A Unified Approach to the Analytical Assessment of Multi-user Diversity with Imperfect Channel State Information

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***Abstract*: In this paper, we explore the combined use of spatial and multi-user diversity in a cellular system where the channel state information (CSI) available at the base station (BS) is subject to imperfections. To do that, we consider a general statistical approach to describe the degree of CSI imperfection. By doing so, performance assessment is conducted for the generalized case in terms of ergodic system capacity, for which closed-form expressions are derived. In order to gain some insight, practical examples are presented and spatial vs. multi-user diversity trade-offs are analytically assessed.**

1. Introduction

Multi-user diversity (MUD) concepts, first introduced by Knopp and Humblet in [1], rely on the assumption that different users in a wireless multi-user system experience independent fading processes. In those circumstances, the aggregated cell throughput can be substantially increased by scheduling in each time slot the user with the most favorable channel conditions. Besides, in such fading environments the exploitation of transmit spatial diversity (e.g. by means of orthogonal space-time block coding, OSTBC) makes transmission links more robust with low complexity receivers [2][3]. Both schemes aimed at exploiting either multi-user or transmit spatial diversity have been proposed for packet data services in 3G wireless networks. For that reason, much attention has been recently paid to their combined use and the associated trade-offs.

Previous Work: In [4], [5] and [6] the inclusion of OSTBC in multiuser schemes was analyzed. It was shown that in a multi-user context Single-Input Single-Output (SISO) schemes outperform OSTBC-based ones in terms of aggregated cell capacity. Certainly, spatial diversity helps reduce the probability of deep fades but, by averaging over different diversity branches, SNR peaks (those that multi-user diversity can exploit) are suppressed as well. As a result, the resulting system capacity is lower. It is worth noting that with perfect Channel State Information (CSI) at the transmitter, spatial diversity can be efficiently exploited in a multi-user context, for instance via optimal transmit beamforming [7]. Unfortunately, perfect CSI is seldom available at the Base Station (BS).

Although multi-user diversity can be efficiently exploited in the presence of *partial* CSI, in FDD systems this involves the use of feedback channels which are often subject to a number of impairments. Recently, several studies show that the increased robustness of OSTBC schemes against imperfect CSI provides significant capacity gains with respect to those of SISO approaches. For instance, in [8] and [9] the authors analyze the impact of *delays* in the feedback channel by means of system-level computer simulations and numerical integration, respectively. The consequences of *bandwidth restrictions* were explored in [10], where the authors analyzed the impact of introducing OSTBC and antenna selection mechanisms in Selective-MUD environments[11].

Contributions: In this paper, we *analytically* assess the existing trade-offs in the combined use of multi-user and transmit spatial diversity in scenarios where CSI at the scheduler is subject to impairments. In particular, we conduct an analytical study of the impact of imperfect CSI at the scheduler on SISO and OSTBC-based schemes. To do that, we derive closed-form expressions of the ergodic system capacity associated to both approaches. We do not restrict ourselves to a specific source of imperfections but, instead, we adopt a general statistical approach to its modelling. In order to gain some insight, we then present two practical examples: delayed feedback channel and channel estimation errors. By doing so, we *analytically* prove that using OSTBC pays off in some situations.

Organization: The corresponding system model is presented in Section 2.. In Section 3., the general statistical approach for modelling the degree of imperfections in the CSI at the scheduler is introduced. Closed-form expressions for the ergodic system capacity are derived for homogeneous and non-homogenous systems in Sections 4. and 5., respectively. Next, the particularization of the ergodic capacity expressions to practical scenarios with delayed feedback or channel estimation errors is given in section 6.. Finally, some numerical results and conclusions are provided in Sections 7. and 8., respectively.

2. Signal model and Scheduler

Consider the downlink of a cellular system with one base station equipped with multiple antennas (N_{BS}), and K single-antenna user equipments (UE). For an arbitrary time-slot, the received signal at the k-th terminal can be modelled as:

$$r_k = \mathbf{h}_k^T \mathbf{s} + n_k$$

where $\mathbf{h}_k \in \mathbb{C}^{N_{BS}}$ is the channel vector gain between the BS and the k-th terminal, for which each component is assumed to be independent and identically dis-

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tributed, circularly symmetric Gaussian random variable with zero mean and user-dependent variance $\sigma^2_{h_k}$($\mathbf{h}_k \sim \mathcal{CN}(0, \sigma^2_{h_k}\mathbf{I}_{N_{BS}})$), $\mathbf{s} \in \mathbb{C}^{N_{BS}}$ is the symbol vector broadcasted from the BS and $n_k \in \mathbb{C}$ denotes additive Gaussian noise (AWGN) with zero mean and variance σ^2. The active users in the system are assumed to undergo independent Rayleigh fading processes and so does the signal being transmitted from different antennas in the BS. Further, we consider quasi-static fading, i.e, the channel response remains constant during one time-slot and, then, it abruptly changes to a new independent realization. We denote by $\gamma_k = \frac{P_t\|\mathbf{h}_k\|^2}{N_{BS}\sigma^2}$ the *instantaneous* signal-to-noise ratio experienced by user k in a given time-slot and by $\bar{\gamma}_k = \frac{P_t \mathrm{E}[\|\mathbf{h}_k\|^2]}{N_{BS}\sigma^2}$ its long-term average SNR, with P_t standing for the total transmit power. Notice that the total transmitted power is constant and evenly distributed among transmit antennas.

At the BS, we will consider two transmission schemes: a SISO configuration ($N_{BS} = 1$) and an OSTBC scheme with $N_{BS} = 2$ transmit antennas[1], more precisely, the well-known Alamouti scheme [3]. For the SISO scheme, the pdf and CDF of the received SNR take the following expressions:

$$f_{\gamma_k,SISO}(\gamma) = \frac{1}{\bar{\gamma}_k} e^{-\frac{\gamma}{\bar{\gamma}_k}}$$
$$F_{\gamma_k,SISO}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_k}} \tag{1}$$

respectively, whereas for the OSTBC case we can write:

$$f_{\gamma_k,OSTBC}(\gamma) = \frac{4\gamma}{\bar{\gamma}_k^2} e^{-\frac{2\gamma}{\bar{\gamma}_k}}$$
$$F_{\gamma_k,OSTBC}(\gamma) = 1 - e^{-\frac{2\gamma}{\bar{\gamma}_k}}\left(\frac{2\gamma}{\bar{\gamma}_k} + 1\right) \tag{2}$$

Concerning channel state information (CSI), we assume the availability of a low-rate error-free feedback channel to let user terminals convey *partial* CSI to the BS, in particular their instantaneous SNR. However, in this work we consider that the partial CSI at the BS, $\hat{\gamma}_k$, differs from the actual SNR, γ_k (further details are given in the next Section). As for the scheduling process, it is organized in a slot-by-slot basis following a modified version of the Proportional Fair Scheduling [12] rule. In particular, in each time slot the user with the maximum normalized SNR is selected for transmission, that is,

$$k^* = \arg\max_k \left\{ \frac{\hat{\gamma}_1}{\mathrm{E}[\hat{\gamma}_1]}, ..., \frac{\hat{\gamma}_k}{\mathrm{E}[\hat{\gamma}_k]}, ..., \frac{\hat{\gamma}_K}{\mathrm{E}[\hat{\gamma}_K]} \right\} \tag{3}$$

By doing so, users are only allowed to transmit when the instantaneous SNR is near to *its own peak*[12], that is with respect to their average SNR. As a result, multi-user diversity is still exploited and the scheduler will grant access probability of $1/K$ to each user.

3. Modelling CSI imperfections at the transmitter

In this section, we derive a statistical model describing the degree of CSI imperfection at the BS. In particular, we consider that the estimated SNR at the transmitter was obtained from a channel vector gain, $\hat{\mathbf{h}}_k$, which differs from the actual channel response, $\mathbf{h}_k$; these two random variables being related with a gaussian model. In other words, we assume that $\mathbf{h}_k$ conditioned on $\hat{\mathbf{h}}_k$ follows a gaussian distribution:

$$\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathcal{CN}(\eta_k\hat{\mathbf{h}}_k, \mathbf{\Sigma_k}) \tag{4}$$

where $\eta_k\hat{\mathbf{h}}_k$ and $\mathbf{\Sigma_k} = \sigma^2_{h_k}\sigma^2_{\epsilon_k}\mathbf{I}$ are the mean and covariance matrix, respectively. Notice that $\sigma^2_{\epsilon_k}$ models the degree of CSI uncertainty and, hence, it is equal to 0 when $\mathbf{h}_k = \hat{\mathbf{h}}_k$. Whereas, admittedly, this model might not be very accurate for some sources of imperfection, it is very helpful in the analysis and design of communication schemes because of its inherent tractability [[13], Chapter 9, pp. 321].

Under those assumptions, it is straightforward to show from Eq. (4) that the actual SNR, γ_k, conditioned on its estimate, $\hat{\gamma}_k$, follows a non-central chi-square distribution with $2N_{BS}$ degrees of freedom [14]:

$$f_{\gamma_k|\hat{\gamma}_k}(\gamma_k|\hat{\gamma}_k) = \frac{N_{BS}}{\bar{\gamma}_k\sigma^2_{\epsilon_k}}\left(\frac{\gamma_k}{\eta_k^2\hat{\gamma}_k}\right)^{\frac{2N_{BS}-2}{4}} e^{\frac{-N_{BS}(\gamma_k+\eta_k^2\hat{\gamma}_k)}{\bar{\gamma}_k\sigma^2_{\epsilon_k}}}$$
$$\times I_{N_{BS}-1}\left(\frac{2N_{BS}\sqrt{\eta_k^2\gamma\hat{\gamma}_k}}{\bar{\gamma}_k\sigma^2_{\epsilon_k}}\right) \tag{5}$$

with $I_n(\cdot)$ standing for the nth-order modified Bessel function of the first kind.

4. Ergodic system capacity for homogeneous systems

In this section, we restrict ourselves to the case where all the users are statistically identical in terms of CSI imperfections ($\eta_k = \eta$ and $\sigma_{\epsilon_k} = \sigma_\epsilon$) and SNR statistics ($\bar{\gamma}_k = \bar{\gamma}$) and, thus, $\mathrm{E}[\hat{\gamma}_k] = \mathrm{E}[\hat{\gamma}]$. The non-homogeneous case will be addressed later in Section 5..

In a multi-user system, the instantaneous channel capacity achievable by the *scheduled* user k^* over the *equivalent* SISO channel is given by[2]:

$$C_I(\gamma) = \log_2(1+\gamma)$$

where γ stands for the instantaneous *post-scheduling* SNR. Because of channel imperfections, the actual SNR, γ, is not fully known at the BS. However, the distribution of γ conditioned on a SNR estimate $\hat{\gamma}$ *is* certainly known (see Eq.(5) above). As a result, we will use the *expected* channel capacity as a performance measure [[13], Chapter 9, pp. 324], that is:

$$C(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma}) = \mathrm{E}_\gamma[C_I(\gamma)] \tag{6}$$

Finally, by averaging over all possible realizations of $\hat{\gamma}$, the ergodic system capacity follows:

$$\overline{C}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = \mathrm{E}_{\hat{\gamma}}[C(\eta, \sigma_\epsilon, \bar{\gamma}, \hat{\gamma})] \tag{7}$$

[1]For simplicity, we have assumed only two transmit antennas but the analysis can be easily extended to the general case.

[2]In this section, subscript k will be dropped for variables associated with the scheduled user.

Notice that only the last expression depends on the number of users, the reason for that being that the scheduler makes its decisions according to the *set* of SNR estimates ($\hat{\gamma}_k$, k=1..K). Next, we derive the corresponding closed-form expressions for both the SISO and OSTBC cases.

4.1. SISO

For the SISO approach, by recalling Eq. (5) and considering that $N_{BS}=1$, Eq. (6) can be rewritten as:

$$C_{SISO}(\eta,\sigma_\epsilon,\bar{\gamma},\hat{\gamma}) = \mathrm{E}_\gamma[C_I(\gamma)]$$
$$= \int_{\gamma=0}^{\infty} \log_2(1+\gamma) f_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) d\gamma$$
$$= \int_{\gamma=0}^{\infty} \log_2(1+\gamma) \frac{1}{\bar{\gamma}\sigma_\epsilon^2} e^{\frac{-(\gamma+\eta^2\hat{\gamma})}{\bar{\gamma}\sigma_\epsilon^2}} I_0\left(\frac{2\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) d\gamma \tag{8}$$

Before characterizing Eq. (7) for the SISO approach, we have just to derive the pdf of $\hat{\gamma}$. In the homogeneous case considered throughout this section, this can be readily obtained by resorting to order statistics [15]:

$$f_{\hat{\gamma}}(\hat{\gamma}) = K\frac{e^{-\frac{\hat{\gamma}}{\mathrm{E}[\hat{\gamma}]}}}{\mathrm{E}[\hat{\gamma}]}\left(1-e^{-\frac{\hat{\gamma}}{\mathrm{E}[\hat{\gamma}]}}\right)^{K-1}$$
$$= \frac{K}{\mathrm{E}[\hat{\gamma}]}\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k e^{-\frac{\hat{\gamma}(k+1)}{\mathrm{E}[\hat{\gamma}]}} \tag{9}$$

where the second equality follows from the application of the binomial expansion.

Last, by plugging (8) along with (9) into (7), the following integral results:

$$\overline{C}_{SISO}(\eta,\sigma_\epsilon,\bar{\gamma},K) = \mathrm{E}_{\hat{\gamma}}[C_{SISO}(\eta,\sigma_\epsilon,\bar{\gamma},\hat{\gamma})]$$
$$= \int_{\hat{\gamma}=0}^{\infty} C_{SISO}(\eta,\sigma_\epsilon,\bar{\gamma},\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}$$
$$= \frac{K}{\mathrm{E}[\hat{\gamma}]\bar{\gamma}\sigma_\epsilon^2}\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k \int_{\gamma=0}^{\infty}\log_2(1+\gamma)$$
$$\times e^{-\frac{\gamma}{\bar{\gamma}\sigma_\epsilon^2}} \int_{\hat{\gamma}=0}^{\infty} e^{-\hat{\gamma}\left(\frac{\eta^2}{\bar{\gamma}\sigma_\epsilon^2}+\frac{k+1}{\mathrm{E}[\hat{\gamma}]}\right)} I_0\left(\frac{2\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) d\hat{\gamma} d\gamma$$

For the sake of brevity, we show below the final expression (details can be found in the Appendix):

$$\overline{C}_{SISO}(\eta,\sigma_\epsilon,\bar{\gamma},K) = -K\log_2 e\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k$$
$$\times \frac{e^{\frac{k+1}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}}}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2} E_i\left(\frac{k+1}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}\right) \tag{10}$$

4.2. OSTBC

For the OSTBC case, one should bear in mind that the pdf of the post-scheduling estimated SNR reads [4]:

$$f_{\hat{\gamma}}(\hat{\gamma}) = K\frac{4\hat{\gamma}}{\mathrm{E}[\hat{\gamma}]^2}e^{-\frac{2\hat{\gamma}}{\mathrm{E}[\hat{\gamma}]}}\left(1-e^{-\frac{2\hat{\gamma}}{\mathrm{E}[\hat{\gamma}]}}\left(\frac{2\hat{\gamma}}{\mathrm{E}[\hat{\gamma}]}+1\right)\right)^{K-1} \tag{11}$$

Then, analogously to the previous case, equation (5) and the binomial expansion of (11) should be used in (7) for obtaining the ergodic system capacity (see Appendix):

$$\overline{C}_{OSTBC}(\eta,\sigma_\epsilon,\bar{\gamma},K) = 4K\log_2 e\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k$$
$$\times\sum_{n=0}^{k}\binom{k}{n}n!\sum_{m=0}^{n}\binom{n+1}{n-m}\mathrm{E}[\hat{\gamma}]^m\eta^{2m}$$
$$\times\bar{\gamma}^{n-m}\sigma_\epsilon^{2(n-m)}(m+1)e^{\frac{2(k+1)}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}}$$
$$\times\sum_{l=1}^{m+2}\frac{2^{m-l}\Gamma_c(l-m-2,\frac{2(k+1)}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2})}{(k+1)^l(\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2)^{m+n-l+2}} \tag{12}$$

5. Ergodic system capacity for non-homogeneous systems

As a natural extension to the analysis conducted in the previous section, we will now consider a scenario where neither CSI statistics nor the imperfections corresponding to different users are identically distributed. This is where the max-normalized SNR scheduling rule comes actually into play since $\mathrm{E}[\hat{\gamma}_k]$ in Eq. 3 differ now.

First, we focus on the user k and derive the conditional probability that this user has the maximum normalized SNR as:

$$\mathrm{Prob}\left(\max_{n\neq k}\frac{\hat{\gamma}_n}{\mathrm{E}[\hat{\gamma}_n]}\leq\frac{y}{\mathrm{E}[\hat{\gamma}_k]}\middle|\hat{\gamma}_k=y\right)=\prod_{\substack{n=1\\n\neq k}}^{N}F_{\hat{\gamma}_n}\left(y\frac{\mathrm{E}[\hat{\gamma}_n]}{\mathrm{E}[\hat{\gamma}_k]}\right)$$

where the specific expressions of $F_{\hat{\gamma}_n}(y)$ for the different transmission schemes can be found in Eqs. (1-2). Then, it can be proved that the ergodic capacity corresponding to the user k depends on:

$$f'_{\hat{\gamma}_k}(y) = \mathrm{Prob}\left(\max_{n\neq k}\frac{\hat{\gamma}_n}{\mathrm{E}[\hat{\gamma}_n]}\leq\frac{y}{\mathrm{E}[\hat{\gamma}_k]}\middle|\hat{\gamma}_k=y\right)f_{\hat{\gamma}_k}(y)$$

By particularizing the above expression for the different transmission schemes:

$$f'_{\hat{\gamma}_k,SISO}(y) = \frac{e^{-\frac{y}{\mathrm{E}[\hat{\gamma}_k]}}}{\mathrm{E}[\hat{\gamma}_k]}\left(1-e^{-\frac{y}{\mathrm{E}[\hat{\gamma}_k]}}\right)^{K-1} \tag{13}$$

$$f'_{\hat{\gamma}_k,OSTBC}(y) = \frac{4\hat{\gamma}_k}{\mathrm{E}[\hat{\gamma}_k]^2}e^{-\frac{2\hat{\gamma}_k}{\mathrm{E}[\hat{\gamma}_k]}}$$
$$\times\left(1-e^{-\frac{2\hat{\gamma}_k}{\mathrm{E}[\hat{\gamma}_k]}}\left(\frac{2\hat{\gamma}_k}{\mathrm{E}[\hat{\gamma}_k]}+1\right)\right)^{K-1} \tag{14}$$

and comparing Eqs. (13-14) with Eqs. (9-11), one can observe that the capacity results we are interested in can be expressed in terms of Eq. (16-17) respectively as:

$$\overline{C'}_{k,SISO} = \frac{1}{K}\overline{C}_{SISO}(\eta_k,\sigma_{\epsilon_k},\bar{\gamma}_k,K)$$
$$\overline{C'}_{k,OSTBC} = \frac{1}{K}\overline{C}_{OSTBC}(\eta_k,\sigma_{\epsilon_k},\bar{\gamma}_k,K)$$

Clearly, the contribution to the ergodic capacity associated to user k (under a max-normalized SNR scheduling rule) exclusively depends on the *number* of users but not on the other users' statistics. Finally, the overall ergodic system capacity for both approaches can be written in closed form as:

$$\overline{C'}_{SISO} = \frac{1}{K}\sum_{k=1}^{K}\overline{C}_{SISO}(\eta_k, \sigma_{\epsilon_k}, \bar{\gamma}_k, K)$$
$$\overline{C'}_{OSTBC} = \frac{1}{K}\sum_{k=1}^{K}\overline{C}_{OSTBC}(\eta_k, \sigma_{\epsilon_k}, \bar{\gamma}_k, K) \quad (15)$$

6. Practical Cases

So far, we have derived analytical expressions for the general case of channel imperfections. In order to gain some insight, we will now particularize those expressions to two practical cases where the gaussian model applies:

6.1. Delayed feedback channel

Unless reciprocity between the forward and reverse links holds, there always exists a delay between the instant when the SNR is measured at the UE and the actual transmission of data to the scheduled user takes place. Under the assumption of a Jakes' scattering model is adopted and, hence, $\mathbf{h}_k$ and $\hat{\mathbf{h}}_k$ turn out to be samples of the same Gaussian process. In other words, $\mathbf{h}_k$ and $\hat{\mathbf{h}}_k$ follow a joint complex Gaussian distribution with correlation coefficient $\rho_k = J_o(2\pi f_{d_k} T_k)$, where f_{d_k} stands for the Doppler frequency, T_k is the delay in time units, and $J_o(\cdot)$ denotes the zero-order Bessel function of the first kind. As a consequence, the conditioned pdf can be easily obtained by applying Bayes' Theorem [[16], Chapter 10]:

$$f_{\mathbf{h}_k|\hat{\mathbf{h}}_k}(\mathbf{h}_k|\hat{\mathbf{h}}_k) = \frac{f_{\mathbf{h}_k,\hat{\mathbf{h}}_k}(\mathbf{h}_k\hat{\mathbf{h}}_k)}{f_{\hat{\mathbf{h}}_k}(\hat{\mathbf{h}}_k)} = \frac{e^{-(\mathbf{h}_k-\rho_k\hat{\mathbf{h}}_k)^H\mathbf{R}_k^{-1}(\mathbf{h}_k-\rho\hat{\mathbf{h}}_k)}}{\pi^M \det(\mathbf{R}_k)}$$

where $\mathbf{R}_k = \sigma_{h_k}^2(1-\rho_k^2)\mathbf{I}_{N_{BS}}$ is the covariance matrix. Therefore, the delayed feedback channel fits into the gaussian model since we have that:

$$\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathcal{CN}(\rho_k\hat{\mathbf{h}}_k, \mathbf{R}_k)$$

and, hence, the ergodic system capacity can be computed by substituting:

$$\eta_k = \rho_k \qquad \sigma_{\epsilon_k}^2 = 1-\rho_k^2 \qquad \mathrm{E}[\hat{\gamma}_k] = \bar{\gamma}_k \ ,$$

into Eq. 15, where the last equality holds from the fact that $\mathbf{h}$ and $\hat{\mathbf{h}}$ are samples of the same gaussian process.

6.2. Imperfect channel estimation

It is common practice to assume that the channel impulse response is perfectly known at the receiver. However, in practical situations only an estimate of the actual channel is actually available. In the case of a linear MMSE estimator, for instance, we can model the channel estimate as [16]:

$$\hat{\mathbf{h}}_k = \mathbf{h}_k + \mathbf{e}_k$$

where $\mathbf{e}_k \in \mathbb{C}^{N_{BS}}$ is the vector corresponding to the channel estimation error for which each component is assumed to be i.i.d circularly symmetric Gaussian random variable with variance $\sigma_{e_k}^2$ and independent from $\mathbf{h}_k$. Then, by applying the Bayes' Theorem one can find that:

$$\mathbf{h}_k|\hat{\mathbf{h}}_k \sim \mathcal{CN}(\frac{1}{1+\Delta_{e_k}}\hat{\mathbf{h}}_k, \sigma_{h_k}^2\frac{\Delta_{e_k}}{1+\Delta_{e_k}}\mathbf{I}_{N_{BS}})$$

where we have defined $\Delta_{e_k} = \frac{\sigma_{e_k}^2}{\sigma_{h_k}^2}$. Thus, we should take the following parameters into account for the derivation of the ergodic system capacity:

$$\eta_k = \tfrac{1}{1+\Delta_{e_k}} \qquad \sigma_{\epsilon_k}^2 = \tfrac{\Delta_{e_k}}{1+\Delta_{e_k}} \qquad \mathrm{E}[\hat{\gamma}_k] = \bar{\gamma}_k(1+\Delta_{e_k})$$

As a final remark, it should be noted that in this second case, the ergodic capacity expressions derived above, are actually upper bounds. This is because we are only considering the impact of noisy channel estimates on the scheduling process, whereas we disregard its impact on the detection process at the receiver. However, this second issue is out of the scope of this paper since we are interested in the analytical study of the impact caused by incorrect scheduling decisions. For further details, the reader is referred to [17] and [18], where work related to SISO and MIMO channels, respectively, can be found.

7. Numerical results

In this section, we are interested in assessing spatial vs. multi-user trade-offs in situations where the partial CSI available at the BS is subject to imperfections. Due to space constraints and the similarity between the expressions obtained in the homogeneous and non-homogeneous cases, we will restrict ourselves to the former case. In particular, we will consider a system transmitting data packets with an average SNR of $\bar{\gamma} = 10$ dB, and $K = 5$ or 30 active users in the system.

In Fig. 1, we depict the average system capacity as a function of UE speed. As for the CSI delay, we adopt the parameters used in [8] for a High Speed Downlink Packet Access (HSDPA) scenario where the authors justify that scheduling decisions can be made every 2 ms with a time delay of $T = 4$ ms. From the curves, one concludes that in the absence of delay, the SISO approach is far more effective than its OSTBC counterpart. However, when UE speed increases and delays in the feedback channel are comparable to the coherence time, the degradation experienced by the SISO scheme is larger than that of OSTBC. In other words, the single-antenna approach is less robust to channel uncertainty (i.e. deep fades) arising from CSI delays. As the number of active users grows, though, the capability of generating post-scheduling SNR peaks improves faster for SISO configurations and, hence, compensates for such SNR uncertainties (i.e. SISO and OSTBC curves cross each other for higher values of the UE speed). Finally, one can also observe that beyond 50 km/h curves are driven again towards higher values of the ergodic capacity. This is because under a Jakes' scattering model assumption,

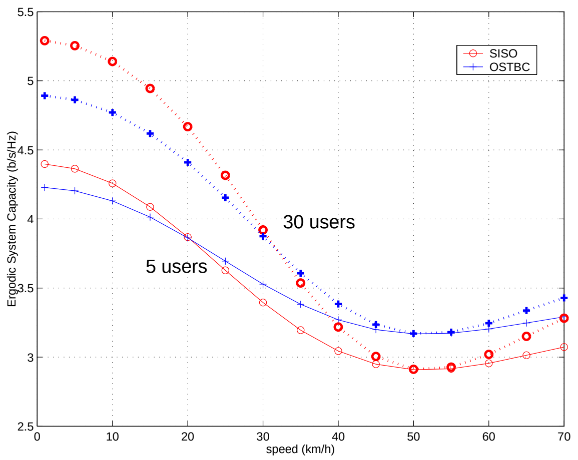

Figure 1: Average system capacity vs. UE speed for the different transmission schemes ($K = 5$ and 30,$\bar{\gamma} = 10$ dB).

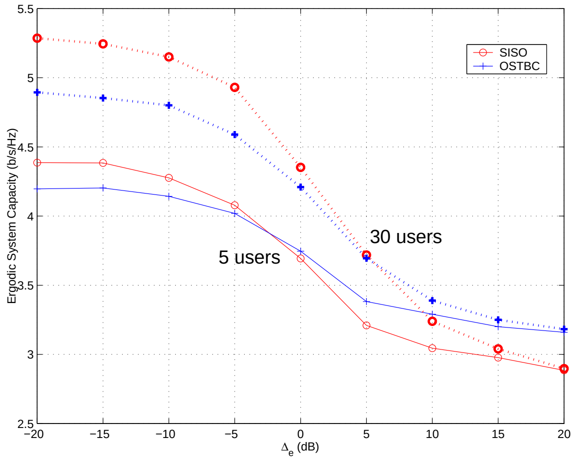

Figure 2: Average system capacity vs. Δ_e for the different transmission schemes ($K = 5$ and 30,$\bar{\gamma} = 10$ dB).

the correlation depends on the zero-order Bessel function of the first kind, which is not a monotonically decreasing function.

Similar conclusions can be drawn from Fig. 2, where ergodic system capacity is plotted as a function of the parameter Δ_e. However, in this case no extra multi-user diversity gain can be extracted as the degradation in the channel estimates increases. For increasing values of Δ_e, capacity curves reach a floor associated with the performance of a round-robin scheduler (i.e. no MUD gain).

8. Conclusions

In this paper, we explored the existing trade-offs in exploiting multi-user and transmit spatial diversity in scenarios where the CSI available at the scheduler is subject to impairments. In particular, the impact of imperfect CSI at the base station was analyzed for SISO and OSTBC-based schemes. To do that, a general statistical approach for modelling the degree of CSI imperfections at the base station was adopted and closed-form expressions for the ergodic system capacity were derived. In order to gain some insight, practical scenarios with delayed feedback channel and channel estimation errors were presented. It was analytically shown that OSTBC-based schemes are more appropriate for scenarios with degradation in the CSI available at the BS, in particular for a reduced number of active users.

9. Appendix

In this appendix, we derive the closed-form solutions of the ergodic system capacity for both the SISO and OSTBC approaches.

9.1. SISO

In order to derive a closed-from expression of the ergodic system capacity with a homogeneous system, one should solve the following expression for the SISO case:

$$\overline{C}_{SISO}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = \frac{K}{\mathrm{E}[\hat{\gamma}]\bar{\gamma}\sigma_\epsilon^2} \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \times \int_{\gamma=0}^{\infty} \log_2(1+\gamma)\, e^{-\frac{\gamma}{\bar{\gamma}\sigma_\epsilon^2}} \times \int_{\hat{\gamma}=0}^{\infty} e^{-\hat{\gamma}\left(\frac{\eta^2}{\bar{\gamma}\sigma_\epsilon^2}+\frac{k+1}{\mathrm{E}[\hat{\gamma}]}\right)} I_0\left(\frac{2\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) d\hat{\gamma}\, d\gamma$$

With the help of identities [[19], Eq. 6.614.3], [[19], Eq. 9.220.2] and [[19],Eq. 9.215.1] one can readily solve the inner integral in the above equation:

$$\overline{C}_{SISO}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = K \log_2 e \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \times \frac{1}{\mathrm{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2} \times \int_{\gamma=0}^{\infty} \ln(1+\gamma) e^{-\frac{\gamma(k+1)}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}} d\gamma$$

After that, by resorting to [[19], Eq. 4.331.2], the latter integral can be solved and written in closed form as:

$$\overline{C}_{SISO}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = -K \log_2 e \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \times \frac{e^{\frac{k+1}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}}}{\mathrm{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2} E_i\left(\frac{k+1}{\mathrm{E}[\hat{\gamma}]\eta^2 + (k+1)\bar{\gamma}\sigma_\epsilon^2}\right) \tag{16}$$

with $E_i(x)$ standing for the exponential integral function ($E_i(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$, for $x < 0$).

9.2. OSTBC

On the other hand, for the OSTBC case we have to plug equation (5) and the binomial expansion of (11) into (7):

$$\overline{C}_{OSTBC}(\eta, \sigma_\epsilon, \bar{\gamma}, K) = \frac{8K}{\mathrm{E}[\hat{\gamma}]^2\eta\bar{\gamma}\sigma_\epsilon^2} \sum_{k=0}^{K-1} \binom{K-1}{k} \times (-1)^k \sum_{n=0}^{k} \binom{k}{n} \frac{2^n}{\mathrm{E}[\hat{\gamma}]^n} \int_{\gamma=0}^{\infty} \log_2(1+\gamma)\, \gamma^{\frac{1}{2}} e^{-\frac{2\gamma}{\bar{\gamma}\sigma_\epsilon^2}} \times \int_{\hat{\gamma}=0}^{\infty} \hat{\gamma}^{n+1/2} e^{-2\hat{\gamma}\left(\frac{\eta^2}{\bar{\gamma}\sigma_\epsilon^2}+\frac{k+1}{\mathrm{E}[\hat{\gamma}]}\right)} I_1\left(\frac{4\sqrt{\eta^2\gamma\hat{\gamma}}}{\bar{\gamma}\sigma_\epsilon^2}\right) d\hat{\gamma}\, d\gamma$$

In order to solve the inner integral in the above equation, one should resort to identities [[19], Eq. 8.406.3] and [[19], Eq. 6.643.4]. Finally, with the help of [[19], Eq. 8.970.1] and [[20], Eq.78], the ergodic system capacity can be expressed analytically in terms of the complementary incomplete gamma function ($\Gamma_c(n,x) = \int_x^\infty e^{-t}t^{n-1}dt$) as:

$$\begin{aligned}\overline{C}_{OSTBC}(\eta,\sigma_\epsilon,\bar{\gamma},K) &= 4K\log_2 e\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\\ &\times\sum_{n=0}^{k}\binom{k}{n}n!\sum_{m=0}^{n}\binom{n+1}{n-m}\mathrm{E}[\hat{\gamma}]^m\eta^{2m}\\ &\times\bar{\gamma}^{n-m}\sigma_\epsilon^{2(n-m)}(m+1)e^{\frac{2(k+1)}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2}}\\ &\times\sum_{l=1}^{m+2}\frac{2^{m-l}\Gamma_c(l-m-2,\frac{2(k+1)}{\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2})}{(k+1)^l(\mathrm{E}[\hat{\gamma}]\eta^2+(k+1)\bar{\gamma}\sigma_\epsilon^2)^{m+n-l+2}}\end{aligned} \tag{17}$$

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