TIME-DELAY ESTIMATION APPLYING THE EXTENDED INVARIANCE PRINCIPLE WITH A POLYNOMIAL ROOTING APPROACH

Felix Antreich[*], *Josef A. Nossek*[⋆], *Gonzalo Seco*[†], *and A. Lee Swindlehurst*[‡]

German Aerospace Center (DLR), Inst. for Communications and Navigation, 82234 Wessling, Germany, phone: +49 8153 28 2119 [*]

Inst. for Circuit Theory & Signal Processing, Munich University of Technology (TUM), Germany[⋆]

Dept. of Telecommunications & Systems Engineering, Universitat Autonoma de Barcelona (UAB), Spain[†]

Dept. of Electrical Engineering & Computer Science, The University of California, Irvine CA, U.S.A.[‡]

e-mail: felix.antreich@dlr.de[*], nossek@nws.ei.tum.de[⋆], gonzalo.seco@uab.es[†], and swindle@uci.edu[‡]

ABSTRACT

This work treats the problem of joint estimation of time-delay and spatial (direction-of-arrival, DOA) parameters of several replicas of a known signal in an unknown spatially correlated field. Unstructured and structured data models have been proposed for maximum likelihood (ML) estimators, whereas the former suffers from a severe performance degradation in some scenarios, and the latter involves huge complexity. In this work it is shown how the extended invariance principle (EXIP) can be applied to obtain estimates with the quality of those of the structured model, but with much lower complexity than directly utilizing the structured model. We present how to improve the quality of the time-delay estimates obtained with an unstructured spatial model by introducing DOA estimates. DOA estimates are derived either directly applying EXIP with a polynomial rooting approach or with decoupled estimators for temporal and spatial parameters applying Unitary ESPRIT. Both methods are compared with respect to estimation accuracy and complexity. Exemplarily, simulation results for time-delay estimation for GPS (Global Positioning System) are shown and confirm that our proposals both approach the Cramer-Rao lower bound (CRLB) of the structured model.

1. INTRODUCTION

Channel estimation is important in many applications as MIMO channel characterization, radar, synchronization, and Global Navigation Satellite Systems (GNSS) like GPS (Global Positioning System). A simple unstructured data model has been used for such problems in order to have low complexity [1] and a solution for an unknown spatial field was given in [2]. On the other hand the structured data model provides better results [3, 4] but has a high complexity and only seems to be easily manageable in the single path case [5]. The latter work already shows how the extended invariance principle (EXIP) can be applied to refine maximum likelihood (ML) estimates for an unstructured model to achieve the performance available using a structured model.

This work extends the approach given in [5] to a multipath case. We present a two-step approach to improve the quality of time-delay estimates obtained with an unstructured spatial model. As a first step we, apply the Space Alternating Generalized Expectation Maximization (SAGE) algorithm [4] to obtain estimates for an unstructured model. As a second step, we adopt EXIP to refine the time-delay estimates. The important new result with respect to [6] is the derivation of a low complexity polynomial rooting approach, an iterative quadratic maximum likelihood (IQML) [7] algorithm in order to solve the resulting direction-of-arrival (DOA) estimation problem. This new approach provides automatic pairing between the DOA estimates and parameter estimates for the unstructured model. In [6] we applied Unitary ESPRIT for the estimation of the DOAs based on the estimates of the spatial covariance matrix and the spatial signatures for the unstructured model which showed that the proposed two-step approach attains the Cramer-Rao lower bound (CRLB) of the structured model even when suboptimal DOA estimates are introduced. However, for this method the pairing between the DOA estimates and the parameter estimates for the unstructured model also has to be estimated. Exemplarily, simulation results for time-delay estimation of the line-of-sight signal (LOSS) for GPS (Global Positioning System) are included and confirm that our proposal approaches the CRLB of the structured model.

2. DATA MODEL

We assume that L narrowband planar wavefronts, $1 \leq \ell \leq L$ are impinging on an antenna array of M isotropic sensor elements. The noise-plus-interference corrupted baseband signal at the antenna output $\mathbf{y}(t) \in \mathbb{C}^{M\times 1}$ can be modelled as a superposition of L wavefronts and additional temporally white Gaussian noise $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$,with zero-mean and unknown spatial covariance matrix $\mathbf{Q} \in \mathbb{C}^{M\times M}$.

2.1. Structured Model

For the structured data model we define the parameter vector $\boldsymbol{\theta} = \left[\mathrm{Re}\{\boldsymbol{\gamma}\}^{\mathrm{T}}, \mathrm{Im}\{\boldsymbol{\gamma}\}^{\mathrm{T}}, \boldsymbol{\phi}^{\mathrm{T}}, \boldsymbol{\tau}^{\mathrm{T}}\right]^{\mathrm{T}}$ on the domain $D_{\boldsymbol{\theta}}$ with the vector of complex amplitudes $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_\ell, \ldots, \gamma_L]^{\mathrm{T}}$, the vector of azimuth angles $\boldsymbol{\phi} = [\phi_1, \ldots, \phi_\ell, \ldots, \phi_L]^{\mathrm{T}}$, and the vector of time-delays $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_\ell, \ldots, \tau_L]^{\mathrm{T}}$. Thus we can write

$$\mathbf{y}(t) = \sum_{\ell=1}^{L} \mathbf{s}_\ell(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{s}_\ell(t)$ is given by

$$\mathbf{s}_\ell(t) = \mathbf{a}_\ell\left(\phi_\ell\right)\ \gamma_\ell\ c(t-\tau_\ell). \tag{2}$$

Here, $\mathbf{a}_\ell\left(\phi_\ell\right)$ denotes the steering vector of an antenna array and $c(t-\tau_\ell)$ denotes the pseudo-noise (PN) binary sequence with delay τ_ℓ.

2.2. Unstructured Model

For the simpler unstructured model we define the parameter vector $\boldsymbol{\xi} = \left[\mathrm{Re}\{\mathrm{vec}\{\mathbf{H}\}\}^{\mathrm{T}}, \mathrm{Im}\{\mathrm{vec}\{\mathbf{H}\}\}^{\mathrm{T}}, \boldsymbol{\tau}^{\mathrm{T}}\right]^{\mathrm{T}}$ on the domain $D_{\boldsymbol{\xi}}$ with the matrix of spatial signatures $\mathbf{H} = [\mathbf{h}_1 \ \cdots \ \mathbf{h}_\ell \ \cdots \ \mathbf{h}_L] \in \mathbb{C}^{M\times L}$. Here, $\mathrm{vec}\{\cdot\}$ denotes the vec operator, which vectorizes a matrix by stacking its columns. Thus we can alternatively write

$$\mathbf{s}_\ell(t) = \mathbf{h}_\ell\ c(t-\tau_\ell). \tag{3}$$

3. MAXIMUM LIKELIHOOD (ML) ESTIMATION FOR THE UNSTRUCTURED MODEL

The spatial observations are collected at N time instances, as $\mathbf{y}[n] = \mathbf{y}(n\cdot T_s)$ with $n = 1, 2, \ldots, N$. The channel parameters are assumed constant during the observation interval. Collecting the samples of the observation interval leads to

$$\mathbf{Y} = [\mathbf{y}[1], \mathbf{y}[2], \ldots, \mathbf{y}[N]] \in \mathbb{C}^{M\times N}, \tag{4}$$
$$\mathbf{N} = [\mathbf{n}[1], \mathbf{n}[2], \ldots, \mathbf{n}[N]] \in \mathbb{C}^{M\times N}, \tag{5}$$
$$\mathbf{S}(\boldsymbol{\xi}) = [\mathbf{s}[1], \mathbf{s}[2], \ldots, \mathbf{s}[N]] \in \mathbb{C}^{M\times N}. \tag{6}$$

Thus, the unstructured signal model can be written in matrix notation

$$\mathbf{Y} = \mathbf{S}(\boldsymbol{\xi}) + \mathbf{N} = \sum_{\ell=1}^{L} \mathbf{S}_\ell(\boldsymbol{\xi}_\ell) + \mathbf{N} = \mathbf{H}\,\mathbf{C} + \mathbf{N}. \tag{7}$$

Here, $\boldsymbol{\xi}_\ell = \left[\mathrm{Re}\{\mathbf{h}_\ell\}^{\mathrm{T}}, \mathrm{Im}\{\mathbf{h}_\ell\}^{\mathrm{T}}, \tau_\ell\right]^{\mathrm{T}}$ contains the parameters of one wave, and $\mathbf{C} = [\mathbf{c}(\tau_1)\cdots\mathbf{c}(\tau_\ell)\cdots\mathbf{c}(\tau_L)]^{\mathrm{T}} \in \mathbb{R}^{L\times N}$ contains the sampled PN binary sequence for each impinging wavefront $\mathbf{c}(\tau_\ell)$. The negative log-likelihood function for the unstructured data model, neglecting additive constants is [5]

$$\begin{aligned}\Lambda(\boldsymbol{\xi}, \mathbf{Q}) &= N\cdot\ \log\ (\det\ (\mathbf{Q})) \\ &\quad +\ \mathrm{tr}\left(\mathbf{Q}^{-1}\left(\mathbf{Y}-\mathbf{S}(\boldsymbol{\xi})\right)\left(\mathbf{Y}-\mathbf{S}(\boldsymbol{\xi})\right)^{\mathrm{H}}\right),\end{aligned} \tag{8}$$

where $(\cdot)^{\mathrm{H}}$ denotes complex conjugate transposition, $\det(\cdot)$ the determinant operation, and $\mathrm{tr}(\cdot)$ the trace operator. The ML estimate for $\mathbf{Q}$ is given by

$$\hat{\mathbf{Q}} = \frac{1}{N}\left(\mathbf{Y}-\mathbf{S}(\boldsymbol{\xi})\right)\left(\mathbf{Y}-\mathbf{S}(\boldsymbol{\xi})\right)^{\mathrm{H}}. \tag{9}$$

4. SPACE-ALTERNATING GENERALIZED EXPECTATION MAXIMIZATION (SAGE) ALGORITHM

In order to obtain the ML estimates $\hat{\boldsymbol{\xi}}$ for the unstructured model we apply an iterative method, the SAGE algorithm [4]. We assume that L is given. The expectation step (E-Step) can be expressed as

$$\hat{\mathbf{X}}_\ell = \mathbf{Y} - \sum_{\substack{\ell'=1\\ \ell'\neq\ell}}^{L} \mathbf{S}_{\ell'}(\hat{\boldsymbol{\xi}}_{\ell'}), \tag{10}$$

$$\hat{\mathbf{Q}} = \frac{1}{N}\left(\mathbf{Y}-\mathbf{S}(\hat{\boldsymbol{\xi}})\right)\left(\mathbf{Y}-\mathbf{S}(\hat{\boldsymbol{\xi}})\right)^{\mathrm{H}}, \tag{11}$$

and for the maximization step (M-Step) we get

$$\hat{\tau}_\ell = \arg\max_{\tau_\ell}\left\{||\,\hat{\mathbf{Q}}^{-\frac{1}{2}}\,\hat{\mathbf{X}}_\ell\,\mathbf{c}^*(\tau_\ell)||_2^2\right\}, \tag{12}$$

$$\hat{\mathbf{h}}_\ell = \frac{\hat{\mathbf{X}}_\ell\,\mathbf{c}^*(\hat{\tau}_\ell)}{\mathbf{c}^{\mathrm{H}}(\hat{\tau}_\ell)\mathbf{c}(\hat{\tau}_\ell)}. \tag{13}$$

The parameters of each wavefront are estimated sequentially. The E-step and the M-step are performed iteratively for each wavefront until the algorithm converges. Instead of solving a L-dimensional non-linear optimization problem only 1-dimensional optimization procedures need to be solved. Initialization of the SAGE algorithm is carried out by successive interference cancellation starting with $\hat{\boldsymbol{\xi}} = [0, \ldots, 0]^{\mathrm{T}}$ as described in [3]. The initial estimate for $\mathbf{Q}$ is

$$\hat{\mathbf{Q}} = \frac{1}{N}\,\mathbf{Y}\,\mathbf{Y}^{\mathrm{H}}. \tag{14}$$

5. EXTENDED INVARIANCE PRINCIPLE (EXIP)

As a second step we invoke the EXIP [5, 8] in order to refine the ML estimates, $\hat{\boldsymbol{\xi}}$, to achieve the performance using an ML estimate for the structured model, $\hat{\boldsymbol{\theta}}$, for which we would need to solve a $2\cdot L$-dimensional non-linear problem, if solved directly. We briefly recall the EXIP following [5].

Assuming that there exists a function f which is one to one, satisfying

$$\boldsymbol{\xi} = f(\boldsymbol{\theta})\ \in D_{\boldsymbol{\xi}}\quad, \forall_{\boldsymbol{\theta}}\ \in D_{\boldsymbol{\theta}}, \tag{15}$$

and

$$\lim_{N\to\infty}\hat{\boldsymbol{\xi}} = \lim_{N\to\infty} f(\hat{\boldsymbol{\theta}}). \tag{16}$$

Then

$$\hat{\hat{\boldsymbol{\theta}}} = \arg\min_{\boldsymbol{\theta}}\left[\hat{\boldsymbol{\xi}} - f(\boldsymbol{\theta})\right]^{\mathrm{T}}\ \mathbf{W}\ \left[\hat{\boldsymbol{\xi}} - f(\boldsymbol{\theta})\right], \tag{17}$$

is asymptotically (for large N) equivalent to the structured ML estimate $\hat{\boldsymbol{\theta}}$, where

$$\mathbf{W} = \mathrm{E}\left\{\frac{\partial\Lambda(\boldsymbol{\xi},\mathbf{Q})}{\partial\boldsymbol{\xi}\,\partial\boldsymbol{\xi}^{\mathrm{T}}}\right\}\Bigg|_{\substack{\boldsymbol{\xi}=\hat{\boldsymbol{\xi}}\\ \mathbf{Q}=\hat{\mathbf{Q}}}}\ , \tag{18}$$

and $\mathrm{E}\{\cdot\}$ denotes expectation.

For the problem at hand we can write

$$[\hat{\boldsymbol{\xi}}-f(\boldsymbol{\theta})]=\left[\left[\begin{array}{c}\mathrm{Re}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\\ \mathrm{Im}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\\ \hat{\boldsymbol{\tau}}\end{array}\right]-\left[\begin{array}{c}\boldsymbol{\Psi}(\boldsymbol{\phi})\left[\begin{array}{c}\mathrm{Re}\{\boldsymbol{\gamma}\}\\ \mathrm{Im}\{\boldsymbol{\gamma}\}\end{array}\right]\\ \boldsymbol{\tau}\end{array}\right]\right], \quad (19)$$

where

$$\boldsymbol{\Psi}(\boldsymbol{\phi})=\left[\begin{array}{cc}\mathrm{Re}\{\mathbf{I}_L\,\square\,\mathbf{A}\} & -\mathrm{Im}\{\mathbf{I}_L\,\square\,\mathbf{A}\}\\ \mathrm{Im}\{\mathbf{I}_L\,\square\,\mathbf{A}\} & \mathrm{Re}\{\mathbf{I}_L\,\square\,\mathbf{A}\}\end{array}\right], \quad (20)$$

and, $\square$ denotes the Khatri-Rao product, $\mathbf{I}_L$ is a $L\times L$ identity matrix, and $\mathbf{A}=[\mathbf{a}_1(\phi_1)\ \cdots\ \mathbf{a}_\ell(\phi_\ell)\ \cdots\ \mathbf{a}_L(\phi_L)]\in\mathbb{C}^{M\times L}$. The weighting matrix $\mathbf{W}$ can be expressed as

$$\mathbf{W}=\left[\begin{array}{ccc}\mathrm{Re}\{\mathbf{W}_1\} & -\mathrm{Im}\{\mathbf{W}_1\} & \mathrm{Re}\{\mathbf{W}_3\}\\ \mathrm{Im}\{\mathbf{W}_1\} & \mathrm{Re}\{\mathbf{W}_1\} & \mathrm{Im}\{\mathbf{W}_3\}\\ \mathrm{Re}\{\mathbf{W}_3\}^{\mathrm{T}} & \mathrm{Im}\{\mathbf{W}_3\}^{\mathrm{T}} & \mathrm{Re}\{\mathbf{W}_2\}\end{array}\right], \quad (21)$$

with

$$\mathbf{W}_1=2\cdot\left(\mathbf{C}\,\mathbf{C}^{\mathrm{H}}\right)\otimes\hat{\mathbf{Q}}^{-1}, \quad (22)$$

$$\mathbf{W}_2=2\cdot(\boldsymbol{\Upsilon}^{\mathrm{H}}\,\boldsymbol{\Upsilon})\odot(\hat{\mathbf{H}}^{\mathrm{H}}\,\hat{\mathbf{Q}}^{-1}\,\hat{\mathbf{H}}), \quad (23)$$

$$\mathbf{W}_3=\left[\begin{array}{c}2\cdot(\hat{\mathbf{Q}}^{-1}\,\hat{\mathbf{H}})\,\mathrm{diag}\{(\mathbf{c}^{\mathrm{H}}(\hat{\tau}_1)\,\boldsymbol{\Upsilon}\}\\ \vdots\\ 2\cdot(\hat{\mathbf{Q}}^{-1}\,\hat{\mathbf{H}})\,\mathrm{diag}\{(\mathbf{c}^{\mathrm{H}}(\hat{\tau}_\ell)\,\boldsymbol{\Upsilon}\}\\ \vdots\\ 2\cdot(\hat{\mathbf{Q}}^{-1}\,\hat{\mathbf{H}})\,\mathrm{diag}\{(\mathbf{c}^{\mathrm{H}}(\hat{\tau}_L)\,\boldsymbol{\Upsilon}\}\end{array}\right]. \quad (24)$$

Here, $\otimes$ denotes the Kronecker product, $\mathrm{diag}\{\cdot\}$ defines a diagonal matrix, and

$$\boldsymbol{\Upsilon}=\left[\frac{\partial\mathbf{c}(\hat{\tau}_1)}{\partial\hat{\tau}_1}\cdots\frac{\partial\mathbf{c}(\hat{\tau}_\ell)}{\partial\hat{\tau}_\ell}\cdots\frac{\partial\mathbf{c}(\hat{\tau}_L)}{\partial\hat{\tau}_L}\right]. \quad (25)$$

Thus, introducing (19) in (17) and then minimizing (17) with respect to $\boldsymbol{\tau}$, for fixed $\boldsymbol{\gamma}$ and $\boldsymbol{\phi}$ leads to the refined time-delay estimates

$$\hat{\hat{\boldsymbol{\tau}}}=\hat{\boldsymbol{\tau}}+\mathrm{Re}\{\mathbf{W}_2\}^{-1}\left[\mathrm{Re}\{\mathbf{W}_3\}^{\mathrm{T}}\,\mathrm{Im}\{\mathbf{W}_3\}^{\mathrm{T}}\right]\cdot\left[\left[\begin{array}{c}\mathrm{Re}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\\ \mathrm{Im}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\end{array}\right]-\boldsymbol{\Psi}(\boldsymbol{\phi})\left[\begin{array}{c}\mathrm{Re}\{\boldsymbol{\gamma}\}\\ \mathrm{Im}\{\boldsymbol{\gamma}\}\end{array}\right]\right] \quad (26)$$

Note that the second term in (26) acts like a correction of the unstructured ML estimates $\hat{\boldsymbol{\tau}}$.

Replacing (26) in (19) and then minimizing (17) with respect to $\boldsymbol{\gamma}$ leads to

$$\left[\begin{array}{c}\mathrm{Re}\{\hat{\boldsymbol{\gamma}}\}\\ \mathrm{Im}\{\hat{\boldsymbol{\gamma}}\}\end{array}\right]=\left(\boldsymbol{\Psi}^{\mathrm{T}}(\boldsymbol{\phi})\,\boldsymbol{\Omega}\,\boldsymbol{\Psi}(\boldsymbol{\phi})\right)^{-1}\cdot\boldsymbol{\Psi}^{\mathrm{T}}(\boldsymbol{\phi})\,\boldsymbol{\Omega}\left[\begin{array}{c}\mathrm{Re}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\\ \mathrm{Im}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\end{array}\right], \quad (27)$$

where $\boldsymbol{\Omega}$ denotes the Schur complement of block $\mathrm{Re}\{\mathbf{W}_2\}$ in matrix $\mathbf{W}$, namely

$$\boldsymbol{\Omega}=\left[\left[\begin{array}{cc}\mathrm{Re}\{\mathbf{W}_1\} & -\mathrm{Im}\{\mathbf{W}_1\}\\ \mathrm{Im}\{\mathbf{W}_1\} & \mathrm{Re}\{\mathbf{W}_1\}\end{array}\right]-\left[\begin{array}{c}\mathrm{Re}\{\mathbf{W}_3\}\\ \mathrm{Im}\{\mathbf{W}_3\}\end{array}\right]\mathrm{Re}\{\mathbf{W}_2\}^{-1}\left[\mathrm{Re}\{\mathbf{W}_3\}^{\mathrm{T}}\,\mathrm{Im}\{\mathbf{W}_3\}^{\mathrm{T}}\right]\right]. \quad (28)$$

Finally, $\hat{\hat{\boldsymbol{\phi}}}$ can be obtained by substituting $\boldsymbol{\tau}$ and $\boldsymbol{\gamma}$ in (17) for (26) and (27)

$$\hat{\hat{\boldsymbol{\phi}}}=\arg\min_{\boldsymbol{\phi}}\left\{\left[\begin{array}{c}\mathrm{Re}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\\ \mathrm{Im}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\end{array}\right]^{\mathrm{T}}\left(\boldsymbol{\Omega}-\boldsymbol{\Omega}\,\boldsymbol{\Psi}(\boldsymbol{\phi})\cdot\left(\boldsymbol{\Psi}^{\mathrm{T}}(\boldsymbol{\phi})\,\boldsymbol{\Omega}\,\boldsymbol{\Psi}(\boldsymbol{\phi})\right)^{-1}\boldsymbol{\Psi}^{\mathrm{T}}(\boldsymbol{\phi})\,\boldsymbol{\Omega}\right)\left[\begin{array}{c}\mathrm{Re}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\\ \mathrm{Im}\{\mathrm{vec}\{\hat{\mathbf{H}}\}\}\end{array}\right]\right\}. \quad (29)$$

Hence, (29) provides estimates of the DOAs that are asymptotically equivalent to the ML estimates of the structured model. The estimate $\hat{\hat{\boldsymbol{\phi}}}$ is introduced in (27) and (26) improving the delay estimates, thus deriving $\hat{\hat{\boldsymbol{\tau}}}$.

In the following we first derive a complex formulation of (29) and then we develop an IQML approach [7] in order to obtain low complexity ML DOA estimates. The problem given in (29) can be transformed to a complex formulation using the unitary matrices

$$\mathbf{T}_1=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}\mathbf{I}_{LM} & \mathrm{j}\,\mathbf{I}_{LM}\\ \mathbf{I}_{LM} & -\mathrm{j}\,\mathbf{I}_{LM}\end{array}\right], \quad (30)$$

and

$$\mathbf{T}_2=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}\mathbf{I}_{L} & \mathrm{j}\,\mathbf{I}_{L}\\ \mathbf{I}_{L} & -\mathrm{j}\,\mathbf{I}_{L}\end{array}\right]. \quad (31)$$

We define the complex matrices $\tilde{\boldsymbol{\Omega}}=\mathbf{T}_1\,\boldsymbol{\Omega}\,\mathbf{T}_1^{\mathrm{H}}$ with $\tilde{\boldsymbol{\Omega}}\in\mathbb{C}^{2LM\times 2LM}$ and $\tilde{\boldsymbol{\Psi}}(\phi)=\mathbf{T}_1\,\boldsymbol{\Psi}(\phi)\,\mathbf{T}_2^{\mathrm{H}}$ with $\tilde{\boldsymbol{\Psi}}(\phi)\in\mathbb{C}^{2LM\times 2L}$. Thus, (29) can be reformulated as

$$\hat{\hat{\boldsymbol{\phi}}}=\arg\min_{\boldsymbol{\phi}}\left\{\mathrm{tr}\left(\mathbf{P}^{\perp}_{\tilde{\boldsymbol{\Omega}}^{\frac{1}{2}}\tilde{\boldsymbol{\Psi}}(\boldsymbol{\phi})}\,\tilde{\boldsymbol{\Omega}}^{\frac{1}{2}}\left[\begin{array}{c}\mathrm{vec}\{\hat{\mathbf{H}}\}\\ \mathrm{vec}\{\hat{\mathbf{H}}^*\}\end{array}\right]\left[\begin{array}{c}\mathrm{vec}\{\hat{\mathbf{H}}\}\\ \mathrm{vec}\{\hat{\mathbf{H}}^*\}\end{array}\right]^{\mathrm{H}}\tilde{\boldsymbol{\Omega}}^{\frac{1}{2}}\right)\right\}, \quad (32)$$

whereas

$$\mathbf{P}^{\perp}_{\tilde{\boldsymbol{\Omega}}^{\frac{1}{2}}\tilde{\boldsymbol{\Psi}}(\boldsymbol{\phi})}=\mathbf{I}_{2LM}-\tilde{\boldsymbol{\Omega}}^{\frac{1}{2}}\,\tilde{\boldsymbol{\Psi}}(\boldsymbol{\phi})\left(\tilde{\boldsymbol{\Psi}}^{\mathrm{H}}(\boldsymbol{\phi})\,\tilde{\boldsymbol{\Omega}}\,\tilde{\boldsymbol{\Psi}}(\boldsymbol{\phi})\right)^{-1}\tilde{\boldsymbol{\Psi}}^{\mathrm{H}}(\boldsymbol{\phi})\tilde{\boldsymbol{\Omega}}^{\frac{1}{2}}. \quad (33)$$

In case of an uniform linear array (ULA), where the array steering matrix $\mathbf{A}$ has a Vandermonde structure we can solve (32) applying a polynomial reparametrization approach [7]. Therefore, we define

$$\boldsymbol{\Xi}=\left[\begin{array}{cccccc}\mathbf{B}_1^{\mathrm{H}} & & & & & \\ & \ddots & & & & \\ & & \mathbf{B}_L^{\mathrm{H}} & & & \\ & & & \mathbf{B}_1^{\mathrm{T}} & & \\ & & & & \ddots & \\ & & & & & \mathbf{B}_L^{\mathrm{T}}\end{array}\right]\in\mathbb{C}^{2L(M-1)\times 2LM}, \quad (34)$$

such that

$$\mathbf{\Xi}^{\mathrm{H}}\,\tilde{\mathbf{\Psi}}(\boldsymbol{\phi}) = \mathbf{0}, \tag{35}$$

whereas

$$\mathbf{B}_\ell^{\mathrm{H}} = \begin{bmatrix} b_{\ell,1} & b_{\ell,0} & & \mathbf{0} \\ & \ddots & \ddots & \\ \mathbf{0} & & b_{\ell,1} & b_{\ell,0} \end{bmatrix} \in \mathbb{C}^{(M-1)\times M}, \tag{36}$$

and

$$\mathbf{B}_\ell^{\mathrm{H}}\mathbf{a}_\ell(\phi_\ell) = \mathbf{0}. \tag{37}$$

Here, $\mathbf{b}_\ell = [b_{\ell,0}, b_{\ell,1}]^{\mathrm{T}} \in \mathbb{C}^{2\times 1}$ with $\ell = 1,\ldots,L$ comprises the coefficients of a first order polynomial. Due to the very special structure of $\tilde{\mathbf{\Psi}}(\boldsymbol{\phi})$ given in (20), we use a first order polynomial for each of the L wavefronts for reparametrization. Since $\mathbf{\Xi}^{\mathrm{H}}\,\tilde{\mathbf{\Omega}}^{-\frac{\mathrm{H}}{2}} \in \mathbb{C}^{2LM\times 2L(M-1)}$ has rank $2L(M-1)$ and $\tilde{\mathbf{\Omega}}^{\frac{1}{2}}\,\tilde{\mathbf{\Psi}}(\boldsymbol{\phi}) \in \mathbb{C}^{2LM\times 2L}$ has rank $2L$ it follows from (35) that $\mathbf{\Xi}^{\mathrm{H}}\,\tilde{\mathbf{\Omega}}^{-\frac{\mathrm{H}}{2}}$ spans the null space of $\tilde{\mathbf{\Omega}}^{\frac{1}{2}}\,\tilde{\mathbf{\Psi}}(\boldsymbol{\phi})$. Thus,

$$\mathbf{P}^{\perp}_{\tilde{\mathbf{\Omega}}^{\frac{1}{2}}\tilde{\mathbf{\Psi}}(\phi)} = \mathbf{P}_{\tilde{\mathbf{\Omega}}^{-\frac{1}{2}}\mathbf{\Xi}} = \tilde{\mathbf{\Omega}}^{-\frac{1}{2}}\,\mathbf{\Xi}\left(\mathbf{\Xi}^{\mathrm{H}}\,\tilde{\mathbf{\Omega}}^{-1}\,\mathbf{\Xi}\right)^{-1}\mathbf{\Xi}^{\mathrm{H}}\,\tilde{\mathbf{\Omega}}^{-\frac{\mathrm{H}}{2}}. \tag{38}$$

Hence, (32) can be rewritten in terms of the coefficients of the L first order polynomials as

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{b}} \mathrm{tr}\left\{\left(\mathbf{\Xi}^{\mathrm{H}}\,\tilde{\mathbf{\Omega}}^{-1}\,\mathbf{\Xi}\right)^{-1}\mathbf{\Xi}^{\mathrm{H}}\begin{bmatrix}\mathrm{vec}\{\hat{\mathbf{H}}\}\\ \mathrm{vec}\{\hat{\mathbf{H}}^*\}\end{bmatrix}\begin{bmatrix}\mathrm{vec}\{\hat{\mathbf{H}}\}\\ \mathrm{vec}\{\hat{\mathbf{H}}^*\}\end{bmatrix}^{\mathrm{H}}\mathbf{\Xi}\right\} \tag{39}$$

subject to

$$\forall_{\ell=1}^{L} b_{\ell,0} = 1, \tag{40}$$

with

$$\mathbf{b} = [\mathbf{b}_1,\ldots,\mathbf{b}_L,\mathbf{b}_1^*,\ldots,\mathbf{b}_L^*]^{\mathrm{T}}. \tag{41}$$

This problem finally can be solved with very low complexity by applying an IQML algorithm [7] which results to iteratively solving a quadratic problem with linear constraints. The estimates of the DOAs can derived by $\hat{\hat{\phi}}_\ell = -\arcsin\left(\frac{\arg\{-\hat{b}_{\ell,1}\}}{\pi}\right)$. The IQML algorithm is initialized with $\mathbf{b} = [1,0,1,0,\ldots,1,0]^{\mathrm{T}}$. Thus, (39) provides estimates of the DOAs that are asymptotically equivalent to the ML estimates of the structured model, and hence are asymptotically efficient. The estimate $\hat{\hat{\boldsymbol{\phi}}}$ is introduced to (27) and (26) improving the delay estimates $\hat{\boldsymbol{\tau}}$.

6. SIMULATION RESULTS

We assume a centro-symmetric [9] uniform linear array (ULA) with M = 8 isotropic sensor elements with half-wavelength spacing. The one-sided bandwidth of the signal is B = 1.023 MHz. For the PN sequence we apply Gold codes [10] as used for the GPS C/A code with code period T = 1 ms, 1023 chips per code period each with a time duration T_c = 977.52 ns. Signal-to-noise ratio (SNR) denotes the LOSS-to-noise ratio. The effective SNR in dB can be obtained by

$$\mathrm{SNR} = \mathrm{C/N_0} - 10\cdot\log_{10}(2\cdot B) + 10\cdot\log_{10}(N_c), \tag{42}$$

whereas $\mathrm{C/N_0}$ in dB-Hz denotes the carrier-to-noise density ratio and $N_c \in \mathbb{N}$ is the number of code periods within the observation period. We assume $\mathrm{C/N_0}$ = 40.3 dB-Hz and $N_c = 6$ which leads to SNR = -15.03 dB. In this analysis we consider a single reflective multipath as a function of its relative delay to the LOSS ($L = 2$). In the following parameters with the subscript 1 refer to the LOSS and parameters with the subscript 2 refer to the reflection. The reflected multipath and the LOSS are considered to be in-phase, which means $\arg(\gamma_1) = \arg(\gamma_2)$, and the signal-to-multipath ratio (SMR) is 5 dB. Further, we assume a temporally white Gaussian interference with interference-to-signal ratio (ISR) of 40 dB which is responsible for the spatial covariance of the noise plus interference field. The interference is uncorrelated with the signals and the noise. The DOAs for the LOSS and the multipath are $\phi_1 = -30°$ and $\phi_2 = 62°$, and for the interference is $\phi_I = 10°$. Further, we define the relative time-delay between the LOSS and the multipath as $\Delta\tau = |\tau_1 - \tau_2|/T_c$.

In this work a two-step approach is assessed. As a first step the SAGE algorithm is used in order to obtain estimates for the unstructured data model as described in section 4. In the second step EXIP is applied in order to refine the time-delay estimates for the unstructured data model as given in (26) by introducing DOA estimates. The DOA estimates are derived either directly applying EXIP with an IQML algorithm as given in (39) or with decoupled estimators for temporal and spatial parameters applying Unitary ESPRIT as discussed in [6]. However, for the latter approach the pairing between the estimates derived for the unstructured model and the DOA estimates obtained with Unitary ESPRIT additionally has to be estimated [6].

For the SAGE algorithm the quantization precision is chosen to 0.1955 ns ($0.0002\cdot T_c$) for the time-delay estimates $\hat{\boldsymbol{\tau}}$ and a maximum of 10 iteration cycles is set. Determination of the parameter vector $\hat{\boldsymbol{\xi}}_\ell$ which refers to the LOSS is achieved by evaluating the detection criterion

$$\max_{\ell=1,\ldots,L} \frac{||\hat{\mathbf{Q}}^{-\frac{1}{2}}\,\hat{\mathbf{h}}_\ell||_2^2}{M}. \tag{43}$$

In order to derive DOA estimates solving (29) we use the proposed IQML algorithm as given in (39). The convergence condition is satisfied either if $\forall_{\ell=1}^{L}\,|\hat{\hat{\phi}}_\ell^{(k-1)} - \hat{\hat{\phi}}_\ell^{(k)}| < 0.01°$ or if $k > 50$, whereas $k \in \mathbb{N}$ denotes the k-th iteration of the IQML algorithm. Detection of the parameter vector $\hat{\boldsymbol{\theta}}_\ell$ referring to the LOSS is achieved through (43), as for the estimates for the unstructured model.

Instead of solving (29) we apply the approach as described in [6] using Unitary ESPRIT with structured least squares (SLS) with one iteration [9] and spatial smoothing. The prewhitened channel estimate

$$\hat{\mathbf{R}} = \hat{\mathbf{Q}}^{-\frac{1}{2}}\,\hat{\mathbf{H}}\,(\hat{\mathbf{Q}}^{-\frac{1}{2}}\,\hat{\mathbf{H}})^{\mathrm{H}}, \tag{44}$$

is used to follow the covariance approach as given in [9]. For the spatial smoothing with $L = 2$ we define the following selection matrices for two subarrays with maximum overlap

[9]:

$$\mathbf{J}_1 = [\mathbf{0} \;\; \mathbf{I}_{M-1}] \in \mathbb{R}^{M-1\times M}, \;\; \mathbf{J}_2 = [\mathbf{I}_{M-1} \;\; \mathbf{0}] \in \mathbb{R}^{M-1\times M}. \tag{45}$$

Thus, the spatially smoothed covariance matrix is given as

$$\hat{\mathbf{R}}_{ss} = \frac{1}{2}\left(\mathbf{J}_1 \hat{\mathbf{R}} \mathbf{J}_1^{\mathrm{T}} + \mathbf{J}_2 \hat{\mathbf{R}} \mathbf{J}_2^{\mathrm{T}}\right), \tag{46}$$

which is to be used by Unitary ESPRIT with SLS for the DOA estimation. We detect the parameter vector $\hat{\boldsymbol{\theta}}_\ell$ which refers to the LOSS using the detection criterion (43) as for the estimates for the unstructured model and the estimates derived by the IQML algorithm.

In Fig. 1 the probability of miss-detection of the parameter vector $\boldsymbol{\theta}_\ell$ referring to the LOSS is depicted. The decision on the LOSS is taken by evaluating (43). Here, IQML denotes the approach solving (39) and ESPRIT denotes the approach using Unitary ESPRIT for DOA estimation with decoupled estimators for temporal and spatial parameters. Miss-detection of the LOSS for the unstructured model using the SAGE algorithm did not occur.

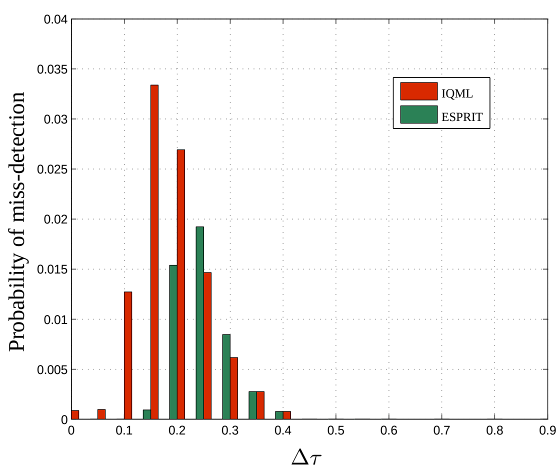

Fig. 1. Probability of miss-detection of LOSS versus $\Delta\tau$

Miss-detection of the LOSS occurs due to several reasons. This includes miss-match of the pairing of the estimates for the unstructured model and the DOA estimates derived by Unitary ESPRIT [6], non-convergence of the IQML algorithm when $k > 50$, and in some rare cases failure of resolving the LOSS. All these cases of miss-detection of the LOSS are excluded from the error statistics which are presented in the following. In a practical implementation miss-detection can be avoided by consistency check of the DOA estimates of the LOSS and for GNSS applications the navigation almanac data can be used to avoid miss-detection. However, simulation results confirmed that miss-detection of the LOSS did not have a significant impact on the time-delay estimates of the LOSS.

In Fig. 2 the root mean square error (RMSE) of $\hat{\tau}_1$ and $\hat{\hat{\tau}}_1$ are depicted. RMSE $\hat{\hat{\tau}}_1$, ESPRIT denotes the approach where the DOA estimates are obtained by Unitary ESPRIT and RMSE $\hat{\hat{\tau}}_1$, IQML denotes the case where the DOA estimates are obtained by solving (39).

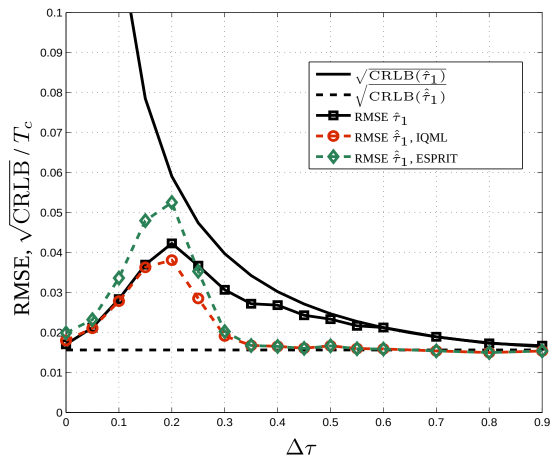

Fig. 2. RMSE of $\hat{\tau}_1$ and $\hat{\hat{\tau}}_1$ versus $\Delta\tau$

In Fig. 2 the SAGE algorithm for small $\Delta\tau$ becomes biased and the RMSE $\hat{\tau}_1$ is below $\sqrt{\mathrm{CRLB}(\hat{\tau}_1)}$, since the interference cancellation in the E-step (10) is not able to separate the two waves if $\Delta\tau$ is very small [4]. In these cases the estimate of one hidden data space, $\hat{\mathbf{X}}_1$ is an estimate for a superposition of the two wavefronts and the other, $\hat{\mathbf{X}}_2$ includes only noise.

In Fig. 3 and Fig. 4 the RMSE $\hat{\hat{\phi}}_1$ and the RMSE $\hat{\hat{\phi}}_2$ for the two methods are depicted.

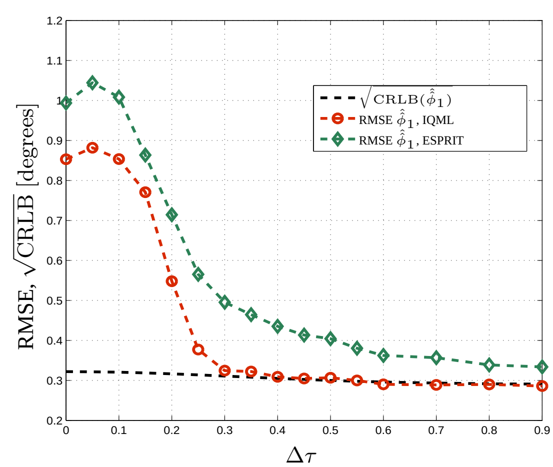

Fig. 3. RMSE of $\hat{\hat{\phi}}_1$ versus $\Delta\tau$

In case we solve (39) with the proposed IQML algorithm, the DOA of the multipath can not be resolved for $\Delta\tau < 0.3$ as depicted in Fig. 4 . This is consistent with the behavior of the SAGE algorithm, which towards small $\Delta\tau$ more and more

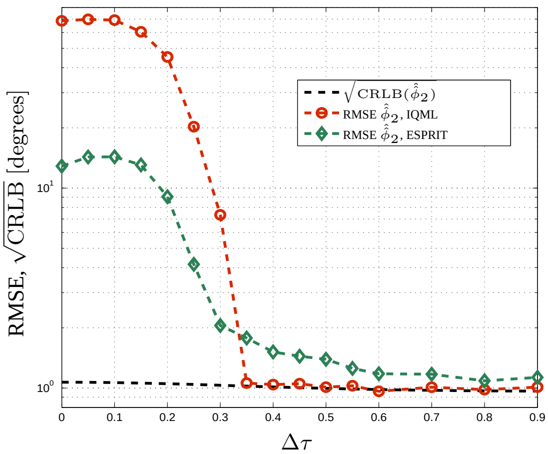

Fig. 4. RMSE of $\hat{\hat{\phi}}_2$ versus $\Delta\tau$

loses its capability of resolving the two paths as described above (cf. Fig. 2). Unitary ESPRIT shows better capability to resolve the DOA of the multipath due to the use of spatial smoothing. However, this does not improve the corresponding RMSE $\hat{\tau}$.

7. CONCLUSIONS

In this work we proposed a two-step approach to achieve estimates for a structured model for the multipath case in an unknown spatially colored field. As a first step the SAGE algorithm is used to obtain estimates for the unstructured data model. In the second step EXIP is applied in order to refine the time-delay estimates for the unstructured data model as given in (26) by introducing DOA estimates. The DOA estimates are derived either directly applying EXIP with an IQML algorithm as given in (39) or with decoupled estimators for temporal and spatial parameters applying Unitary ESPRIT as discussed in [6]. Exemplarily, simulation results for time-delay estimation in GPS are shown which confirm that the proposed two-step approach attains the CRLB of the structured model. The developed low complexity IQML algorithm to solve the DOA estimation problem resulting from EXIP shows better estimation accuracy for the time-delay of the LOSS than the approach using Unitary ESPRIT, however its probability of miss-detection of the LOSS being slightly higher compared to the other approach.

References

[1] A. L. Swindlehurst, "Time Delay and Spatial Signature Estimation Using Known Asynchronous Signals," *IEEE Transactions on Signal Processing*, vol. 46, no. 2, February 1998.

[2] G. Seco, A. L. Swindlehurst, and D. Astely, "Exploiting Antenna Arrays for Synchronization," in *Signal Processing Advances in Wireless and Mobile Communications, Volume 2: Trends in Single- and Multi-User Systems*, G. B. Giannakis, P. Stoica, Y. Hua, and L. Tong, Eds., pp. 403 – 430. Prentice-Hall, 2000.

[3] F. Antreich, J. A. Nossek, and W. Utschick, "Maximum Likelihood Delay Estimation in a Navigation Receiver for Aeronautical Applications," *Aerospace Science and Technology, doi:10.1016/j.ast.2007.06.005*, 2008.

[4] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. I. Pedersen, "Channel Parameter Estimation in Mobile Radio Environments Using the SAGE Algorithm," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 3, March 1999.

[5] A. L. Swindlehurst and P. Stoica, "Maximum Likelihood Methods in Radar Array Signal Processing," *Proceedings of the IEEE*, vol. 86, no. 2, February 1998.

[6] F. Antreich, J. A. Nossek, G. Seco, and L. A. Swindlehurst, "Time delay estimation in a spatially structured model using decoupled estimators for temporal and spatial parameters," in *Proceedings of the International ITG Workshop on Smart Antennas WSA*, Darmstadt, Germany, February 2008.

[7] P. Stoica and K. Sharman, "Maximum Likelihood Methods for Direction-of-Arrival Estimation," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 7, July 1990.

[8] P. Stoica and T. Söderström, "On Reparametrization of Loss Functions Used in Estimation and the Invariance Principle," *Signal Processing*, vol. 17, pp. 383-387, August 1989.

[9] M. Haardt, *Efficient One-, Two-, and Multidimensional High-Resolution Array Signal Processing*, Ph.D. thesis, Department Electrical Engineering of Munich University of Technology, 1996.

[10] B. W. Parkinson and J. J. Spilker, *Global Positioning System: Theory and Applications*, vol. 1, Progress in Astronautics and Aeronautics, 1996.