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A unified approach to the analytical assessment of multi-user diversity with imperfect channel state information: ergodic capacity and robustness analysis†

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SUMMARY

In this paper, we explore the combined use of spatial and multi-user diversity (MUD) in a cellular system where the channel state information (CSI) available at the base station (BS) is subjected to imperfections. To do that, we consider a general statistical approach to describe the degree of CSI imperfection. By doing so, performance assessment is conducted for the generalised case in terms of ergodic system capacity, for which closed-form expressions are derived. In order to gain some insight, practical examples are presented and spatial versus MUD trade-offs are analytically assessed. Next, we analyse several design trade-offs in terms of increased ergodic (long term) system capacity versus robustness to short-term SNR fluctuations for the transmission schemes under consideration. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

Multi-user diversity (MUD) concepts, first introduced by Knopp and Humblet in Reference [1], rely on the assumption that different users in a wireless multi-user system experience independent fading processes. In those circumstances, the aggregated cell throughput can be substantially increased by scheduling in each time-slot the user with the most favorable channel conditions. Besides, in such fading environments the exploitation of transmit spatial diversity (e.g. by means of orthogonal space–time block coding, OSTBC) makes transmission links more robust with low complexity receivers [2, 3]. Both schemes aimed at exploiting either multi-user or transmit spatial diversity have been proposed for packet data services in 3G wireless networks. For that reason, much attention has been recently paid to their combined use and the associated trade-offs.

Previous work: In References [4–6] the inclusion of OSTBC in multi-user schemes was analysed. It was shown that in a multi-user context single-input single-output (SISO) schemes outperform OSTBC-based ones in terms of aggregated cell capacity. Certainly, spatial diversity helps reduce the probability of deep fades but, by averaging over different diversity branches, SNR peaks (those that MUD can exploit) are suppressed as well. As a result, the resulting system capacity is lower. Similar conclusions were drawn for the case of Nakagami fading channels in Reference [7].

Nonetheless, it was proven in References [7, 8] that with perfect channel state information (CSI) at the transmitter, spatial diversity can be efficiently exploited in a multi-user context via optimal transmit beamforming. Unfortunately, perfect CSI is seldom available at the base station (BS). Alternatively, a scheme that concentrates all the power in the transmit antenna with the largest gain was also proposed. For this second approach only low-rate partial CSI is needed but its performance is considerably sensitive to imperfections in the feedback channel.

Recently, several studies show that the increased robustness of OSTBC schemes against imperfect CSI provides significant capacity gains with respect to those of SISO approaches. For instance, in References [9, 10] the authors analyse the impact of *delays* in the feedback

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channel by means of system-level computer simulations and numerical integration, respectively. The consequences of *bandwidth restrictions* were explored in Reference [11], where the authors analysed the impact of introducing OSTBC and antenna selection mechanisms in Selective-MUD environments [12].

Contributions: In this paper, we *analytically* assess the existing trade-offs in the combined use of multi-user and transmit spatial diversity in scenarios where CSI at the scheduler is subjected to impairments. In particular, we conduct an analytical study of the impact of imperfect CSI at the scheduler on SISO and OSTBC-based schemes. To do that, we derive closed-form expressions of the ergodic system capacity associated to both approaches. We do not restrict ourselves to a specific source of imperfections but, instead, we adopt a general statistical approach to its modelling. In order to gain some insight, we then present two practical examples: delayed feedback channel and channel estimation errors. By doing so, we *analytically* prove that using OSTBC pays off in some situations.

Finally, we also assess spatial versus multi-user trade-offs by using mean versus standard deviation plots inspired by theory of modern *portfolio* [13, 14]. This kind of representation is originally used in financial market theory with the aim of assessing the expected profit versus associated risk of an investment. Bartolome introduced this methodology in a multi-antenna communications context in Reference [15] in order to analyse a number of sum-rate versus user fairness trade-offs arising when different bit-allocation schemes are used in combination with transmit zero-forcing beamforming. Other studies were conducted in order to compare optimal and zero-forcing beamforming [16] and orthogonal linear pre-coding [17, 18]. In this paper, we use such a view since both the degree of robustness to short-term SNR fluctuations and its impact in terms of system performance can be easily quantified for the different transmission schemes.

Organisation: The corresponding system model is presented in Section 2. In Section 3, the general statistical approach for modelling the degree of imperfections in the CSI at the scheduler is introduced. Closed-form expressions for the ergodic system capacity are derived for homogeneous and non-homogenous systems in Sections 4 and 5, respectively. Next, the particularisation of the ergodic capacity expressions to practical scenarios with delayed feedback or channel estimation errors is given in Section 6. In Section 7, some numerical results are provided to evaluate the performance of the different schemes in terms of ergodic capacity. After that, the robustness of the different transmission schemes is illustrated in Section 8 by using mean versus standard deviation plots. Finally, in Section 9, the summary and conclusions of this paper are presented.

2. SIGNAL MODEL AND SCHEDULER

Consider the downlink of a wireless system with one BS equipped with multiple antennas ($N_{\rm BS}$), and K single-antenna mobile stations (MS). For an arbitrary time-slot, the received signal at the kth terminal can be modelled as:

$$r_k = \mathbf{h}_k^T \mathbf{s} + n_k$$

where $\mathbf{h}_k \in \mathbb{C}^{N_{\rm BS}\times 1}$ is the channel vector gain between the BS and the kth terminal, for which each component is assumed to be independent and identically distributed, circularly symmetric Gaussian random variable with zero mean and user-dependent variance $\sigma_{h_k}^2 (\mathbf{h}_k \sim \mathcal{CN}(0, \sigma_{h_k}^2 \mathbf{I}_{N_{\rm BS}}))$, $\mathbf{s} \in \mathbb{C}^{N_{\rm BS}\times 1}$ is the symbol vector broadcasted from the BS and $n_k \in \mathbb{C}$ denotes additive Gaussian noise (AWGN) with zero mean and variance σ^2. The active users in the system are assumed to undergo independent Rayleigh fading processes and so does the signal being transmitted from different antennas in the BS. Further, we consider quasi-static fading, i.e the channel response remains constant during one time-slot and, then, it abruptly changes to a new independent realisation. We denote by $\gamma_k = \frac{P_{\rm t}\|\mathbf{h}_k\|^2}{N_{\rm BS}\sigma^2}$ the *instantaneous* signal-to-noise ratio experienced by user k in a given time-slot and by $\bar{\gamma}_k = \frac{P_{\rm t}\mathbb{E}[\|\mathbf{h}_k\|^2]}{N_{\rm BS}\sigma^2} = \frac{P_t \sigma_{h_k}^2}{\sigma^2}$ its long-term average SNR, with $\rm P_t$ standing for the total transmit power. Notice that the total transmit power is constant and evenly distributed among transmit antennas.

At the BS, we will consider two transmission schemes: a SISO configuration ($N_{\rm BS} = 1$) and an OSTBC scheme with $N_{\rm BS} = 2$ transmit antennas[†], more precisely, the well-known Alamouti scheme [3]. For the SISO scheme, the pdf and CDF of the received SNR take the following expressions:

$$f_{\gamma_k,\rm SISO}(\gamma) = \frac{1}{\bar{\gamma}_k} e^{-\frac{\gamma}{\bar{\gamma}_k}}$$
$$F_{\gamma_k,\rm SISO}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_k}} \quad (1)$$

[†] For simplicity, we have assumed only two transmit antennas but the analysis can be easily extended to the general case. Notice, however, that full rate can only be achieved with the proposed configuration [2].

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respectively, whereas for the OSTBC case we can write:

$$f_{\gamma_k,\text{OSTBC}}(\gamma) = \frac{4\gamma}{\bar{\gamma}_k^2} e^{-\frac{2\gamma}{\bar{\gamma}_k}}$$
$$F_{\gamma_k,\text{OSTBC}}(\gamma) = 1 - e^{-\frac{2\gamma}{\bar{\gamma}_k}} \left(\frac{2\gamma}{\bar{\gamma}_k} + 1 \right) \tag{2}$$

Concerning CSI, we assume the availability of a low-rate error-free feedback channel to let user terminals convey *partial* CSI to the BS, in particular their instantaneous SNR. However, in this work we consider that the partial CSI available at the BS, $\hat{\gamma}_k$, differs from the actual SNR, γ_k (further details are given in the next section). As for the scheduling process, it is organised in a slot-by-slot basis following a modified version of the Proportional Fair Scheduling [19] rule. In particular, in each time-slot the user with the maximum normalised SNR is selected for transmission, that is,

$$k^* = \arg\max_k \left\{ \frac{\hat{\gamma}_1}{\mathbb{E}[\hat{\gamma}_1]}, \ldots, \frac{\hat{\gamma}_k}{\mathbb{E}[\hat{\gamma}_k]}, \ldots, \frac{\hat{\gamma}_K}{\mathbb{E}[\hat{\gamma}_K]} \right\} \tag{3}$$

By doing so, users are only allowed to transmit when their instantaneous SNR is near *its own peak* [19], that is with respect to their average SNR. As a result, multi-user diversity can still be exploited and the scheduler will grant access probability of $1/K$ to each user.

3. MODELLING CSI IMPERFECTIONS AT THE TRANSMITTER

In this section, we derive a statistical model describing the degree of CSI imperfection at the BS. In particular, we consider that the SNR estimates available at the transmitter were obtained from a channel vector gain, $\hat{\mathbf{h}}_k$, which differs from the actual channel response, $\mathbf{h}_k$; these two random variables being related with a Gaussian model. In other words, we assume that $\mathbf{h}_k$ conditioned on $\hat{\mathbf{h}}_k$ follows a Gaussian distribution:

$$\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathcal{CN}(\eta_k \hat{\mathbf{h}}_k, \mathbf{\Sigma}_k) \tag{4}$$

where $\eta_k \hat{\mathbf{h}}_k$ and $\mathbf{\Sigma}_k = \sigma_{h_k}^2 \sigma_{\epsilon_k}^2 \mathbf{I}$ are the mean and covariance matrix, respectively. Notice that parameters η_k and $\sigma_{\epsilon_k}^2$ are used to model different source of impairments (some examples are given in Section 6). As an example, the parameter set $\eta_k = 1$ and $\sigma_{\epsilon_k}^2 = 0$ models the case where the actual channel response is perfectly known from its estimate. Conversely, $\eta_k = 0$ and $\sigma_{\epsilon_k}^2 = 1$ implies that the estimate does not give any additional information about the actual channel response (i.e. $\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathbf{h}_k$). Admittedly, this model might not be very accurate for some sources of error but, still, it is very helpful in the analysis and design of communication schemes because of its mathematical tractability [20].

Under those assumptions, it is straightforward to show from Equation (4) that the actual SNR, γ_k, conditioned on its estimate, $\hat{\gamma}_k$, follows a non-central chi-square distribution with $2N_{\text{BS}}$ degrees of freedom [21]:

$$f_{\gamma_k|\hat{\gamma}_k}(\gamma_k|\hat{\gamma}_k) = \frac{N_{\text{BS}}}{\bar{\gamma}_k \sigma_{\epsilon_k}^2} \left(\frac{\gamma_k}{\eta_k^2 \hat{\gamma}_k} \right)^{\frac{2N_{\text{BS}}-2}{4}} e^{\frac{-N_{\text{BS}}(\gamma_k + \eta_k^2 \hat{\gamma}_k)}{\bar{\gamma}_k \sigma_{\epsilon_k}^2}} \times I_{N_{\text{BS}}-1}\left(\frac{2N_{\text{BS}}\sqrt{\eta_k^2 \gamma_k \hat{\gamma}_k}}{\bar{\gamma}_k \sigma_{\epsilon_k}^2} \right) \tag{5}$$

with $I_n(\cdot)$ standing for the nth-order modified Bessel function of the first kind.

In Figure 1, we plot the conditioned pdf in Equation (5) for $\eta_k = 1$ and different values of σ_{ϵ_k}. For small values of σ_{ϵ_k}, γ_k takes values close to $\hat{\gamma}_k$ with high probability (i.e. the knowledge on the actual SNR is accurate). Indeed, in the limiting case where $\sigma_{\epsilon_k} = 0$, the conditioned pdf turns into a delta function centred at $\gamma = \hat{\gamma}_k$, that is, the actual channel response matches the estimated one with probability one.

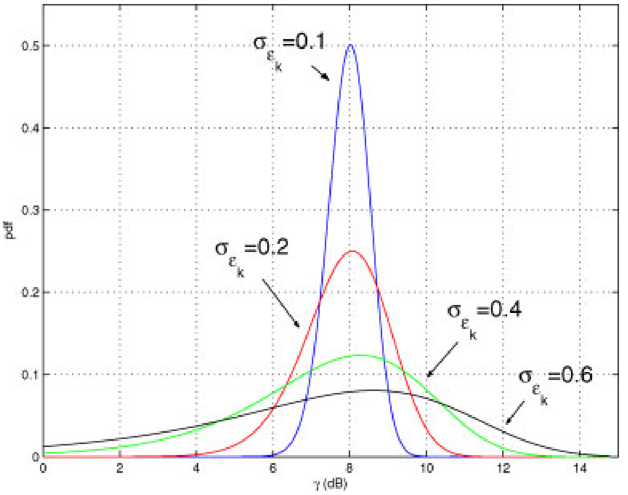

Figure 1. Probability density function of the actual SNR, γ_k, conditioned on its estimate, $\hat{\gamma}_k$, for different values of σ_{ϵ_k}. ($\bar{\gamma}_k = 10\,\text{dB}$, $\hat{\gamma}_k = 8\,\text{dB}$, $\eta_k = 1$, $N_{\text{BS}} = 2$).

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4. ERGODIC SYSTEM CAPACITY FOR HOMOGENEOUS SYSTEMS

In this section, we restrict ourselves to the case where all the users are statistically identical in terms of CSI imperfections ($\eta_k = \eta$ and $\sigma_{\epsilon_k} = \sigma_\epsilon$) and SNR statistics ($\bar{\gamma}_k = \bar{\gamma}$) and, thus, $\mathbb{E}[\hat{\gamma}_k] = \mathbb{E}[\hat{\gamma}]$. The non-homogeneous case will be addressed later, in Section 5.

In a multi-user system, the instantaneous rate achievable by the *scheduled* user k^* over the *equivalent* SISO channel is given by the expression of the channel capacity[‡]:

$$C_I(\gamma) = \log_2(1+\gamma)$$

where γ stands for the instantaneous *post-scheduling* SNR. Because of channel imperfections, the actual SNR, γ, is not fully known at the BS. However, the distribution of γ conditioned on an SNR estimate $\hat{\gamma}$ *is* indeed known (see Equation (5) above). As a result, we will use the *expected* channel capacity conditioned to the estimated SNR as a performance measure [20], that is:

$$C(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma}) = \mathbb{E}_{\gamma|\hat{\gamma}}[C_I(\gamma)] \tag{6}$$

Finally, by averaging over all possible realisations of $\hat{\gamma}$, the ergodic system capacity follows:

$$\overline{C}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = \mathbb{E}_{\hat{\gamma}}[C(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma})] \tag{7}$$

Notice that only the last expression depends on the number of users, the reason for that being that the scheduler makes its decisions according to the *set* of SNR estimates ($\hat{\gamma}_k$, $k = 1, \ldots, K$). Next, we derive the corresponding closed-form expressions for both the SISO and OSTBC cases.

4.1. SISO

For the SISO approach, by recalling Equation (5) and considering that $N_{\mathrm{BS}} = 1$, we can re-write Equation (6) as

$$\begin{aligned} C_{\mathrm{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma}) &= \mathbb{E}_{\gamma|\hat{\gamma}}[C_I(\gamma)] \\ &= \int_{\gamma=0}^{\infty} \log_2(1+\gamma)\, f_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) \mathrm{d}\gamma \\ &= \int_{\gamma=0}^{\infty} \log_2(1+\gamma)\, \frac{1}{\bar{\gamma}\sigma_\epsilon^2} \mathrm{e}^{\frac{-(\gamma+\eta^2\hat{\gamma})}{\bar{\gamma}\sigma_\epsilon^2}} I_0\left(\frac{2\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) \mathrm{d}\gamma \end{aligned} \tag{8}$$

[‡] In this section, subscript k will be dropped for variables associated with the scheduled user.

Before particularising Equation (7) to this case, we must derive the pdf of $\hat{\gamma}$. In the homogeneous case considered throughout this section, the above-mentioned pdf can be readily obtained by resorting to order statistics [22]:

$$\begin{aligned} f_{\hat{\gamma}}(\hat{\gamma}) &= K \frac{\mathrm{e}^{-\frac{\hat{\gamma}}{\mathbb{E}[\hat{\gamma}]}}}{\mathbb{E}[\hat{\gamma}]} \left(1 - \mathrm{e}^{-\frac{\hat{\gamma}}{\mathbb{E}[\hat{\gamma}]}}\right)^{K-1} \\ &= \frac{K}{\mathbb{E}[\hat{\gamma}]} \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \mathrm{e}^{-\frac{\hat{\gamma}(k+1)}{\mathbb{E}[\hat{\gamma}]}} \end{aligned} \tag{9}$$

where the second equality follows from the application of the binomial expansion.

Last, by plugging Equation (8) along with (9) into (7), the following integral results:

$$\begin{aligned} \overline{C}_{\mathrm{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) &= \mathbb{E}_{\hat{\gamma}}[C_{\mathrm{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma})] \\ &= \int_{\hat{\gamma}=0}^{\infty} C_{\mathrm{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) \mathrm{d}\hat{\gamma} \\ &= \frac{K}{\mathbb{E}[\hat{\gamma}]\bar{\gamma}\sigma_\epsilon^2} \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \int_{\gamma=0}^{\infty} \log_2(1+\gamma) \\ &\quad \times \mathrm{e}^{-\frac{\gamma}{\bar{\gamma}\sigma_\epsilon^2}} \int_{\hat{\gamma}=0}^{\infty} \mathrm{e}^{-\hat{\gamma}\left(\frac{\eta^2}{\bar{\gamma}\sigma_\epsilon^2} + \frac{k+1}{\mathbb{E}[\hat{\gamma}]}\right)} I_0\left(\frac{2\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) \mathrm{d}\hat{\gamma}\mathrm{d}\gamma \end{aligned}$$

For the sake of brevity, we show below the final expression only (details can be found in the Appendix):

$$\begin{aligned} \overline{C}_{\mathrm{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) &= K \log_2 \mathrm{e} \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^{k+1} \\ &\quad \times \frac{\mathrm{e}^{\frac{k+1}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}}}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2} E_i\left(\frac{k+1}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}\right) \end{aligned} \tag{10}$$

with $E_i(x)$ standing for the exponential integral function ($E_i(x) = -\int_{-x}^{\infty} \frac{\mathrm{e}^{-t}}{t} \mathrm{d}t$, for $x < 0$).

4.2. *OSTBC*

For the OSTBC case, one should bear in mind that the pdf of the post-scheduling estimated SNR reads [4]:

$$f_{\hat{\gamma}}(\hat{\gamma}) = K\frac{4\hat{\gamma}}{\mathbb{E}[\hat{\gamma}]^2}\mathrm{e}^{-\frac{2\hat{\gamma}}{\mathbb{E}[\hat{\gamma}]}}\left(1-\mathrm{e}^{-\frac{2\hat{\gamma}}{\mathbb{E}[\hat{\gamma}]}}\left(\frac{2\hat{\gamma}}{\mathbb{E}[\hat{\gamma}]}+1\right)\right)^{K-1} \quad (11)$$

Then, analogously to the previous case, Equation (5) and the binomial expansion of (11) should be used in (7) for obtaining the ergodic system capacity (see Appendix):

$$\begin{aligned}\overline{C}_{\mathrm{OSTBC}}(\eta,\sigma_\epsilon,\bar{\gamma},K) &= 4K\log_2\mathrm{e}\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\\ &\times\sum_{n=0}^{k}\binom{k}{n}n!\sum_{m=0}^{n}\binom{n+1}{n-m}\mathbb{E}[\hat{\gamma}]^m\eta^{2m}\\ &\times\bar{\gamma}^{n-m}\sigma_\epsilon^{2(n-m)}(m+1)\mathrm{e}^{\frac{2(k+1)}{\mathbb{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}}\\ &\times\sum_{l=1}^{m+2}\frac{2^{m-l}\Gamma_c\left(l-m-2,\frac{2(k+1)}{\mathbb{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}\right)}{(k+1)^l(\mathbb{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2)^{m+n-l+2}}\end{aligned} \quad (12)$$

where $\Gamma_c(n,x)$ stands for the complementary incomplete gamma function ($\Gamma_c(n,x)=\int_x^\infty \mathrm{e}^{-t}t^{n-1}\mathrm{d}t$).

5. ERGODIC SYSTEM CAPACITY FOR NON-HOMOGENEOUS SYSTEMS

As a natural extension to the analysis conducted in the previous section, we will now consider a scenario where neither CSI statistics nor the imperfections corresponding to different users are identically distributed. This is where the max-normalised SNR scheduling rule comes into play since the terms $\mathbb{E}[\hat{\gamma}_k]$ in Equation (3) differ now.

First, we focus on user k and derive the conditional probability for this user to have the maximum normalised SNR as:

$$\mathrm{Prob}\left(\max_{n\neq k}\frac{\hat{\gamma}_n}{\mathbb{E}[\hat{\gamma}_n]}\leqslant\frac{y}{\mathbb{E}[\hat{\gamma}_k]}\middle|\hat{\gamma}_k=y\right)=\prod_{\substack{n=1\\n\neq k}}^{N}F_{\hat{\gamma}_n}\left(y\frac{\mathbb{E}[\hat{\gamma}_n]}{\mathbb{E}[\hat{\gamma}_k]}\right)$$

where the specific expressions of $F_{\hat{\gamma}_n}(y)$ (and $f_{\hat{\gamma}_n}(y)$) for the different transmission schemes can be found in Equations (1) and (2). Then, it can be proved that the ergodic capacity corresponding to user k depends on [23]:

$$f'_{\hat{\gamma}_k}(y)=\mathrm{Prob}\left(\max_{n\neq k}\frac{\hat{\gamma}_n}{\mathbb{E}[\hat{\gamma}_n]}\leqslant\frac{y}{\mathbb{E}[\hat{\gamma}_k]}\middle|\hat{\gamma}_k=y\right)f_{\hat{\gamma}_k}(y)$$

By particularising the above expression for the different transmission schemes:

$$f'_{\hat{\gamma}_k,\mathrm{SISO}}(y)=\frac{\mathrm{e}^{-\frac{y}{\mathbb{E}[\hat{\gamma}_k]}}}{\mathbb{E}[\hat{\gamma}_k]}\left(1-\mathrm{e}^{-\frac{y}{\mathbb{E}[\hat{\gamma}_k]}}\right)^{K-1} \quad (13)$$

$$\begin{aligned}f'_{\hat{\gamma}_k,\mathrm{OSTBC}}(y) &= \frac{4\hat{\gamma}_k}{\mathbb{E}[\hat{\gamma}_k]^2}\mathrm{e}^{-\frac{2\hat{\gamma}_k}{\mathbb{E}[\hat{\gamma}_k]}}\\ &\times\left(1-\mathrm{e}^{-\frac{2\hat{\gamma}_k}{\mathbb{E}[\hat{\gamma}_k]}}\left(\frac{2\hat{\gamma}_k}{\mathbb{E}[\hat{\gamma}_k]}+1\right)\right)^{K-1}\end{aligned} \quad (14)$$

and comparing Equations (13) and (14) with Equations (9) and (11), one can observe that the capacity results we are interested in can be expressed in terms of Equations (16) and (17), respectively as [23]:

$$\overline{C'}_{k,\mathrm{SISO}}=\frac{1}{K}\overline{C}_{\mathrm{SISO}}(\eta_k,\sigma_{\epsilon_k},\bar{\gamma}_k,K)$$

$$\overline{C'}_{k,\mathrm{OSTBC}}=\frac{1}{K}\overline{C}_{\mathrm{OSTBC}}(\eta_k,\sigma_{\epsilon_k},\bar{\gamma}_k,K)$$

Clearly, the contribution to the ergodic capacity associated to user k (under a max-normalised SNR scheduling rule) exclusively depends on the *number* of users but not on the other users' statistics. Finally, the overall ergodic system capacity for both approaches can be written in closed form as:

$$\overline{C'}_{\mathrm{SISO}}=\frac{1}{K}\sum_{k=1}^{K}\overline{C}_{\mathrm{SISO}}(\eta_k,\sigma_{\epsilon_k},\bar{\gamma}_k,K)$$

$$\overline{C'}_{\mathrm{OSTBC}}=\frac{1}{K}\sum_{k=1}^{K}\overline{C}_{\mathrm{OSTBC}}(\eta_k,\sigma_{\epsilon_k},\bar{\gamma}_k,K) \quad (15)$$

6. PRACTICAL EXAMPLES

So far, we have derived analytical expressions for the general case of channel imperfections. In order to gain some insight, we now particularise those expressions to two practical situations where the Gaussian model applies:

6.1. Delayed feedback channel

Unless reciprocity between the forward and reverse links holds, there always exists a delay between the instants when the SNR is measured at the MS and the actual transmission of data to the scheduled user takes place. Under the assumption of a Jakes' scattering model, $\mathbf{h}_k$ and $\hat{\mathbf{h}}_k$ turn out to be samples of the same Gaussian process. In other words, $\mathbf{h}_k$ and $\hat{\mathbf{h}}_k$ follow a joint complex Gaussian distribution with correlation coefficient $\rho_k = J_o(2\pi f_{d_k} T_k)$, where f_{d_k} stands for the Doppler frequency, T_k is the delay in time units, and $J_o(\cdot)$ denotes the zero-order Bessel function of the first kind. As a consequence, the conditioned pdf can be easily obtained by applying Bayes' Theorem [24 Chapter 10]:

$$f_{\mathbf{h}_k|\hat{\mathbf{h}}_k}(\mathbf{h}_k|\hat{\mathbf{h}}_k) = \frac{f_{\mathbf{h}_k,\hat{\mathbf{h}}_k}(\mathbf{h}_k\hat{\mathbf{h}}_k)}{f_{\hat{\mathbf{h}}_k}(\hat{\mathbf{h}}_k)} = \frac{\mathrm{e}^{-(\mathbf{h}_k-\rho_k\hat{\mathbf{h}}_k)^H\mathbf{R}_k^{-1}(\mathbf{h}_k-\rho\hat{\mathbf{h}}_k)}}{\pi^{N_{\mathrm{BS}}}\det(\mathbf{R}_k)}$$

where $\mathbf{R}_k = \sigma_{h_k}^2(1-\rho_k^2)\mathbf{I}_{N_{\mathrm{BS}}}$ is the covariance matrix. Therefore, the delayed feedback channel fits into the Gaussian model since we have that:

$$\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathcal{CN}(\rho_k\hat{\mathbf{h}}_k, \mathbf{R}_k)$$

and, hence, the ergodic system capacity can be computed by substituting:

$$\eta_k = \rho_k \quad \sigma_{\epsilon_k}^2 = 1-\rho_k^2 \quad \mathbb{E}[\hat{\gamma}_k] = \bar{\gamma}_k$$

into Equation (15), where the last equality holds from the fact that $\mathbf{h}$ and $\hat{\mathbf{h}}$ are samples of the same Gaussian process.

6.2. Imperfect channel estimation

It is common practice to assume that the channel impulse response is perfectly known at the receiver. However, in practical situations only an estimate of the actual channel is available. In the case of a linear MMSE estimator, for instance, we can model the channel estimate as [24]:

$$\hat{\mathbf{h}}_k = \mathbf{h}_k + \mathbf{e}_k$$

where $\mathbf{e}_k \in \mathbb{C}^{N_{\mathrm{BS}}}$ denotes the (vector) channel estimation error for which each component is assumed to be i.i.d circularly symmetric Gaussian random variable with variance $\sigma_{e_k}^2$ and independent from $\mathbf{h}_k$. Then, by applying the Bayes' Theorem one can find that:

$$\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathcal{CN}\left(\frac{1}{1+\Delta_{e_k}}\hat{\mathbf{h}}_k, \sigma_{h_k}^2\frac{\Delta_{e_k}}{1+\Delta_{e_k}}\mathbf{I}_{N_{\mathrm{BS}}}\right)$$

where we have defined $\Delta_{e_k} = \frac{\sigma_{e_k}^2}{\sigma_{h_k}^2}$, which can be interpreted as the inverse of the SNR of the estimation process. Thus, we should take the following parameters into account for the derivation of the ergodic system capacity:

$$\eta_k = \frac{1}{1+\Delta_{e_k}} \quad \sigma_{\epsilon_k}^2 = \frac{\Delta_{e_k}}{1+\Delta_{e_k}} \quad \mathbb{E}[\hat{\gamma}_k] = \bar{\gamma}_k(1+\Delta_{e_k})$$

As a final remark, it should be noted that in this second case, the ergodic capacity expressions derived above, are actually upper bounds. This is because we are only considering the impact of noisy channel estimates on the scheduling process, whereas we disregard its impact on the detection process at the receiver. However, this second issue is out of the scope of this paper since we are interested in the analytical study of the impact caused by incorrect scheduling decisions. For further details, the reader is referred to References [25, 26], where work related to SISO and MIMO channels, respectively, can be found.

7. NUMERICAL EVALUATION

Here we are interested in assessing some spatial versus multi-user trade-offs by evaluating the expressions obtained in the previous sections. Due to space constraints and the similarity between the expressions obtained in the homogeneous and non-homogeneous cases, we restrict ourselves to the former case. In particular, we consider where data packets are sent with an average SNR of $\bar{\gamma} = 10\,\mathrm{dB}$, and $K = 5$ or 30 active users in the system. In Figure 2, we depict the ergodic capacity as a function of MS speed. As for the CSI delay, we adopt the parameters used in Reference [9] for a high speed downlink packet access (HSDPA) scenario where the authors justify that scheduling decisions can be made every 2 ms with a time delay of $T = 4\,\mathrm{ms}$. From the curves, one concludes that for $v = 0\,\mathrm{km/h}$ (i.e. no channel mismatch), the ergodic capacity of SISO is higher than that of OSTBC. However, when the MS speed increases, the degradation experienced by the SISO scheme is larger than that of OSTBC. In other words, the single-antenna approach is less robust to channel uncertainty (i.e. deep fades) arising from delays in the feedback channel. As the number of active users grows, though, the capability of generating post-scheduling SNR peaks improves faster

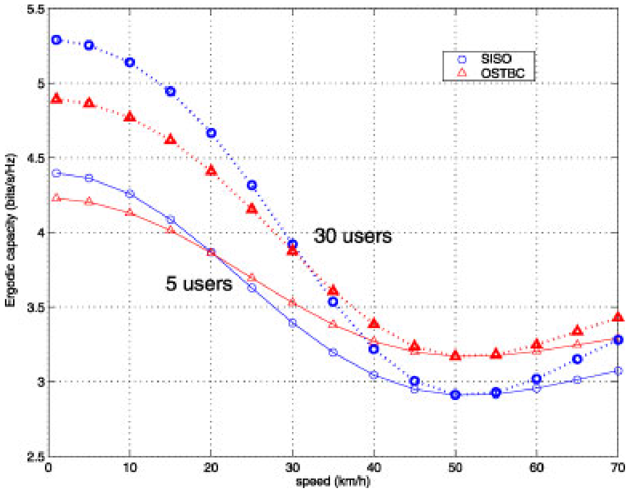

Figure 2. Ergodic capacity versus MS speed for the different transmission schemes (symbols: simulated results, curves: analytical expressions, $K = 5$ and 30 users, $\bar{\gamma} = 10\,\text{dB}$).

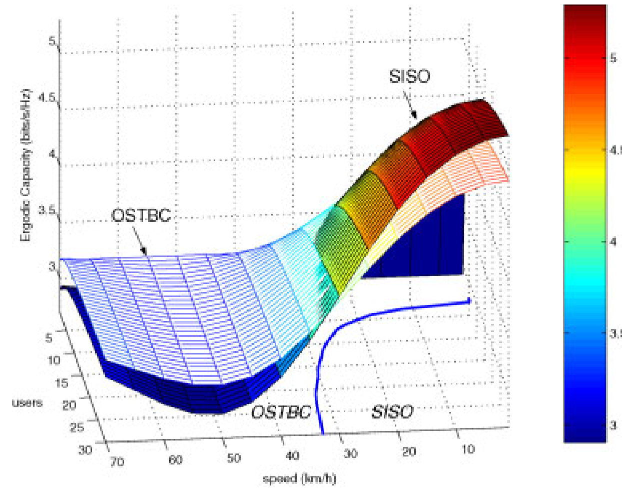

Figure 4. Ergodic capacity versus MS speed and number of active users. The curve on the XY plane defines two regions where one of the schemes outperforms the other ($\bar{\gamma} = 10\,\text{dB}$).

for SISO configurations and, hence, compensates for such SNR uncertainties (i.e. SISO and OSTBC curves cross each other for higher values of the MS speed). Finally, one can also observe that beyond 50 km/h curves are driven again towards higher values of the ergodic capacity. This is because under a Jakes' scattering model assumption, the correlation depends on the zero-order Bessel function of the first kind, which is not a monotonically decreasing function.

Similar conclusions can be drawn from Figure 3, where ergodic capacity is plotted as a function of the parameter Δ_e. However, in this case no extra multi-user diversity gain can be extracted as the degradation in the channel estimates increases. For increasing values of Δ_e, capacity curves reach a floor associated with the performance of a round-robin scheduler (i.e. no MUD gain).

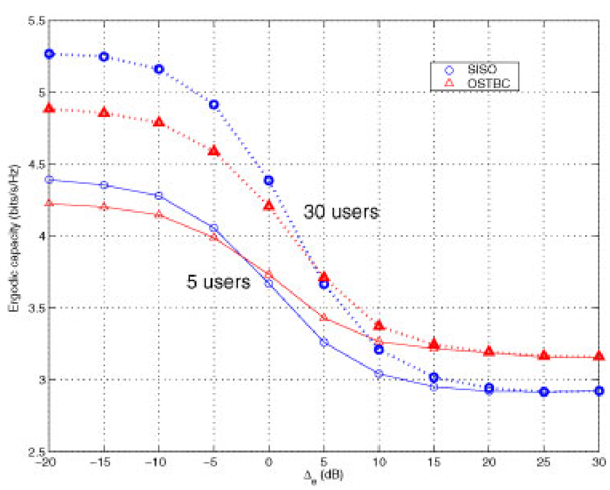

Figure 3. Ergodic capacity versus Δ_e for the different transmission schemes (symbols: simulated results, curves: analytical expressions, $K = 5$ and 30 users, $\bar{\gamma} = 10\,\text{dB}$).

Finally, in order to provide the reader with a complementary point of view, we plot in Figure 4 the ergodic system capacity as a function of the number of users and the MS speed for both transmissions schemes. Results corresponding to the case with imperfect channel estimation are quite similar and are omitted for brevity. We also depict a projection of the intersection of both surfaces onto the XY plane, this curve defining two regions where either SISO or OSTBC performance dominates. For a given number of active users, there always exists a MS speed beyond which the OSTBC-based scheme outperforms a SISO configuration, since OSTBC is inherently robust against delay-induced scheduling errors. In conclusion, OSTBC-based schemes are more appropriate for high mobility scenarios, in particular for a reduced number of active users.

8. ASSESSMENT OF CAPACITY VERSUS ROBUSTNESS TRADE-OFFS

In the previous section, we have shown that the suppression of SNR peaks associated to the stabilising effect of OSTBC penalises system performance. Nonetheless, such stabilising effect provides additional robustness against unfavourable fading conditions resulting from incorrect scheduling decisions. Therefore, there exists a trade-off

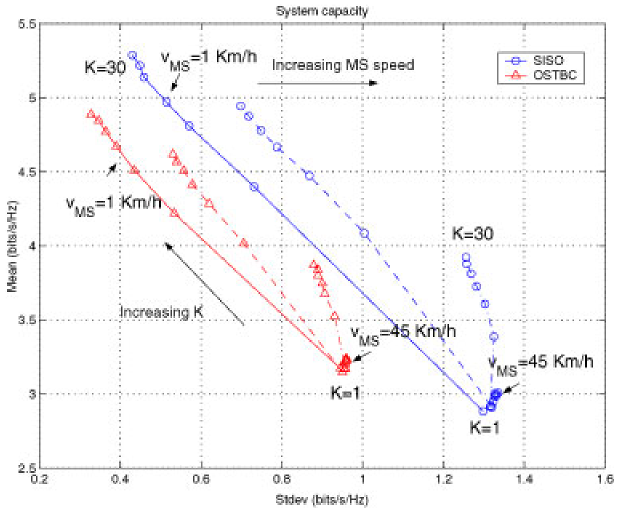

Figure 5. System capacity: mean (average) vs. standard deviation plot as a function of the transmission scheme (SISO/OSTBC), number of users ($K = 1$, 5, 10, 15, 20, 25, 30 users), and MS speed ($\upsilon_{MS} = 1$, 15, 30, 45 km/h). $\bar{\gamma}$=10 dB.

in terms of the degree of robustness to short-term SNR fluctuations and its impact in terms of system performance. In this section, we illustrate such a trade-off with the help of mean versus standard deviation plots inspired by modern *portfolio* theory. By representing in the same plot both the mean and standard deviation of the capacity, one can visually quantify a number of spatial versus MUD trade-offs. Such a view is useful when considering delay-limited services where the short-term fluctuations of channel capacity become relevant and, hence, one should take a closer look at the standard deviation of the ergodic capacity.

In Figure 5, we plot some results for a varying number of users ($K = 1, \ldots, 30$), users' speed ($\upsilon_{MS} = 1, 15, 30, 45$ km/h), and transmission schemes (SISO/OSTBC). For the case of $K = 1$ users and $\upsilon_{MS} = 1$ km/h, the ergodic capacity is higher for OSTBC than for SISO and, simultaneously, the standard deviation is lower (i.e. higher capacity and more stable communication links). However, as soon as the number of users increases beyond $K = 1$ and for mid to low values of υ_{MS}, SISO links outperform OSTBC ones in terms of capacity whereas OSTBC links remain more stable than SISO ones (or, alternatively, the data-rate dispersion among active users for a *short* period of time is lower). One can also observe that for increasing values of the MS speed, both SISO and OSTBC links become less stable (to different extents) since, in those conditions, the number of incorrect scheduling decisions increases. For low and moderate values of MS speed (say $\upsilon_{MS} \leqslant 45$ km/h) this can be partially compensated by increasing the number of active users. Although the CSI is outdated, the scheduler is still capable of selecting the maximum of a set of channel gains. Beyond that point, the available CSI is so degraded that the max-SNR scheduler at the BS turns in practice into a round-robin one, the performance of which does not depend on the number of users.

In summary, a number of non-trivial trade-offs in terms of ergodic capacity versus robustness to short-term fluctuations arise when considering different transmission schemes, impairments in the CSI and terminal count. As usual, design decisions at the cell level will be closely related with the QoS requirements of the services under consideration.

9. CONCLUSIONS

In this paper, we have explored a number of trade-offs resulting from the joint exploitation of multi-user and transmit spatial diversity in scenarios where the CSI available at the scheduler is subjected to impairments. In particular, the impact of imperfect CSI at the BS has been analysed for SISO and OSTBC-based schemes. To do that, a general statistical approach for modelling the degree of CSI innacuracy at the BS has been adopted and closed-form expressions for the ergodic system capacity were derived. In order to gain some insight, practical scenarios with delayed feedback channel and channel estimation errors have been presented. It has been analytically shown that OSTBC-based schemes are more appropriate for scenarios with CSI degradation, in particular for a reduced number of active users.

Also, a novel approach has been used to assess spatial versus multi-user trade-offs. More precisely, the robustness of the different transmission schemes against impairments in the CSI has been assessed by means of mean versus standard deviation plots. It has been shown that, when the information available at the scheduler is degraded, both SISO and OSTBC links become less stable. In particular, a lower degradation is observed for the OSTBC approaches due to the spatial averaging effect. By increasing the number of users, the system becomes more robust to impairments in the CSI, but such an effect vanishes as the available CSI is further degraded.

APPENDIX

In this appendix, we derive the closed-form solutions of the ergodic system capacity for both the SISO and OSTBC approaches.

SISO

In order to derive a closed-form expression of the ergodic system capacity with a homogeneous system, one should solve the following expression for the SISO case:

$$\overline{C}_{\text{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = \frac{K}{\mathbb{E}[\hat{\gamma}]\bar{\gamma}\sigma_\epsilon^2} \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \times \int_{\gamma=0}^{\infty} \log_2(1+\gamma)\, e^{-\frac{\gamma}{\bar{\gamma}\sigma_\epsilon^2}} \times \int_{\hat{\gamma}=0}^{\infty} e^{-\hat{\gamma}\left(\frac{\eta^2}{\bar{\gamma}\sigma_\epsilon^2} + \frac{k+1}{\mathbb{E}[\hat{\gamma}]}\right)} I_0\left(\frac{2\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) d\hat{\gamma} d\gamma$$

With the help of identities [27, Equation 6.614.3], [27, Equation 9.220.2], and [27, Equation 9.215.1] one can readily solve the inner integral in the above equation:

$$\overline{C}_{\text{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = K \log_2 e \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \times \frac{1}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2} \times \int_{\gamma=0}^{\infty} \ln(1+\gamma) e^{-\frac{\gamma(k+1)}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}} d\gamma$$

After that, by resorting to [27, Equation 4.331.2], the latter integral can be solved and written in closed-form as:

$$\overline{C}_{\text{SISO}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = K \log_2 e \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^{k+1} \times \frac{e^{\frac{k+1}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}}}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2} E_i\left(\frac{k+1}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}\right) \tag{16}$$

with $E_i(x)$ standing for the exponential integral function ($E_i(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$, for $x < 0$).

OSTBC

On the other hand, for the OSTBC case we have to plug Equation (5) and the binomial expansion of (11) into (7):

$$\overline{C}_{\text{OSTBC}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = \frac{8K}{\mathbb{E}[\hat{\gamma}]^2\eta\bar{\gamma}\sigma_\epsilon^2} \sum_{k=0}^{K-1} \binom{K-1}{k} \times (-1)^k \sum_{n=0}^{k} \binom{k}{n} \frac{2^n}{\mathbb{E}[\hat{\gamma}]^n} \int_{\gamma=0}^{\infty} \log_2(1+\gamma)\, \gamma^{\frac{1}{2}} e^{-\frac{2\gamma}{\bar{\gamma}\sigma_\epsilon^2}} \times \int_{\hat{\gamma}=0}^{\infty} \hat{\gamma}^{n+1/2} e^{-2\hat{\gamma}\left(\frac{\eta^2}{\bar{\gamma}\sigma_\epsilon^2} + \frac{k+1}{\mathbb{E}[\hat{\gamma}]}\right)} I_1\left(\frac{4\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) d\hat{\gamma} d\gamma$$

In order to solve the inner integral in the above equation, one should resort to identities [27, Equation 8.406.3] and [27, Equation 6.643.4]. Finally, with the help of [27, Equation 8.970.1] and [28, Equation78], the ergodic system capacity can be expressed analytically in terms of the complementary incomplete gamma function ($\Gamma_c(n, x) = \int_x^{\infty} e^{-t} t^{n-1} dt$) as:

$$\overline{C}_{\text{OSTBC}}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = 4K \log_2 e \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \times \sum_{n=0}^{k} \binom{k}{n} n! \sum_{m=0}^{n} \binom{n+1}{n-m} \mathbb{E}[\hat{\gamma}]^m \eta^{2m} \times \bar{\gamma}^{n-m} \sigma_\epsilon^{2(n-m)} (m+1) e^{\frac{2(k+1)}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}} \times \sum_{l=1}^{m+2} \frac{2^{m-l}\Gamma_c\left(l-m-2, \frac{2(k+1)}{\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}\right)}{(k+1)^l (\mathbb{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2)^{m+n-l+2}} \tag{17}$$

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