ANALYTICAL ASSESSMENT OF CAPACITY VS. ROBUSTNESS TRADE-OFFS IN SYSTEMS WITH SELECTIVE MULTI-USER DIVERSITY

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ABSTRACT

In this paper, we explore spatial vs. multi-user diversity tradeoffs in a cellular system with selective feedback. We first derive closed-form expressions of the average system capacity for both SISO and STBC transmission schemes in order to *analytically* assess the impact of the number of terminals and bandwidth restrictions in the feedback channel. Next, we analyze several design trade-offs in terms of increased average (long term) system capacity vs. robustness to short-term SNR fluctuations for both transmission schemes under consideration.

1. INTRODUCTION

Multi-user diversity (MUD) concepts, first introduced by Knopp and Humblet in [1], rely on the assumption that different users in a wireless multi-user system experience independent fading processes. In those circumstances, the aggregated cell throughput can be substantially increased by scheduling in each time slot the user with the most favorable channel conditions.

Besides, in such fading environments the exploitation of spatial diversity (e.g. by means of space-time block coding, STBC) makes transmission links more robust [2][3] and, for that reason, much attention has been recently paid to the combined use of multi-user and spatial diversity. In [4], for instance, the authors show that in a multi-user context Single-Input Single-Output (SISO) schemes outperform STBC-based ones in terms of aggregated cell capacity. Certainly, spatial diversity helps reduce the probability of deep fades but, by averaging over different diversity branches, SNR peaks (those that multi-user diversity can exploit) are suppressed as well. As a result, the resulting system capacity is lower.

However, in order to exploit multi-user diversity some partial channel state information (CSI) must be made available to the scheduler at the Base Station (BS). In FDD systems, this involves the use of feedback channels which are often subject to a number of impairments. For instance, in [5] and [6] the authors analyzed the impact of *delays* in the feedback channel. The consequences of *bandwidth restrictions* were explored in [7] by Gesbert and Alouini where a bandwidth-efficient selective-MUD scheduler was presented. In [8], the authors analyzed the impact of introducing antenna selection mechanisms in such Selective-MUD environments, which revealed moderately useful in both SISO and STBC contexts unless the feedback load is dramatically reduced. Also, the increased robustness of STBC schemes against fading provided significant capacity gains with respect to those of SISO approaches is such limited-feedback systems.

Continuing the work in [8], in this paper we explore and quantify spatial vs. multi-user diversity trade-offs for a cellular system operating in a Selective-MUD scenario. More precisely, we derive closed-form expressions for the system capacity associated to a STBC transmission scheme. To the best of authors' knowledge, this has not been done before for multi-user systems with a Selective-MUD scheduler. We also assess spatial vs. multi-user trade-offs by using mean vs. standard deviation plots [9], inspired by modern *portfolio* theory [10]. By doing so, both the degree of robustness to short-term SNR fluctuations and its impact in terms of system performance can be easily quantified for the different transmission schemes.

2. SIGNAL MODEL AND SCHEDULER

Consider the downlink of a cellular system with one base station equipped with multiple antennas (N_{BS}), and K single-antenna terminals. For an arbitrary time instant, the received signal at the k-th terminal is given by:

$$r_k = \mathbf{h}_k^T \mathbf{s} + n_k$$

where $\mathbf{h}_k \in \mathbb{C}^{N_{BS}}$ is the channel vector gain between the BS and the k-th terminal, for which each component is assumed to be independent and identically distributed, circularly symmetric Gaussian random variable with zero mean and unit variance ($\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{I}_{N_{BS}})$), $\mathbf{s} \in \mathbb{C}^{N_{BS}}$ is the symbol vector broadcasted from the BS and $n_k \in \mathbb{C}$ denotes additive Gaussian noise (AWGN) with zero mean and variance σ^2. The active users in the system are assumed to undergo independent Rayleigh fading processes and so does the signal being transmitted from different antennas in the BS. Further, we consider quasi-static fading, i.e, the channel response remains constant during one time-slot and, then, it abruptly changes to a new independent realization. Concerning channel state information (CSI), we assume perfect knowledge for *each* user at the receive side, and the availability of a low-rate error-free feedback channel to convey partial CSI to the transmitter. Finally, we denote by $\gamma_k(s)$ the instantaneous signal-to-noise ratio experienced by user k during time-slot s and by $\bar{\gamma}_k = \frac{P_t}{\sigma^2}$ its average SNR, with P_t standing for the total transmitted power, which is constant and evenly distributed among transmit antennas. Throughout this work, we will assume identical average SNRs for all the active users (i.e. $\bar{\gamma} = \bar{\gamma}_k$).

At the BS, we will consider two transmission schemes: a SISO configuration ($N_{BS} = 1$) and an STBC (i.e. Alamouti) scheme [3] with $N_{BS} = 2$ transmit antennas. As for the scheduling process, it is organized in a slot-by-slot basis following a *max-SNR* (*greedy*) rule.

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In order to reduce bandwidth requirements in the feedback channel, a $Selective$ Multi-user Diversity (SMUD) approach is adopted[7]. In other words, only terminals experiencing SNRs above a pre-defined threshold (γ_{th}) in a specific time slot s are allowed to report their channel state information to the BS. Thus, the max-SNR scheduler conducts the search over such a subset of the active users only, that is,

$$k^*(s) = \arg\max_k \{\gamma_k(s) \text{ s.t. } \gamma_k(s) > \gamma_{th}\}$$

Conversely, when all the users remain silent (i.e. in the event of a *scheduling outage*) the scheduling rule amounts to:

$$k^*(s) = \text{rand}\{1, ..., k, ..., K\}$$

that is, one of the users is randomly selected for transmission. In the sequel, subscript s will be dropped for the ease of notation.

3. SNR STATISTICS IN A SELECTIVE-MUD SYSTEM

In this section, we revise the statistics of the *post-scheduling* SNRs, that is, the signal-to-noise-ratio experienced by the scheduled user. Both pdf and CDF functions will be used later to derive closed-form expressions of the average system capacity.

- *SISO (Single-Input, Single-Output):*

With one single antenna at the BS, the received SNR for user k becomes $\gamma_{k,SISO} = \bar{\gamma}_k |h_{1,k}|^2$. Hence, the received SNR is distributed as a chi-square random variable with two degrees of freedom, χ_2^2:

$$f_{\gamma_{SISO}}(\gamma) = \tfrac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \qquad F_{\gamma_{SISO}}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}}}$$

where f_γ and F_γ stand for the pdf and CDF density functions, respectively (subscript k has been dropped for brevity). As for the *post-scheduling* SNR, γ^*, the analysis must be conducted for two different SNR regions: $\gamma \le \gamma_{th}$ (i.e. all users remain silent), and $\gamma > \gamma_{th}$ (at least one user reports its CSI to the BS). For the $\gamma \le \gamma_{th}$ case and by recalling that all users experience i.i.d fading, we have:

$$\begin{aligned} F_{\gamma^*_{SISO}}(\gamma) &= \text{Prob}(\gamma^* \le \gamma, \gamma_k \le \gamma_{th} \text{ for all } k = 1...K) \\ &= \left(F_{\gamma_{SISO}}(\gamma_{th})\right)^{K-1} F_{\gamma_{SISO}}(\gamma) \end{aligned}$$

On the other hand, for $\gamma > \gamma_{th}$, the CDF function can be expressed as:

$$F_{\gamma^*_{SISO}}(\gamma) = \text{Prob}(\gamma_k \le \gamma, \text{ for all } k) = \left(F_{\gamma_{SISO}}(\gamma)\right)^K$$

Therefore, the pdf expressions of the *post-scheduling* SNR are given by:

$$\begin{aligned} f_{\gamma^*_{SISO}}(\gamma) &= \frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}} \left(1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\right)^{K-1} & \gamma \le \gamma_{th} \\ f_{\gamma^*_{SISO}}(\gamma) &= K \frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}} \left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}\right)^{K-1} & \gamma > \gamma_{th} \end{aligned} \quad (1)$$

- *STBC (Space-Time Block Coding):*

In this case and bearing in mind that power is evenly allocated to transmit antennas, the received SNR for user k turns out to be

$$\gamma_{k,STBC} = \frac{\bar{\gamma}_k}{2}\left(|h_{1,k}|^2 + |h_{2,k}|^2\right) = \frac{\bar{\gamma}_k}{2} |h_k|^2$$

Now, the signal-to-noise ratio becomes a chi-square random variable with four degrees of freedom, χ_4^2, i.e.,

$$f_{\gamma_{STBC}}(\gamma) = \frac{4\gamma}{\bar{\gamma}^2} e^{-\frac{2\gamma}{\bar{\gamma}}}$$

$$F_{\gamma_{STBC}}(\gamma) = 1 - e^{-\frac{2\gamma}{\bar{\gamma}}}\left(\frac{2\gamma}{\bar{\gamma}} + 1\right)$$

Finally, the pdf of the *post-scheduling* SNR can be expressed as:

$$\begin{aligned} f_{\gamma^*_{STBC}}(\gamma) &= \frac{4\gamma}{\bar{\gamma}^2} e^{-\frac{2\gamma}{\bar{\gamma}}} \left(1 - e^{-\frac{2\gamma_{th}}{\bar{\gamma}}}\left(\frac{2\gamma_{th}}{\bar{\gamma}} + 1\right)\right)^{K-1} & \gamma \le \gamma_{th} \\ f_{\gamma^*_{STBC}}(\gamma) &= K \frac{4\gamma}{\bar{\gamma}^2} e^{-\frac{2\gamma}{\bar{\gamma}}} \left(1 - e^{-\frac{2\gamma}{\bar{\gamma}}}\left(\frac{2\gamma}{\bar{\gamma}} + 1\right)\right)^{K-1} & \gamma > \gamma_{th} \end{aligned} \quad (2)$$

4. CAPACITY ANALYSIS

In a multi-user system, the instantaneous channel capacity achievable by the scheduled user k^* over the *equivalent* SISO channel is given by

$$C^* = \log_2(1 + \gamma^*)$$

and, consequently, the average (ergodic) system capacity achievable under a max-SNR scheduling policy can be expressed as

$$\overline{C} = \text{E}_{\gamma^*}[C^*] = \int_0^\infty \log_2(1 + \gamma) f_{\gamma^*}(\gamma) \, d\gamma$$

By plugging Eqs. (1) and (2) into the above expression, the corresponding capacity for the SISO and STBC schemes can be obtained, respectively. For the SISO case, one should resort to the binomial expansion, integrate by parts and then find out that

$$\begin{aligned} &\overline{C}_{SISO}(K, \gamma_{th}) = \log_2(e)\left(1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\right)^{K-1} \\ &\times \left[e^{\frac{1}{\bar{\gamma}}}\left(E_i\left(-\frac{1+\gamma_{th}}{\bar{\gamma}}\right) - E_i\left(-\frac{1}{\bar{\gamma}}\right)\right) - e^{-\frac{\gamma_{th}}{\bar{\gamma}}} \ln(1+\gamma_{th}) \right] \\ &\quad + K \log_2(e) \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{(-1)^k}{k+1} \\ &\times \left[e^{-\frac{\gamma_{th}}{\bar{\gamma}}(k+1)} \ln(1+\gamma_{th}) - e^{\frac{k+1}{\bar{\gamma}}} E_i\left(-\frac{1+\gamma_{th}}{\bar{\gamma}}(k+1)\right) \right] \end{aligned}$$

with $E_i(x)$ standing for the exponential integral function ($E_i(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$, for $x < 0$)[[11], Eq. 8.211.1][1]. Deriving a closed-form expression of the average capacity for the STBC scheme is somewhat more involved. In particular, one should integrate the following expression:

$$\begin{aligned} \overline{C}_{STBC}(K, \gamma_{th}) &= \frac{4}{\bar{\gamma}^2} \log_2(e) \left(1 - e^{-\frac{2\gamma_{th}}{\bar{\gamma}}}\left(\frac{2\gamma_{th}}{\bar{\gamma}} + 1\right)\right)^{K-1} \\ &\times \int_0^{\gamma_{th}} \ln(1+\gamma)\gamma e^{-\frac{2\gamma}{\bar{\gamma}}} d\gamma \\ &+ K \log_2(e) \sum_{k=0}^{K-1} \binom{K-1}{k} (-1)^k \sum_{n=0}^{k} \binom{k}{n} \left(\frac{2}{\bar{\gamma}}\right)^{n+2} \\ &\times \int_{\gamma_{th}}^{\infty} \ln(1+\gamma)\gamma^{n+1} e^{-\frac{2\gamma}{\bar{\gamma}}(k+1)} d\gamma \end{aligned}$$

Both integrals in the above expression are solved in the Appendix where the interested reader can find the details. By using Eqs. (5)

[1] Albeit *apparently* different, this expression is equivalent to that derived in [7]. Such a difference results from the fact that a simpler expression of the post-scheduling pdf (for $\gamma > \gamma_{th}$) was used here.

and (6) in such Appendix, the average system capacity can be expressed in closed-form as:

$$
\begin{aligned}
&\overline{C}_{STBC}(K,\gamma_{th}) = \log_2(e)\left(1-e^{-\frac{2\gamma_{th}}{\bar{\gamma}}}\left(\frac{2\gamma_{th}}{\bar{\gamma}}+1\right)\right)^{K-1} \\
&\times \sum_{i=1}^{2}\left(\frac{\bar{\gamma}}{2}\right)^{i-2}\Big[e^{\frac{2}{\bar{\gamma}}}\Gamma(-2+i,\frac{2}{\bar{\gamma}}) - e^{-\frac{2\gamma_{th}}{\bar{\gamma}}}\gamma_{th}^{2-i} \\
&\times \Big(\ln(1+\gamma_{th}) + \sum_{p=0}^{2-i}\binom{2-i}{p}\left(\frac{1+\gamma_{th}}{\gamma_{th}}\right)^{p} e^{\frac{2}{\bar{\gamma}}(1+\gamma_{th})} \\
&\times p!\Gamma\left(-p,\frac{2}{\bar{\gamma}}(1+\gamma_{th})\right)\Big)\Big] \\
&+ K\log_2(e)\sum_{k=0}^{K-1}\binom{K-1}{k}(-1)^k\sum_{n=0}^{k}\binom{k}{n} \\
&\times \sum_{i=1}^{n+2}\frac{(n+2-1)!}{(n+2-i)!}\left(\frac{\bar{\gamma}}{2}\right)^{i-n-2}\frac{e^{-\frac{2}{\bar{\gamma}}(k+1)\gamma_{th}}\gamma_{th}^{n+2-i}}{(k+1)^{i}} \\
&\times \Big[\ln(1+\gamma_{th}) + \sum_{p=0}^{n+2-i}\binom{n+2-i}{p}\left(\frac{1+\gamma_{th}}{\gamma_{th}}\right)^{p} \\
&\times e^{\frac{2}{\bar{\gamma}}(k+1)(1+\gamma_{th})}p!\Gamma\left(-p,\frac{2}{\bar{\gamma}}(k+1)(1+\gamma_{th})\right)\Big]
\end{aligned}
$$

where the first term in the summation accounts for contributions to capacity due to random scheduling (i.e. in the case of *scheduling outage*), whereas the second term reflects contributions coming from max-SNR scheduling.

5. NUMERICAL RESULTS AND DISCUSSION

Throughout this section, we will consider a system with a number of active users in the range $K = 1..30$, transmitting data packets with an average SNR of $\bar{\gamma} = 10$ dB. In Fig. 1, we depict the average system capacity as a function of the number of active users and different feedback loads[2] ($\bar{F} = 0.01...1$). First of all, it is worth noting the close match of the curves associated with the analytical expressions derived in the previous section with the corresponding computer simulation results (markers on those curves). Apart from that, in the case of full feedback load ($\bar{F} = 1$) one can also observe that the SISO approach is far more effective than its STBC counterpart in exploiting multi-user diversity. In other words, the suppression of SNR peaks due to the SNR-stabilizing effect associated to STBC penalizes system performance. Conversely, when the average feedback load per user is reduced, the degradation experienced by the SISO-based schemes is larger than that exhibited by the STBC ones. This follows from the fact that STBC approaches provide additional robustness against unfavorable fading conditions resulting from random user selection.

However, results in terms of *average* (ergodic) capacity give only a partial view concerning system performance. Such a view is relevant to services where data rate is a priority, regardless of packet delay or delay jitter. Conversely, for delay-limited services where channel coding is conducted over a (potentially) low number of frames, the short-term fluctuations of channel capacity become more relevant [12]. In other words, when QoS requirements are given in terms of outage probability (i.e. the probability that a pre-defined data rate cannot be supported), one should take a closer look at the standard deviation of system capacity. In Fig. 2 above, we depict the mean (average) vs. the standard deviation of system capacity for a varying number of users ($K = 1..30$), feedback loads ($\bar{F} = 1..0.01$), and transmission schemes (SISO/STBC). To start with, consider the $K = 1$, $\bar{F} = 1$ case: as expected, the average capacity is higher for STBC than for SISO and, simultaneously, the standard deviation is lower (i.e. higher capacity and more stable communication links). However, as soon as the number of users increases beyond $K = 1$ and for mid to high values of $\bar{F}$, SISO links outperform STBC ones in terms of capacity whereas STBC links remain more stable than SISO ones (or, alternatively, the data-rate dispersion among active users for a *short* period of time is lower[3]).

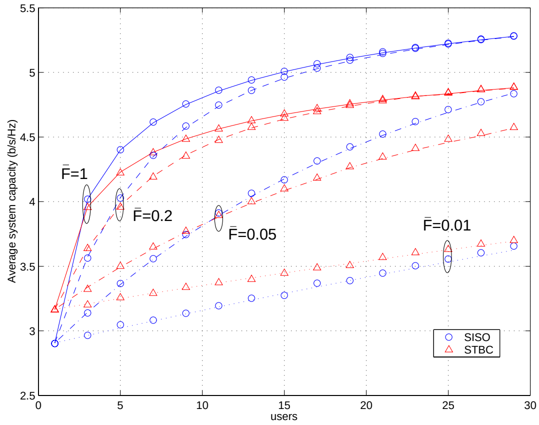

Fig. 1. Average system capacity vs. number of users for the different transmission schemes and feedback loads $\bar{F} = 1 \dots 0.01$. A close match can be observed between analytical expressions (curves) and computer simulation results (markers). $\bar{\gamma} = 10dB$.

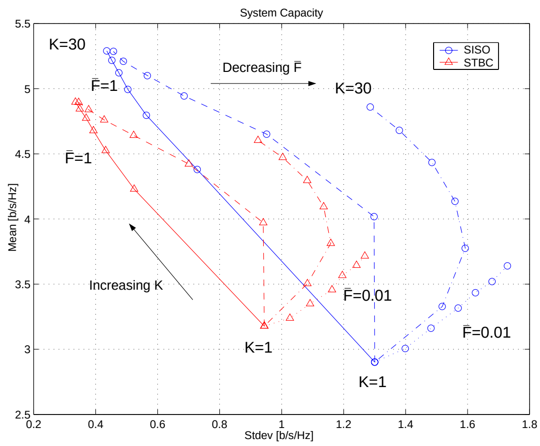

Fig. 2. Aggregated system capacity: mean (average) vs. standard deviation plot as a function of the transmission scheme (SISO/STBC), number of users ($K = 1, 5, 10, 15, 20, 25, 30$), and feedback load ($\bar{F} = 1, 0.2, 0.05, 0.01$). $\bar{\gamma}$=10dB.

[2] The normalized average feedback load, $\bar{F}$, can be interpreted as the fraction of times a user is allowed to report its SNR [8]. For a given $\bar{F}$, different SNR thresholds (γ_{th}) result for specific transmission schemes.

[3] Note that, being the average SNR identical for all users, both the max-SNR and random schedulers will grant access probability of $1/K$ to each user.

One can also observe that for decreasing values of the feedback load, both SISO and STBC links become less stable (to different extents) since, in those conditions, the number of random scheduling decisions increases. For high and moderate values of $\bar{F}$ this can be partially compensated by increasing the number of active users. In those conditions, the likelihood of having at least one user above γ_{th} is higher and, hence, the reduced number of random scheduling decisions drives those curves again towards the low standard deviation region. Nonetheless, such an effect vanishes as the feedback load is further reduced (i.e $\bar{F} = 0.01$).

In summary, a number of non-trivial trade-offs in terms of average capacity vs. robustness to short-term variations arise when considering different transmission schemes, feedback loads and terminal count. As usual, design decisions at the cell level will be closely linked to the QoS requirements of the services under consideration.

6. APPENDIX

In order to derive a closed-from expression of the average system capacity with STBC one should solve the following two integrals:

$$\mathcal{A}(a,m,\mu) = \int_a^\infty \ln(1+t)t^{m-1}e^{-\mu t}dt \quad (3)$$
$$\mathcal{B}(a,m,\mu) = \int_0^a \ln(1+t)t^{m-1}e^{-\mu t}dt$$
$$\mu > 0; m = 1, 2, ...$$

In [13], the authors solved the integral $\mathcal{A}(a,m,\mu)$ for the case $a = 0$. Hence, we only need to calculate $\mathcal{A}(a,m,\mu)$ for finite values of a since, clearly, $\mathcal{B}(a,m,\mu) = \mathcal{A}(0,m,\mu) - \mathcal{A}(a,m,\mu)$. By conducting an integration by parts, Eq. (3) can be conveniently rewritten as:

$$\mathcal{A}(a,m,\mu) = \int_a^\infty u dv = \lim_{t\longrightarrow\infty}(uv) - \lim_{t\longrightarrow a}(uv) - \int_a^\infty v du. \quad (4)$$

Then, we let:

$$u = \ln(1+t) \quad dv = t^{m-1}e^{-\mu t}$$
$$du = \tfrac{dt}{1+t} \quad v = -e^{-\mu t}\sum_{i=1}^{m}\frac{(m-1)!}{(m-i)!}\frac{t^{m-i}}{\mu^i}$$

where the last equality results from [[11], Eq. 2.321.2]. If we use the expressions above in Eq. (4) and note that the first term goes to zero, the following intermediate expression results:

$$\mathcal{A}(a,m,\mu) = \ln(1+a)e^{-\mu a}\sum_{i=1}^{m}\frac{(m-1)!}{(m-i)!}\frac{a^{m-i}}{\mu^i} + \sum_{i=1}^{m}\frac{(m-1)!}{(m-i)!}\frac{1}{\mu^i}\int_a^\infty \frac{t^{m-i}}{1+t}e^{-\mu t}dt$$

By using the change of variables $x = t - a$ and the binomial expansion, the integral in the above equation can be re-written as:

$$\int_a^\infty \frac{t^{m-i}}{1+t}e^{-\mu t}dt = e^{-\mu a}\int_0^\infty \frac{(x+a)^{m-i}e^{-\mu x}}{1+a+x}dx = e^{-\mu a}\sum_{p=0}^{m-i}\binom{m-i}{p}a^{m-i-p}\int_0^\infty \frac{x^p e^{-\mu x}}{1+a+x}dx$$

Next, with the help of [[11], Eq. 3.383.10] and after some manipulation, $\mathcal{A}(a,m,\mu)$ can be expressed in terms of the complementary incomplete gamma function ($\Gamma(\alpha,x) = \int_x^\infty e^{-t}t^{\alpha-1}dt$) [[11], Eq. 8.350.2]:

$$\mathcal{A}(a,m,\mu) = \sum_{i=1}^{m}\frac{(m-1)!}{(m-i)!}\frac{e^{-\mu a}a^{m-i}}{\mu^i}\Bigg[\ln(1+a) + \sum_{p=0}^{m-i}\binom{m-i}{p}\left(\frac{1+a}{a}\right)^p e^{\mu(1+a)}p!\Gamma\left(-p,\mu(1+a)\right)\Bigg] \quad (5)$$

Finally and by resorting to [[13], Eq.78] and Eq. (5), we can write $\mathcal{B}(a,m,\mu)$ in closed-form as well:

$$\mathcal{B}(a,m,\mu) = \sum_{i=1}^{m}\frac{(m-1)!}{\mu^i}\Bigg[e^{\mu}\Gamma(-m+i,\mu) - \frac{e^{-\mu a}a^{m-i}}{(m-i)!} \times \Bigg(\ln(1+a) + \sum_{p=0}^{m-i}\binom{m-i}{p}\left(\frac{1+a}{a}\right)^p e^{\mu(1+a)} \times p!\Gamma\left(-p,\mu(1+a)\right)\Bigg)\Bigg] \quad (6)$$

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