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ADAPTIVE BEAM SELECTION TECHNIQUES FOR OPPORTUNISTIC BEAMFORMING

Jose Lopez Vicario
Centre Tecnològic de
Telecomunicacions de Catalunya
SPAIN
jose.vicario@cttc.es

Roberto Bosisio, Umberto Spagnolini
Politecnico de Milano
ITALY
rbosisio@elet.polimi.it
spagnoli@elet.polimi.it

Carles Anton-Haro
Centre Tecnològic de
Telecomunicacions de Catalunya
SPAIN
carles.anton@cttc.es

ABSTRACT

In multi-antenna downlink systems the optimization of linear precoding is severely hampered by the amount of feedback. An efficient solution consists in opportunistic schemes, which generate M random beams and schedule the users with the highest signal-to-noise-plus-interference ratios (SINRs), which can be made available to the transmitter with very little feedback. Although this technique has been shown to be optimal for asymptotically large number of users, severe performance degradation occurs in practical system where the number of users is limited. In this paper we propose an enhancement of this strategy based on an adaptive beam selection procedure. Instead of transmitting all the generated beams, the scheduler picks the optimum subset of beams that maximizes the system sum-rate according to the feedback information. We propose and compare several beam selection algorithms according to different complexity requirements. In particular, we show that the proposed approaches give substantial gains with respect to conventional opportunistic schemes.

I. INTRODUCTION

In the downlink of a Single-Input Single-Output (SISO) multi-user wireless system, it is well known that the average cell throughput can be increased when in each slot the user with better channel conditions is scheduled [1]. Conversely, this is not the optimum strategy for the Multiple-Input Multiple-Output (MIMO) Broadcast Channel, given that multi-antenna capabilities are not exploited to serve several users simultaneously. In particular, the capacity region of the gaussian MIMO broadcast channel can be achieved with dirty paper coding (DPC) [2]. However, DPC may not be considered an appropriate scheme for real applications, since it is not easy implementable due to the successive encodings and decodings.

In [3], it was proposed the Zero Forcing (ZF) scheme as an alternative to DPC. It was shown that with this suboptimal (but very simple for implementation) strategy, the same capacity growth rate as in DPC can be achieved when the number of users is large. Unfortunately, the ZF scheme (like DPC) requires *perfect* channel side information (CSI), which is seldom available at the base station. For that reason, opportunistic schemes with multiple beams based on partial CSI at the base station (BS) have recently attracted interest [4]. The main idea is to generate a random set of beamformers at the BS to schedule users according to signal-to-interference-and-noise ratio (SINR) measurements. In this scheme, users only have to report SINR relative to the selected precoding and, then, the amount of information to be sent in the feedback is considerably reduced. Performance has been shown to be very effective for systems with a large number of users, as the sum capacity for partial CSI has the same capacity growth rate as for the case of perfect CSI at the BS [4].

However, performance can be far from satisfactory in systems with a practical number of users. Kountoris and Gesbert proposed two techniques with the aim of improving opportunistic beamfoming with multiple beams in sparse networks. In [5], they proposed an approach where channel time correlation is exploited to search the optimal set of random beamvectors. In order not to depend excessively on the properties for the channel, a different approach was proposed in [6]. More precisely, it was proposed a scheme where a low rate feedback is used for selecting the best group of users, for which more efficient beamforming techniques are applied: MMSE beamformer and iterative power allocation with full and partial CSI at the transmitter, respectively.

In this paper, we propose approaches aimed at improving opportunistic beamfoming with multiple beams in sparse networks. However, we focus our attention on the simple philosophy of the original approach where users are served with uniform power allocation. As we show in this paper, using all the available beams active may not be the optimum solution for scarcely populated cells with constant transmit power at the base station. For that reason, we propose beam selection procedures where the optimum subset of beams is selected for transmission. In particular, we propose different beam selection techniques where the only required information are the channel gains or the SINR.

II. SIGNAL MODEL

Consider the downlink of a cellular system with one base station equipped with M antennas, and K single-antenna Mobile Stations (MS). In order to serve multiple users in the same time-slot, a linear precoding matrix is used at the base station. In particular, we follow a random beamforming strategy [4]. More precisely, in each time slot we construct a random matrix $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_M]$, where $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$, $i = 1..M$, are random orthonormal vectors generated according to an isotropic distribution [7]. Then, these vectors are used for transmitting information to the users with the highest SINRs. Unlike the approach in [4], we do not necessarily transmit with all the beams vectors $\mathbf{w}_i$. That is, the transmission is made with a subset of active beams $\mathcal{B} \subset \mathcal{W} = \{\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_M\}$. Further details about the beam selection procedure will be given in Section V. Therefore, the received signal at the k-th MS is given by:

$$r_k = \mathbf{h}_k^T \mathbf{W}_{\mathcal{B}} \mathbf{s}_{\mathcal{B}} + n_k$$

where the time index has been dropped for the ease of notation, $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ is the channel vector gain between the BS and the k-th MS, for which each component is assumed to be independent and identically distributed, circularly symmetric Gaussian random variable with zero mean and unit variance ($\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{I}_M)$), $\mathbf{W}_{\mathcal{B}} \in \mathbb{C}^{M \times B}$ is the precoding matrix constructed with the columns of $\mathbf{W}$ corresponding to the subset of active beams $\mathcal{B}$, $\mathbf{s}_{\mathcal{B}} \in \mathbb{C}^{B \times 1}$ is the symbol vector broadcasted from the BS, $B = \text{card}(\mathcal{B}) \leq M$ is the number of simultaneous served users and $n_k \in \mathbb{C}$ denotes additive Gaussian noise (AWGN) with zero mean and variance σ^2. The active users in the system are assumed to undergo independent Rayleigh fading processes. Further, we consider quasi-static fading, i.e, the channel response remains constant during one time-slot and it changes to a new independent realization in the subsequent one.

Concerning channel state information, we assume perfect CSI knowledge for *each* user at the receive side, and the availability of a low-rate error-free feedback channel to convey partial CSI to the transmitter. Finally, the total transmit power, P_t, is constant and evenly distributed among active beams, i.e., $\mathbb{E}\{\mathbf{s}_{\mathcal{B}}^H \mathbf{s}_{\mathcal{B}}\} = P_t$. Then, we define $\rho = \frac{P_t}{\sigma^2}$ as the average SNR of the system.

III. Post-scheduling SINR statistics

According to the signal model presented in the previous section, the SINR associated to user k with beam i can be expressed as follows:

$$\mathsf{SINR}_{k,i}(\mathcal{B}) = \frac{|\mathbf{h}_k^T \mathbf{w}_i|^2}{B/\rho + \sum_{\substack{j \in \mathcal{B} \\ j \neq i}} |\mathbf{h}_k^T \mathbf{w}_j|^2} = \frac{z}{B/\rho + y} \quad (1)$$

Since we assume that all users experience i.i.d Rayleigh fading and the beamformers are orthonormal to each other, z and y become independent chi-square random variables, $z \sim \chi_2^2$ and $y \sim \chi_{2B-2}^2$ [4]. As a consequence, both the CDF and pdf of the SINR can be expressed as:

$$F_{\mathsf{SINR}}(\gamma) = 1 - \frac{e^{-\frac{\gamma B}{\rho}}}{(1+\gamma)^{B-1}} \quad (2)$$

$$f_{\mathsf{SINR}}(\gamma) = \frac{e^{-\frac{\gamma B}{\rho}}}{(1+\gamma)^B} \left(\frac{B}{\rho}(1+\gamma) + B - 1 \right)$$

Notice that in a i.i.d Rayleigh fading scenario the SINR statistics depend on the *number* of interfering beams but not on the number of *transmit antennas*, M, as long as the number of active beams is lower than M.

The scheduling process is organized in a slot-by-slot basis following a *max*-SINR (*greedy*) rule. That is, for beam i, the scheduler selects the active user k_i^* satisfying:

$$k_i^* = \arg \max_{k=1..K} \{\mathsf{SINR}_{k,i}\}$$

where it is assumed that a different user is selected for each beam[1]. Since all users experience i.i.d Rayleigh fading, the CDF of the *post-scheduling* SINR, $F_{\mathsf{SINR}^*}(\gamma)$, i.e. the SINR experienced by the scheduled user can be readily expressed in terms of Eq. (2) as:

$$F_{\mathsf{SINR}^*}(\gamma) = (F_{\mathsf{SINR}}(\gamma))^K = \left(1 - \frac{e^{-\frac{\gamma B}{\rho}}}{(1+\gamma)^{B-1}} \right)^K$$

Finally, by simply differentiating the above expression the pdf can be written as:

$$f_{\mathsf{SINR}^*}(\gamma) = K \frac{e^{-\frac{\gamma B}{\rho}}}{(1+\gamma)^B} \left(\frac{B}{\rho}(1+\gamma) + B - 1 \right) \times \left(1 - \frac{e^{-\frac{\gamma B}{\rho}}}{(1+\gamma)^{B-1}} \right)^{K-1}$$

These expressions will be used in the following section in order to compute the resulting sum-rate.

IV. Asymptotic behavior of the Sum-rate

According to the proposed scheduling policy, the sum-rate achievable when B beams are active is given by:

$$R(\mathcal{B}) = \mathbb{E}\left\{ \sum_{i \in \mathcal{B}} \log_2 \left(1 + \max_{1 \leq k \leq K} \mathsf{SINR}_{k,i}(\mathcal{B}) \right) \right\} = B \int_{\gamma=0}^{\infty} \log_2(1+\gamma) f_{\mathsf{SINR}^*}(\gamma) d\gamma \quad (3)$$

In [4], Sharif and Hassibi deived a closed-form expression for the asymptotic case ($K \to \infty$) which exhibits the same sum-rate growth as DPC. For a practical scenario with a finite number of users, though, resorting to numerical integration is needed. Still, this expression is tractable when the average SNR of the system is arbitrarily high ($\rho \to \infty$) as shown by the authors in [8]. In this case, the pdf of the post-scheduling SINR can be written as follows:

$$f_{high,\mathsf{SINR}^*}(\gamma) = K \frac{B-1}{(1+\gamma)^B} \left(1 - \frac{1}{(1+\gamma)^{B-1}} \right)^{K-1}$$

As a consequence, the integral in Eq. (3) becomes considerably simpler and a closed-form expression can be derived for the sum-rate (see [8] for further details):

$$R_{high} \approx \frac{B}{B-1} \log_2(e) \sum_{k=1}^{K} \frac{1}{k} \quad (4)$$

where the term $\sum_{k=1}^{K} \frac{1}{k}$ accounts for the multi-user gain. Two main conclusions can be drawn from the expression above. First, the sum-rate tends to infinity when only one beam is used since, in this case, the sum-rate capacity grows logarithmically with the average SNR. Besides, the sum-rate decreases with the number of active beams due to the term $B/(B-1)$. In summary, using only one active beam is the optimum strategy in the high-SNR regime.

[1] The probability that one user achieves the highest SINR over more than one beam is negligible when the number of users is large compared with the number of active beams ($K >> B$) [4].

Table 1: Sum-rate performance for low-SNR scenarios (200 users, ρ=-5 dB).

	B=1	B=2	B=3	B=4
Simulation	1.485	1.759	1.836	1.893
Eq. (5)	1.485	1.848	2.071	2.179

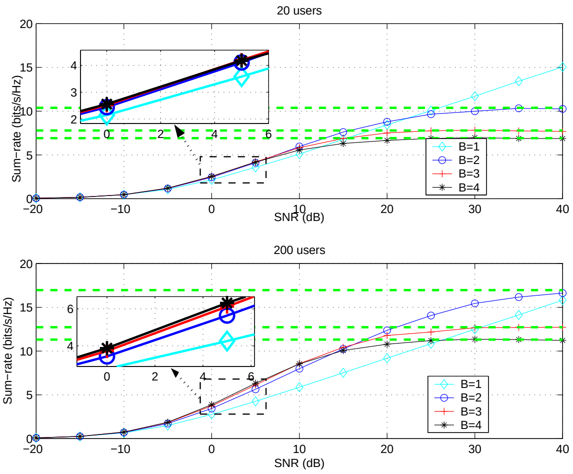

Figure 1: Sum-rate vs. average SNR for a different number of active beams (B). Dotted lines correspond to the approximate results for the high-SNR regime given by Eq. (4). Top: 20 users, bottom: 200 users.

In a low-SNR regime (i.e., $\rho \to 0$), a totally different scenario results. By neglecting the interference term in eq. (1), the post-scheduling pdf can be expressed as:

$$f_{low,\mathsf{SINR}^*}(\gamma) = K\frac{e^{-\frac{\gamma B}{\rho}}}{\rho}\left(1 - e^{-\frac{\gamma B}{\rho}}\right)^{K-1}$$

Finally, after some algebraic manipulations the sum-rate can expressed as:

$$R_{low} \approx -BK\log_2 e \sum_{k=0}^{K-1}\binom{K-1}{k}\frac{(-1)^k}{k+1}e^{B\frac{(k+1)}{\rho}} \times E_i\left(-B\frac{(k+1)}{\rho}\right) \quad (5)$$

with $E_i(x)$ standing for the exponential integral function ($E_i(x) = -\int_{-x}^{\infty}\frac{e^{-t}}{t}dt$, for $x < 0$) [9, Eq. 8.211.1]. Table 1 and Fig. 1 illustrate the accuracy of the approximate sum-rate expressions for the low- and high-SNR regimes, respectively[2]. The following conclusions are in line. First, in noise-limited scenarios a higher number of active beams turns out to be beneficial. In particular, this is more relevant when the population of users is high (see top and bottom plots in Fig. 1). Second, in interference limited scenarios with a moderate number of users, the use of multiple beams does not pay off. Instead, a careful selection of *the* active beams gives better results.

V. Beam Selection Algorithms

In the previous section, we have shown that using all the set of active beams may not be the optimal transmission configuration. For that reason, we now derive several beam selection algorithms capable of identifying the best subset of beams (and users) according to scenario conditions. In addition, the number of users in the system can be *virtually* increased by using beam selection because the number of SINR combinations is augmented[3], which is quite interesting for sparse networks. In particular, we propose the following beam selection techniques:

A. *Optimum Beam Selection*

This algorithm conducts an exhaustive search over all the possible subsets of beams and users. For a fixed number of active beams B, a total of $\binom{M}{B}KB$ SINRs must be computed in order to find the best transmission configuration. Next, by considering all the possible number of active beams, a total of $\sum_{B=1}^{M}\binom{M}{B}KB = KM2^{M-1}$ SINR computations results. Finally, it is worth noting that this algorithm requires all the gains $|\mathbf{h}_k^T\mathbf{w}_i|^2$ to be known. Therefore, it is necessary for each user to report M integer numbers to the base station over the feedback channel.

In order to reduce the computational complexity of the beam selection procedure, next we present some sub-optimum approaches:

B. *Bottom-up Trellis Beam Selection*

The first sub-optimum methodology is based on a bottom-up procedure. More precisely, the algorithm is started by selecting the best user for each beam. After that, the selected users in the first step are combined (with their associated beams) in order to find the best combination with two active beams. The algorithm is iterated until the combination where all the beams are active ($B = M$) is reached. Basically, the objective is to reduce the computational cost by focusing on the users achieving the highest gains with only one active beam. By doing so, KM computations are needed in the first level but only $\binom{M}{B}B$ operations are required in subsequent ones. As a result, complexity drops to $M2^{M-1} + M(K-1)$ SINR computations.

C. *Top-down Trellis Beam Selection*

In the bottom-up procedure, we are restricting the search to those users maximizing system performance when only one beam is activated. However, this subset of users may not be adequate when the number of beams increases and interference comes into play (as a result of spatial multiplexing). For that reason, we propose a similar approach starting on the maximum number of active beams ($B = M$), for which the best subset of users is found. Then, the algorithm is iterated by removing one beam in each step. Again, user-beam pairs selected in the first step are kept. The number

[2] See how both the simulated and approximated results in Table 1 reflect the same trend for a growing number of beams.

[3] In the single beam case (B=1), the number of equivalent users it is equal to MK. This is because $\mathsf{SINR}_{k,i}$ for $k = 1..K$ and $i = 1..M$ are i.i.d distributed in this case.

of required computations is the same as in the bottom-up approach.

D. Greedy Beam Selection

Both the top-down and bottom-up procedures restrict the search to the best user subset when only one or all the beams are active, respectively. In order to extend the search to a larger set of users, we propose a greedy beam selection procedure. Specific details about the proposed greedy algorithm can be found in Table 2 but the basic idea consists in selecting in each step the pair user-beam leading to a higher increase of sum-rate. The algorithm is iterated until the configuration with all the active beams is reached and, then, the best sub-set with $B = j^*$ active beams is selected. In the first iteration, MK computations are needed to find the best user-beam pair. Next, K SINRs should be computed when a new beam is added and this operation must be repeated for the remaining beams. Therefore, the algorithm performs $(M-B-1)K$ computations in each iteration with a total computational cost of $\sum_{B=1}^{M}(M-B-1)K = \frac{KM}{2}(M+1)$ SINR operations.

E. Enhanced Greedy Beam Selection

In the greedy beam selection scheme proposed above, the overall performance depends on the user-beam pair of the first iteration (in the absence of inter-user interference). Instead, we can defer such decision to the second iteration where some inter-user interference is already present. In other words, we initialize the algorithm by identifying the $best$ user for each beam $i = 1..M$ (i.e., in the absence of interference). Then, we run the greedy algorithm M times taking as a starting point each user-beam pair obtained in the initialization. In this case, the total computational complexity amounts to $\frac{KM}{2}(M^2-M+2)$ SINR computations.

F. Restricted Beam Selection

Finally, we present a methodology where the optimum beam selection procedure is restricted to a predetermined number of active beams B. In other words, all the possible transmission configurations with B active beams are tested. By doing so, the number of SINR computations is reduced to $\binom{M}{B}KB = \frac{KM!}{(M-B)!(B-1)!}$ operations. This strategy is very appropriate for those situations where the optimum number of active beams can be known beforehand. For instance, it was shown in Section IV. that the optimum strategy is using a single active beam when the SNR is considerably high. Besides, when the number of possible transmission configurations $\binom{M}{B}$ is equal or lower than M, each user can report the highest SINRs for each configuration instead of all the gains $|\mathbf{h}_k^T\mathbf{w}_i|^2$. Then, by sending only SINRs associated to a limited number of transmission configurations, sub-optimum approaches can be derived in terms of system performance vs. feedback requirements constraints. In the sequel, these algorithms will be called BSX, where X will be the number of active beams.

Table 2: Greedy Beam Selection Algorithm

1. Set j=1, $\mathcal{K}_1$={$1,\ldots,K$} and $\mathcal{B}_1$={$\mathbf{w}_1,\ldots,\mathbf{w}_M$}.
2. Compute the best user-beam pair for the case with only one active beam as:
   $(k_1,i_1) = argmax_{(k,i)}\rho|\mathbf{h}_k^T\mathbf{w}_i|^2,\ \forall(k,i)\in\mathcal{K}_1\times\mathcal{B}_1$
3. Compute $R_1 = \log_2(1+\rho|\mathbf{h}_{k_1}^T\mathbf{w}_{i_1}|^2)$.
4. Set $j = j+1$, $\mathcal{K}_j$=$\mathcal{K}_{j-1} - \{k_{j-1}\}$ and $\mathcal{B}_j$=$\mathcal{B}_{j-1} - \{\mathbf{w}_{i_{j-1}}\}$.
5. Compute the best user-beam pair that can be added to the system as:
   $$(k_j,i_j) = \\ = argmax_{(k,i)}\left\{\log_2\left(\frac{|\mathbf{h}_k^T\mathbf{w}_i|^2}{j/\rho+\sum_{s=1}^{j-1}|\mathbf{h}_k^T\mathbf{w}_{i_s}|^2}\right)\right. \\ \left.+\sum_{p=1}^{j-1}\log_2\left(\frac{|\mathbf{h}_{k_p}^T\mathbf{w}_{i_p}|^2}{j/\rho+|\mathbf{h}_{k_p}^T\mathbf{w}_i|^2+\sum_{\substack{s=1\\s\neq p}}^{j-1}|\mathbf{h}_{k_p}^T\mathbf{w}_{i_s}|^2}\right)\right\}, \\ \forall(k,i)\in\mathcal{K}_j\times\mathcal{B}_j$$
6. Compute
   $$R_j = \sum_{p=1}^{j}\log_2\left(\frac{|\mathbf{h}_{k_p}^T\mathbf{w}_{i_p}|^2}{j/\rho+\sum_{\substack{s=1\\s\neq p}}^{j}|\mathbf{h}_{k_p}^T\mathbf{w}_{i_s}|^2}\right)$$
7. If $j < M$, go to step 4. Otherwise go to step 8.
8. Set $j^* = argmax_j R_j$.
9. The algorithm is finished and the set of selected beams and users is the following:
   $$(k_1,\mathbf{w}_{i_1}),\ldots,(k_{j^*},\mathbf{w}_{i_{j^*}})$$

VI. Simulation Results and Discussion

We consider a system with a number of active users in the range $K = 10..100$. Since Fig. 1 revealed that little improvement can be obtained with four active beams, we restrict the analysis of the proposed beam selection algorithms to a scenario with $M = 3$ transmit antennas.

In Fig. 2, performance in terms of sum-rate vs. number of users for the different beam selection methodologies is compared in a low SNR regime (SNR=0 dB). As expected, the best performance is obtained with the optimum approach. Regarding the sub-optimum approaches, performance losses can be observed for both the bottom-up and greedy methodologies, whereas most of the sum-rate gains can be achieved with the top-down and enhanced trellis approaches. This is because using several active beams may be beneficial when the SNR is low and, then, incorrect decisions made in the first step of the greedy and bottom-up algorithms penalize system performance. This effect is even clearer when the number of users increases. As for the restricted beam selection procedures, it is observed that the best results are obtained with the BS2 approach.

When the SNR is increased (see Figs. 3 and 4), the system becomes interference limited. As a result, the impact of wrong users selection on the denominator of the SINR (see eq. (1)) is emphasized. It can also be observed that

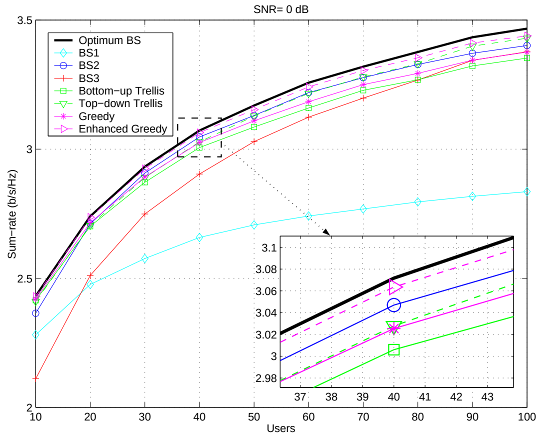

Figure 2: Sum-rate vs. number of users for the different beam selection procedures (SNR=0 dB).

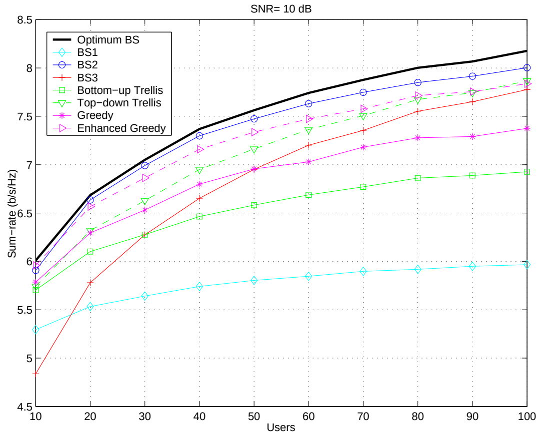

Figure 3: Sum-rate vs. number of users for the different beam selection procedures. (SNR=10 dB)

results associated to BS1 are improved, whereas BS3 worsens its performance. This is because the optimum solution tends to use a reduced number of active beams when the SNR is increased. This effect can be clearly observed in a scenario with SNR=50 dB. Due to space constraints results corresponding to this scenario are not included, but it is noteworthy that the same results are obtained with the optimum, bottom-up, greedy, enhanced greedy and BS1 approaches.

In order to provide the reader with a complementary point of view, computational complexity requirements in terms of SINR computations are shown in Table 3. Considerably computational savings are obtained with the suboptimum approaches except with the enhanced greedy approach. This algorithm executes M times the greedy approach, where in each step of algorithm the set of users and beams in the search are reduced but all the elements of the set must be tested. For that reason, it is observed that the enhanced greedy approach begins to be computationally efficient with respect to the optimal approach when the number of transmit antennas is increased.

In summary, in the proposed scenario with a practical number of users and constant transmit power at the base station, it is shown that substantial gains can be obtained

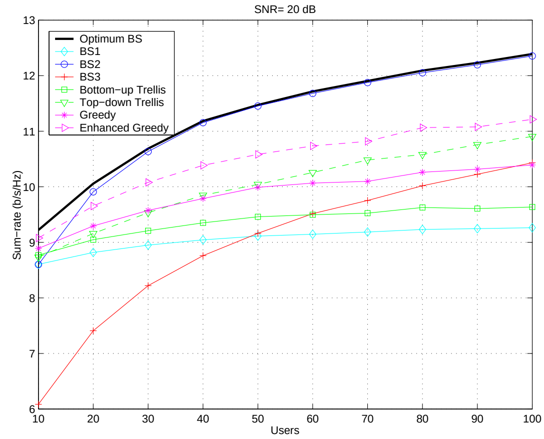

Figure 4: Sum-rate vs. number of users for the different beam selection procedures. (SNR=20 dB.)

Table 3: Computational Complexity for the different approaches in terms of SINR computations (20 users).

	M=1	M=2	M=3	M=4
Optimum BS	20	80	240	640
Bottom-up/Top-down	20	42	69	108
Greedy	20	60	120	200
Enhanced Greedy	20	80	240	560
BS-1	20	40	60	80
BS-2	-	40	120	240
BS-3	-	-	60	240
BS-4	-	-	-	80

with the BS2 approach. In particular, most of the capacity gains can be achieved with a computational complexity considerably lower with respect to the optimum solution.

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