all GNSS use Direct-Sequence Spread-Spectrum modulations, the data used to compute the beamforming weights can be obtained either before or after the despreading process [24]. The current techniques can be mainly grouped into those that exploit knowledge of the LOSS spatial signature, as in [25]–[29], into those that exploit knowledge of the LOSS temporal signature, as in [30]–[33], or into the blind ones, that only exploit some specific properties of the signals involved [34]–[40]. Although all these techniques are very useful in GNSS scenarios, they have serious limitations when correlated multipaths are present. A great example is that of Capon [41], which works extraordinarily well in the presence of uncorrelated interferences, but suffers from direct signal cancellation in the presence of correlated multipaths.

Recently, a modification of the Capon beamformer has been proposed exploiting the known power of the direct signal at the receiver, and it is referred to as Power-Based Capon (PBC) beamforming [42]. By estimating some particular cross-correlation terms and subtracting them from the spatial correlation matrix of the received signal, this novel technique prevents great part of the cancellation effects between direct signal and correlated multipaths. In this work, we review the most important features of the PBC and propose more practical and better performing implementations. In Section II, we formally state the problem to be addressed. In Section III, we derive an equivalent formulation of the PBC that leads to enhanced implementations. In Section IV, we show numerical examples that support the theoretical discussions. Finally, we draw conclusions about the work in Section V.

II. Problem Statement

Let us assume that the n-th sample of the post-despreading baseband equivalent signal received by an arbitrary L-element array can be written as follows:

$$\mathbf{x}[n] = \mathbf{a}s[n] + \sum_{k=1}^{D} \mathbf{b}_k m_k[n] + \mathbf{v}[n] \quad (1)$$

where $s[n] \in \mathbb{C}$ is the LOSS, $\mathbf{a} \in \mathbb{C}^L$ is its corresponding spatial signature, $m_k[n] \in \mathbb{C}$ is the k-th multipath reflection, $\mathbf{b}_k \in \mathbb{C}^L$ is its corresponding spatial signature, and $\mathbf{v}[n] \in \mathbb{C}^L$ is the received spatially white noise with identical power at each sensor. The signals involved in (1) impinge on the array from different directions, so that $\mathbf{a}, \mathbf{b}_1, \ldots, \mathbf{b}_D$ are linearly independent. The vector $\mathbf{a}$ is considered to be known, whereas $\mathbf{b}_1, \ldots, \mathbf{b}_D$ are unknown. In the scenario of interest, the multipath reflections can be either correlated or uncorrelated with the direct signal, and verify $D < L$. Specifically, they can be written as $m_k[n] = \beta_k s[n - \tau_k] e^{-j\varphi_k}\ \forall\, k \in \{1, \ldots, D\}$, where

Manuscript received April 14, 2021; revised June 16, 2021; accepted June 21, 2021. Date of publication June 24, 2021; date of current version July 29, 2021. This work was supported in part by the Ministry of Science and Innovation Grant TEC2017-89925-R and in part by the ICREA Academia Program. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Jun Liu. *(Corresponding author: Martí Mañosas-Caballú.)*

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Digital Object Identifier 10.1109/LSP.2021.3092229

$\beta_k \in \mathbb{R}$, $\tau_k \in \mathbb{R}$ and $\varphi_k \in \mathbb{R}$ are the corresponding amplitude, delay and phase relative to the LOSS.

At the receiver, a reference signal $c[n, \tau, \varphi]$ is also available. It is a non-synchronized replica of the direct signal $s[n]$, and can be tuned through a delay τ and phase φ:

$$c[n, \tau, \varphi] = s[n + \tau - \tau_e] e^{j(\varphi - \varphi_e)}$$

where τ_e and φ_e are the unknown delay and phase synchronization errors. The spatial cross-correlation vector between $\mathbf{x}[n]$ and $c[n, \tau, \varphi]$ can be easily calculated, which is defined as $\mathbf{r}_{\mathbf{x}c}(\tau, \varphi) = \mathrm{E}\{\mathbf{x}[n] c[n, \tau, \varphi]^*\}$ and can be written as:

$$\mathbf{r}_{\mathbf{x}c}(\tau, \varphi) = \left(\mathbf{a}\, r_s(\tau - \tau_e) + \mathbf{B}\mathbf{r}_{\mathbf{m}s}(\tau - \tau_e)\right) e^{-j(\varphi - \varphi_e)} \quad (2)$$

where $r_s(\tau) = \mathrm{E}\{s[n]s[n+\tau]^*\}$, $\mathbf{r}_{\mathbf{m}s}(\tau) = \mathrm{E}\{\mathbf{m}[n]s[n+\tau]^*\}$, $\mathbf{m}[n] = [m_1[n] \ldots m_D[n]]^T$ and $\mathbf{B} = [\mathbf{b}_1 \ldots \mathbf{b}_D]$. In addition, the spatial correlation matrix of $\mathbf{x}[n]$ can also be calculated. It is defined as $\mathbf{R}_{\mathbf{xx}} = \mathrm{E}\{\mathbf{x}[n]\mathbf{x}[n]^H\}$ and can be written as:

$$\mathbf{R}_{\mathbf{xx}} = \sigma_s^2 \mathbf{a}\mathbf{a}^H + \mathbf{a}\mathbf{r}_{\mathbf{m}s}^H \mathbf{B}^H + \mathbf{B}\mathbf{r}_{\mathbf{m}s}\mathbf{a}^H + \mathbf{B}\mathbf{R}_{\mathbf{m}}\mathbf{B}^H + \sigma_v^2 \mathbf{I}$$

where σ_s^2 is the known power of $s[n]$, σ_v^2 is the power of $\mathbf{v}[n]$, $\mathbf{r}_{\mathbf{m}s} = \mathrm{E}\{\mathbf{m}[n]s[n]^*\}$, $\mathbf{R}_{\mathbf{m}} = \mathrm{E}\{\mathbf{m}[n]\mathbf{m}[n]^H\}$ and $\mathbf{I}$ denotes the identity matrix.

If we evaluate (2) at $\tau = \tau_e$ and $\varphi = \varphi_e$, then $\mathbf{r}_{\mathbf{x}c}(\tau, \varphi)$ is equal to the vector $\mathbf{a}\sigma_s^2 + \mathbf{B}\mathbf{r}_{\mathbf{m}s}$, which in turn can be transformed to $\mathbf{B}\mathbf{r}_{\mathbf{m}s}$ because $\mathbf{a}$ and σ_s^2 are known. This procedure further allows to calculate the cross-correlation terms $\mathbf{B}\mathbf{r}_{\mathbf{m}s}\mathbf{a}^H$ and $\mathbf{a}\mathbf{r}_{\mathbf{m}s}^H\mathbf{B}^H$, and to subtract them to $\mathbf{R}_{\mathbf{xx}}$. The result is a new correlation matrix that would correspond to the scenario where the multipaths are totally uncorrelated with the LOSS. Then, if the Capon beamformer is calculated using the new correlation matrix, the typical cancellation effects between the direct signal and multipaths are avoided. This procedure is known as Power-Based Capon (PBC) beamforming [42], and the estimates of τ_e and φ_e are obtained from the solution to:

$$\min_{\tau, \varphi} \ \mathrm{rank}\left(\mathbf{R}(\tau, \varphi)\right) \quad (3)$$

where $\mathbf{R}(\tau, \varphi)$ is defined as $\mathbf{R}_{\mathbf{xx}} - \sigma_s^2\mathbf{a}\mathbf{a}^H - \sigma_v^2\mathbf{I} - \mathbf{a}\boldsymbol{\alpha}(\tau, \varphi)^H - \boldsymbol{\alpha}(\tau, \varphi)\mathbf{a}^H$, and $\boldsymbol{\alpha}(\tau, \varphi) = \mathbf{r}_{\mathbf{x}c}(\tau, \varphi) - \mathbf{a}\sigma_s^2$. Here, the value of σ_v^2 must be assumed known.

In practice, the discontinuities of the rank function and the need to use a threshold for the calculation of the numerical rank make the minimization problem (3) very difficult to implement. For this reason, the original PBC uses a generalization of the Schatten p-norm to approximate the rank:

$$S(\tau, \varphi) = \left(\sum_k \sigma_k^p(\tau, \varphi)\right)^{\frac{1}{p}} \quad \text{with} \quad 0 < p < \infty$$

where $\sigma_k(\tau, \varphi)$ is the k-th singular value of $\mathbf{R}(\tau, \varphi)$. This cost function is relatively simple to calculate, and does not present neither the discontinuities of the rank nor its numerical limitations. However, it only provides the exact values of τ_e and φ_e when $p \to 0$, and very low values of p are not numerically recommended [42]. With the goal of providing more accurate and reliable results, in the following we develop an equivalent approach to (3) that leads to new implementations of the PBC.

III. Equivalent Approach

A. Theoretical Bases

Let us start by noting that the matrix $\mathbf{R}(\tau, \varphi)$ can be written as follows:

$$\begin{bmatrix}\mathbf{a} & \mathbf{B}\end{bmatrix} \begin{bmatrix} 2\sigma_s^2 - 2\mathcal{R}\{r_s(\bar{\tau})e^{-j\bar{\varphi}}\} & \mathbf{r}_{\mathbf{m}s}^H - \mathbf{r}_{\mathbf{m}s}^H(\bar{\tau})e^{j\bar{\varphi}} \\ \mathbf{r}_{\mathbf{m}s} - \mathbf{r}_{\mathbf{m}s}(\bar{\tau})e^{-j\bar{\varphi}} & \mathbf{R}_{\mathbf{m}} \end{bmatrix} \begin{bmatrix}\mathbf{a}^H \\ \mathbf{B}^H\end{bmatrix}$$

where $\bar{\tau}$ and $\bar{\varphi}$ are used to denote $\tau - \tau_e$ and $\varphi - \varphi_e$ respectively, and $\mathcal{R}\{\}$ denotes the real part operator. A direct consequence of this formulation is that $\mathbf{R}(\tau, \varphi)$ can be considered as the spatial correlation matrix of an hypothetically received signal $\mathbf{x}_0[n]$:

$$\mathbf{x}_0[n] = \mathbf{a}s[n] - \mathbf{a}s[n + \bar{\tau}]e^{j\bar{\varphi}} + \sum_{k=1}^{D} \mathbf{b}_k m_k[n] \quad (4)$$

If we evaluate (4) at $\tau = \tau_e$ and $\varphi = \varphi_e$, then there is no contribution of $\mathbf{a}$ in $\mathbf{x}_0[n]$, and therefore it is natural that in this case the rank of $\mathbf{R}(\tau, \varphi)$ is minimal. Further, when we process $\mathbf{x}_0[n]$ with a beamformer $\mathbf{w} \in \mathbb{C}^L$ that satisfies the distortionless constraint $\mathbf{w}^H\mathbf{a} = 1$, it is expected that the minimum achievable power at the output of the beamformer is zero. In contrast, if $\tau \neq \tau_e$ or $\varphi \neq \varphi_e$, then there is contribution of $\mathbf{a}$ in $\mathbf{x}_0[n]$, and some amount of power must be present at the output unless the direct signal is fully correlated with some multipaths. This observation suggests that minimizing the rank of $\mathbf{R}(\tau, \varphi)$ may provide the same minimizers as:

$$\min_{\tau, \varphi} \ \mathrm{P}_{\min}(\tau, \varphi) \quad (5)$$

where $\mathrm{P}_{\min}(\tau, \varphi)$ is the minimum achievable power at the output of a distortionless beamformer with input equal to $\mathbf{x}_0[n]$, and thus it is defined as the minimum value of $\mathbf{w}^H\mathbf{R}(\tau, \varphi)\mathbf{w}$ subject to $\mathbf{w}^H\mathbf{a} = 1$. The following lemma has been developed to prove the equivalence.

Lemma 1: The minima of problems (3) and (5) are attained at the same values of (τ, φ).

Proof: Let us find first an analytical expression of $\mathrm{P}_{\min}(\tau, \varphi)$. If we exploit the known structure of $\mathbf{R}(\tau, \varphi)$ together with $\mathbf{w}^H\mathbf{a} = 1$, then $\mathbf{w}^H\mathbf{R}(\tau, \varphi)\mathbf{w}$ can be replaced with the following function:

$$\mathrm{P}(\mathbf{w}) = 2\sigma_s^2 - 2\mathcal{R}\{r_s(\bar{\tau})e^{-j\bar{\varphi}}\} + (\mathbf{r}_{\mathbf{m}s}^H - \mathbf{r}_{\mathbf{m}s}^H(\bar{\tau})e^{j\bar{\varphi}})\mathbf{B}^H\mathbf{w} + \mathbf{w}^H\mathbf{B}(\mathbf{r}_{\mathbf{m}s} - \mathbf{r}_{\mathbf{m}s}(\bar{\tau})e^{-j\bar{\varphi}}) + \mathbf{w}^H\mathbf{B}\mathbf{R}_{\mathbf{m}}\mathbf{B}^H\mathbf{w}$$

Thus, using the Lagrange multiplier theorem to find the minimum of $\mathrm{P}(\mathbf{w})$ subject to $\mathbf{w}^H\mathbf{a} = 1$, we shall define:

$$L(\mathbf{w}, \lambda) = \mathrm{P}(\mathbf{w}) + \lambda(\mathbf{w}^H\mathbf{a} - 1) + \lambda^*(\mathbf{a}^H\mathbf{w} - 1)$$

and solve the system:

$$\nabla L(\mathbf{w}, \lambda) = \mathbf{0} \quad (6)$$

$$\mathbf{a}^H\mathbf{w} = 1 \quad (7)$$

where $\lambda \in \mathbb{C}$ is the Lagrange multiplier and ∇ is the complex gradient operator with respect to $\mathbf{w}^H$.

The solution to (6) gives $\lambda = 0$ because the columns of $\mathbf{B}$ and $\mathbf{a}$ are linearly independent, and also as a result:

$$\mathbf{B}\mathbf{R}_{\mathbf{m}}\mathbf{B}^H\mathbf{w} = -\mathbf{B}(\mathbf{r}_{\mathbf{m}s} - \mathbf{r}_{\mathbf{m}s}(\bar{\tau})e^{-j\bar{\varphi}})$$

which can be reduced to:

$$\mathbf{R_m}\mathbf{B}^H\mathbf{w} = -(\mathbf{r}_{\mathbf{m}s} - \mathbf{r}_{\mathbf{m}s}(\bar{\tau})e^{-j\bar{\varphi}})$$

These last two equations are compatible with (7), and if we substitute them into $\mathrm{P}(\mathbf{w})$, and we rewrite $\mathbf{R_m}$ as $\mathbf{R_m}\mathbf{R_m^+}\mathbf{R_m}$ by means of the pseudoinverse $\mathbf{R_m^+}$, we finally obtain that the minimum achievable power is:

$$\mathrm{P}_{\min}(\tau,\varphi) = 2\sigma_s^2 - 2\mathcal{R}\{r_s(\bar{\tau})e^{-j\bar{\varphi}}\} - (\mathbf{r}_{\mathbf{m}s}^H - \mathbf{r}_{\mathbf{m}s}^H(\bar{\tau})e^{j\bar{\varphi}})\,\mathbf{R_m^+}\,(\mathbf{r}_{\mathbf{m}s} - \mathbf{r}_{\mathbf{m}s}(\bar{\tau})e^{-j\bar{\varphi}}) \quad (8)$$

Now note that $\mathrm{P}_{\min}(\tau,\varphi)$ is positive by definition. Therefore its minimum over τ and φ is attained when $\mathrm{P}_{\min}(\tau,\varphi) = 0$. This leads to the condition $2\sigma_s^2 - 2\mathcal{R}\{r_s(\bar{\tau})e^{-j\bar{\varphi}}\} = (\mathbf{r}_{\mathbf{m}s}^H - \mathbf{r}_{\mathbf{m}s}^H(\bar{\tau})e^{j\bar{\varphi}})\,\mathbf{R_m^+}\,(\mathbf{r}_{\mathbf{m}s} - \mathbf{r}_{\mathbf{m}s}(\bar{\tau})e^{-j\bar{\varphi}})$, which is the one obtained in [42] starting from the minimization of the rank of $\mathbf{R}(\tau,\varphi)$ and extended to complex $r_s(\bar{\tau})$. ■

Lemma 1 tells us that the PBC beamformer can be implemented from the points (τ,φ) that minimize $\mathrm{P}_{\min}(\tau,\varphi)$. This is inherently better than minimizing $\mathrm{rank}(\mathbf{R}(\tau,\varphi))$, because $\mathrm{P}_{\min}(\tau,\varphi)$ is continuous, and also because small numerical deviations in $\mathbf{R}(\tau,\varphi)$ do not change $\mathrm{P}_{\min}(\tau,\varphi)$ dramatically.

B. Implementation Aspects

At the receiver, the calculation of $\mathrm{P}_{\min}(\tau,\varphi)$ needs to be performed from an estimate of $\mathbf{R}(\tau,\varphi)$. Then, the Lagrange multiplier theorem can be applied to the function $\mathbf{w}^H\mathbf{R}(\tau,\varphi)\mathbf{w}$ subject to the constraint $\mathbf{w}^H\mathbf{a} = 1$, leading to the system:

$$\mathbf{R}(\tau,\varphi)\mathbf{w} = -\mu\mathbf{a} \quad (9)$$

$$\mathbf{a}^H\mathbf{w} = 1 \quad (10)$$

where $\mu \in \mathbb{C}$ is the associated Lagrange multiplier.

The solution to (9)-(10) is straightforward and well-known when $\mathbf{R}(\tau,\varphi)$ is invertible. However, the matrix $\mathbf{R}(\tau,\varphi)$ may not be full rank in the scenario of interest, and in fact we know that it degenerates for some τ and φ [42]. When $\mathbf{R}(\tau,\varphi)$ is not full rank, two distinct scenarios must be contemplated that depend on the space spanned by $\mathbf{R}(\tau,\varphi)$. If $\mathbf{a} \notin \mathrm{span}\{\mathbf{R}(\tau,\varphi)\}$, then (9) can only be fulfilled when $\mu = 0$, which gives $\mathbf{w} \in \mathrm{null}\{\mathbf{R}(\tau,\varphi)\}$ and leads to $\mathrm{P}_{\min}(\tau,\varphi) = 0$. In contrast, if $\mathbf{a} \in \mathrm{span}\{\mathbf{R}(\tau,\varphi)\}$, then (9) has multiple solutions that together with (10) lead to $\mathrm{P}_{\min}(\tau,\varphi) = (\mathbf{a}^H\mathbf{R}(\tau,\varphi)^+\mathbf{a})^{-1}$. This results into a piecewise definition of $\mathrm{P}_{\min}(\tau,\varphi)$ with sub-domains defined by $\mathrm{span}\{\mathbf{R}(\tau,\varphi)\}$. Unfortunately, these sub-domains are very delicate to calculate numerically.

In order to avoid the need of piecewise definitions, one can attempt to directly find a point (τ_0,φ_0) where $\mathrm{P}_{\min}(\tau,\varphi)$ is minimum by using the sole function:

$$F_1(\tau,\varphi) = (\mathbf{a}^H\mathbf{R}(\tau,\varphi)^+\mathbf{a})^{-1} \quad (11)$$

This function is strictly positive, and must satisfy:

$$\lim_{(\tau,\varphi)\to(\tau_0,\varphi_0)} F_1(\tau,\varphi) = 0$$

because (8) implies that $\mathrm{P}_{\min}(\tau,\varphi)$ must be continuous due to the continuity of all its terms, and because (τ_0,φ_0) verifies $\mathbf{a} \notin \mathrm{span}\{\mathbf{R}(\tau_0,\varphi_0)\}$. As a result, (τ_0,φ_0) can be approached accurately by minimizing $F_1(\tau,\varphi)$. What is certain is that a jump discontinuity exists, since $F_1(\tau_0,\varphi_0) \neq 0$. A discontinuity in $F_1(\tau,\varphi)$ implies a discontinuity in $\mathbf{R}(\tau,\varphi)^+$, and this carries some numerical consequences. In essence, it becomes crucial to select very carefully a threshold for the singular values of $\mathbf{R}(\tau,\varphi)$ that are taken as zero. A threshold that is too large can cause $F_1(\tau,\varphi)$ jump close to the discontinuity value $F_1(\tau_0,\varphi_0)$ when (τ,φ) is still far from (τ_0,φ_0). In contrast, a threshold that is too small allows numerical deviations of the null singular values of $\mathbf{R}(\tau,\varphi)$ to be translated to $F_1(\tau,\varphi)$.

A reasonable approach to avoid the need of a threshold in the computation of (11) is to add a small amount of diagonal loading $\varepsilon > 0$ to the matrix $\mathbf{R}(\tau,\varphi)$. In this way, the function $F_1(\tau,\varphi)$ is approximated by the continuous function $F_2(\tau,\varphi) = (\mathbf{a}^H(\mathbf{R}(\tau,\varphi) + \varepsilon\mathbf{I})^{-1}\mathbf{a})^{-1}$. Not only the use of an standard inverse avoids the discontinuity problem, but also the strictly positive singular values of $\mathbf{R}(\tau,\varphi) + \varepsilon\mathbf{I}$ prevent the numerical instability associated with the null singular values. In order to see how similar $F_2(\tau,\varphi)$ is to the desired function $\mathrm{P}_{\min}(\tau,\varphi)$, the following lemma has been developed.

Lemma 2: The function $F_2(\tau,\varphi) \to \mathrm{P}_{\min}(\tau,\varphi)$ when $\varepsilon \to 0$.

Proof: Let us start by writing $F_2(\tau,\varphi)$ as:

$$\left(\sum_{k=1}^{\rho}\frac{1}{\lambda_k+\varepsilon}|\boldsymbol{\phi}_k^H\mathbf{a}|^2 + \sum_{k=\rho+1}^{L}\frac{1}{\varepsilon}|\boldsymbol{\phi}_k^H\mathbf{a}|^2\right)^{-1} \quad (12)$$

where λ_k and $\boldsymbol{\phi}_k$ are the k-th singular value and k-th singular vector of $\mathbf{R}(\tau,\varphi)$ respectively, and ρ is the rank of $\mathbf{R}(\tau,\varphi)$. The sets $\{\boldsymbol{\phi}_1,\ldots,\boldsymbol{\phi}_\rho\}$ and $\{\boldsymbol{\phi}_{\rho+1},\ldots,\boldsymbol{\phi}_L\}$ form orthonormal bases of $\mathrm{span}\{\mathbf{R}(\tau,\varphi)\}$ and $\mathrm{null}\{\mathbf{R}(\tau,\varphi)\}$ respectively. If $\mathbf{a} \notin \mathrm{span}\{\mathbf{R}(\tau,\varphi)\}$, then $|\boldsymbol{\phi}_k^H\mathbf{a}| \neq 0$ for some $k \in \{\rho+1,\ldots,L\}$, and the limit of (12) when $\varepsilon \to 0$ is zero. In contrast, if $\mathbf{a} \in \mathrm{span}\{\mathbf{R}(\tau,\varphi)\}$, then $|\boldsymbol{\phi}_k^H\mathbf{a}| = 0$ for all $k \in \{\rho+1,\ldots,L\}$, and the limit of (12) when $\varepsilon \to 0$ is $(\sum_{k=1}^{\rho}\frac{1}{\lambda_k}|\boldsymbol{\phi}_k^H\mathbf{a}|^2)^{-1} = (\mathbf{a}^H\mathbf{R}(\tau,\varphi)^+\mathbf{a})^{-1}$. ■

The asymptotic equivalence shown in Lemma 2 means that for a sufficiently small ε, the function $F_2(\tau,\varphi)$ is a good enough estimate of $\mathrm{P}_{\min}(\tau,\varphi)$, so that the minimizers of $F_2(\tau,\varphi)$ are also good enough estimates of the minimizers of $\mathrm{P}_{\min}(\tau,\varphi)$. In addition, $F_2(\tau,\varphi)$ presents a great advantage in terms of simplicity if the value of ε is set to σ_v^2. The noise term that had to be subtracted from $\mathbf{R_{xx}}$ in the calculation of $\mathbf{R}(\tau,\varphi)$ is then reintroduced, eliminating the need to estimate σ_v^2. And since $\sigma_s^2\mathbf{a}\mathbf{a}^H$ can also be added to $\mathbf{R}(\tau,\varphi)$ without influencing the minimization problem (5), we can use:

$$F_2(\tau,\varphi) = (\mathbf{a}^H\bar{\mathbf{R}}(\tau,\varphi)^{-1}\mathbf{a})^{-1} \quad (13)$$

where $\bar{\mathbf{R}}(\tau,\varphi)$ is $\mathbf{R_{xx}} - \mathbf{a}\boldsymbol{\alpha}(\tau,\varphi)^H - \boldsymbol{\alpha}(\tau,\varphi)\mathbf{a}^H$. Similarly to [42], the process to calculate the PBC beamforming weights is then as shown in Algorithm 1.

IV. Simulation Results

In this section, we present some numerical examples related to the implementation of (5). Specifically, we compare the accuracy of the proposed functions to estimate the delay and phase that minimize $\mathrm{P}_{\min}(\tau,\varphi)$, and also analize the response of the corresponding PBC beamformers to multipath and noise.

Throughout all the simulations, we assume that a 5-element array receives a Global Positioning System (GPS) signal and

Algorithm 1: PBC Beamforming.

Given the signals $\mathbf{x}[n]$ and $c[n, \tau, \varphi]$:
1) Calculate $\mathbf{R_{xx}}$ and $\mathbf{r_{x}}_c(\tau, \varphi)$, and then $\bar{\mathbf{R}}(\tau, \varphi)$
2) Solve $(\tau_0, \varphi_0) = \arg\min_{\tau,\varphi} (\mathbf{a}^H \bar{\mathbf{R}}(\tau, \varphi)^{-1}\mathbf{a})^{-1}$
if solution is not unique **then**
take the one with smallest τ and $\varphi = 0$
end if
3) Compute Capon using $\bar{\mathbf{R}}(\tau_0, \varphi_0)$ instead of $\mathbf{R_{xx}}$

two multipath reflections with angles of arrival equal to 30°, −20° and 80° respectively. The post-despreading versions of the received signals are calculated from a triangle function of duration $T_c = 1/1023$ ms. This function is also used as the shape of the autocorrelation of the Gaussian noise. After each integration time $T_{\text{int}} = 20$ ms, 31 samples are taken within an interval approximately $\pm T_c$, centered at the time-delay obtained from a standard Delay-Locked Loop. The relative amplitudes and phases of the multipaths are set to $\beta_1 = 0.95$, $\beta_2 = 0.7$ and $\varphi_1 = -\pi/4$ rad, $\varphi_2 = \pi/2$ rad. The relative delays of the multipaths are defined as $\tau_1 = \xi \cdot 1.5\, T_c$ and $\tau_2 = \xi \cdot 2.5\, T_c$, where $\xi \in [0, 1]$ is a scaling factor. When ξ is zero, all the multipaths are received coherently. When $\xi = 1$, they are received with delays $1.5\, T_c$ and $2.5\, T_c$. Then, the simulation results are represented as a function of ξ, allowing a wide range of correlations to be taken into account. The Carrier-to-Noise-density (C/N$_0$) is set to 45 dBHz, and the observation time of the GPS receiver to $T = 200$ ms.

Fig. 1 and Fig. 2 plot the Root Mean Squared Error (RMSE) of the estimated delay and phase that minimize the functions $S(\tau, \varphi)$, $F_1(\tau, \varphi)$ and $F_2(\tau, \varphi)$, with respect to the desired values τ_e and φ_e. Unlike in [42], the value of p in $S(\tau, \varphi)$ is set here to 0.4. In this way, the resulting mean of the estimates worsens slightly, but the variance improves and so does the RMSE. The threshold needed for the calculation of the pseudoinverse in $F_1(\tau, \varphi)$ is set to 10^{-4}, and the value of ε in $F_2(\tau, \varphi)$ to σ_v^2. Fig. 3 plots the expected value of the Signal-to-Noise-plus-Multipath Ratio (SNMR) at the output of different beamformers. The dotted line corresponds to the traditional Capon (CAP), and each solid line corresponds to a specific implementation of the PBC, with either $S(\tau, \varphi)$, $F_1(\tau, \varphi)$ or $F_2(\tau, \varphi)$. Looking at the plots, Fig. 1 shows that the delays estimated by the distinct functions differ only by a few nanoseconds. Fig. 2 shows that the differences obtained in the estimated phase can be as large as 0.2 rad. Fig. 3 shows that, while CAP suffers serious cancellation effects for $\xi = 0$ and only achieves SNMR $<$ 8 dB for the remaining ξ, all PBC implementations start with SNMR $= 1$ dB and approach SNMR $= 26$ dB as ξ grows. Remarkable enough, $F_2(\tau, \varphi)$ leads to SNMR > 10 dB for $\xi >$ 0.02.

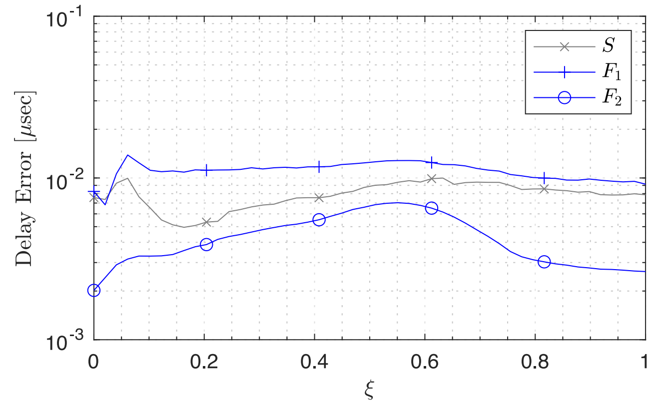

Fig. 1. Root Mean Squared Error of the estimated delay that minimize the functions $S(\tau, \varphi)$, $F_1(\tau, \varphi)$ and $F_2(\tau, \varphi)$, versus delay factor ξ.

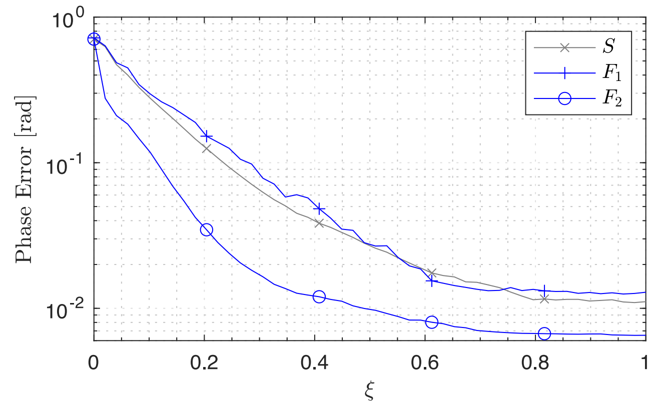

Fig. 2. Root Mean Squared Error of the estimated phase that minimize the functions $S(\tau, \varphi)$, $F_1(\tau, \varphi)$ and $F_2(\tau, \varphi)$, versus delay factor ξ.

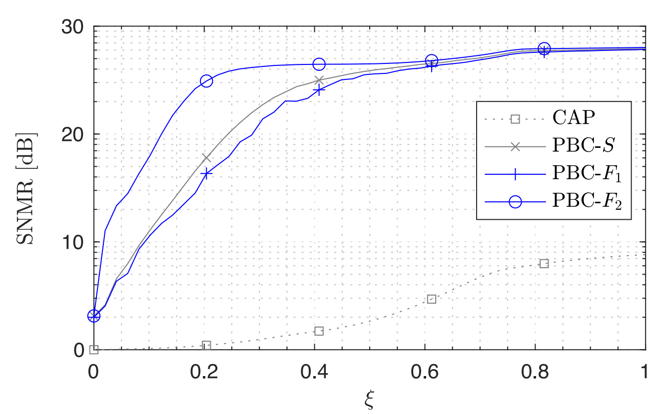

Fig. 3. Expected Signal-to-Noise-plus-Multipath Ratio at the output of different beamformers, versus delay factor ξ.

V. Conclusion

This article reviews the most fundamental aspects of a recently proposed beamforming technique, referred to as Power-Based Capon. In order to overcome the numerical limitations of the original formulation, we have approached the problem to be solved from a different perspective, based on a new cost function that is continuous and less sensitive to numerical deviations in the estimated correlation matrices. The equivalence between the new formulation and the original one is proven, and corresponding implementations are proposed. While the results obtained show that all implementations are valid, they also indicate that $F_2(\tau, \varphi)$ in (13) provides the best results.

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