The effect of imperfect feedback on broadcast oppotunistic beamforming schemes

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Abstract: **In multiantenna systems the optimization of linear spatial precoding is severely hampered by the amount of feedback. An efficient solution consists in the opportunistic beamforming (OB), which randomly generates the precoding and schedules the users according to the corresponding signal-to-noise ratio. Performance of OB can be enhanced by matching the generation of the beamforming to the users spatial covariance as in the recently proposed Cluster-eigenbeamforming (Cluster-EB). In this paper we show that the benefits of Cluster-EB further improve in practical systems when the transmitter has imperfect knowledge of the instantaneous feedback. Cluster-EB capitalizes on the knowledge of the spatial covariance to reduce the sensitivity with respect to the feedback degradation, thus preserving the systems performance. Analytical derivations and simulation results validate the analysis.**

1. Introduction

Opportunistic beamforming (OB) [1][2] has been recently proposed to exploit multiuser diversity (MUD) in a broadcast wireless communication systems when an antenna array is employed at the base station (BS) and when reduced feedback is mandatory compared to other optimal strategies, that require full channel state information (CSI) at the transmitter. The main idea is to generate a random precoding at the BS and to schedule the users according to the signal-to-noise ratio (SNR) and the fairness requirements. Limited feedback is needed as the users have to report only the SNR relative to the transmitted precoding (partial CSI).

The OB scheme can be enhanced by capitalizing on the knowledge at the BS of the channels spatial covariance, also known as long term CSI [3][4]. This can be assumed stationary over large time scale and it can be acquired by the transmitter either directly from measurements on the opposite link or from low-rate feedback. The long term CSI can be exploited to match the beamforming generation to the users spatial patterns. More specifically, eigenbeamforming (EB) technique [3] selects the precoding from the set of leading eigenvectors of the users channel covariance matrices. Cluster-eigenbeamforming (Custer-EB) [4] arranges the users into groups based on their spatial properties and assigns a beamforming configuration to each group. Different clusters are selected in round robin fashion by transmitting the corresponding beamforming vector. At the BS, the scheduler opportunistically selects the user within the cluster that has the largest SNR. A performance comparison among Cluster-EB, OB and EB is provided in [4] under assumption of perfect partial and long term CSI. Cluster-EB is shown to outperform OB and EB as it simultaneously exploits the long term CSI and it achieves MUD gain. Furthermore, Cluster-EB reduces the feedback rate as only the users within the served cluster are required to send back the SNR.

In this paper we focus on the sensitivity of the opportunistic schemes with respect to imperfect knowledge of the multiuser feedback. More specifically, we still assume perfect long term CSI as the estimation of the spatial covariance can be drawn over several training periods and it is not delay sensitive. Differently, we assume that the partial CSI (instantaneous SNR) can be affected by imperfections. In practical systems there exists a delay τ between the channel quality measurement at the user terminal and the downlink transmission of the scheduled user (i.e., in HSPDA is $\tau = 2ms$ [5]). As a consequence, the SNR employed at the scheduler is an outdated version of the actual SNR, thus leading to a MUD degradation as pointed out in [6]. Furthermore, the SNR is estimated at the mobile terminal over a single training period and the estimate can be affected by noise in particular if the training is limited and/or the channel is noisy.

We derive analytical expressions for the performance of Cluster-EB, while simulation results are provided to assess the sensitivity of EB and OB schemes. Cluster-EB is shown to outperform OB and EB for each kind of partial CSI imperfection and the gain increases with the feedback degradation. User clustering based on spatial covariance exploits the reliability of the long term CSI to increase the robustness against partial CSI degradation. Accordingly, Cluster-EB always guarantees the spatial matching between the scheduled user channel and the precoding. The results confirm the efficiency of Cluster-EB and motivate the employment in practical systems, where reduced feedback is mandatory and instantaneous side information can be imperfect.

The outline of the paper is as follows. We introduce the system model in Sect. 2. and we briefly review the clustering algorithm in Sect. 3.. The opportunistic schemes are described in Sect. 4., while in Sect. 5. we assess the sensitivity to the feedback imperfection. Simulation results corroborates our analysis in Sect. 6.. Finally Sect. 7. draws some concluding remarks.

2. System model

Consider the downlink of cellular system with a BS equipped with M antennas and N single-antenna mobile users. Let $\mathbf{x}(t)$ be any $M \times 1$ symbols vector transmitted within time-slot time t by the BS and let $y_i(t)$ be the signal at the $i-th$ receiver

$$y_i(t) = \mathbf{h}_i^T(t)\mathbf{x}(t) + n_i(t), \qquad (1)$$

where $\mathbf{h}_i(t) = [h_i^1(t) \cdots h_i^M(t)]^T$ is the complex channel vector and $n_i(t)$ is the AWGN at the i-th receiver with $n_i \sim CN(0, \sigma_i^2)$. Channel vector is assumed to be zero-mean (Rayleigh fading) Gaussian distributed and it is modelled as

$$\mathbf{h}_i(t) = \mathbf{R}_i^{1/2}\mathbf{w}_i(t), \quad (2)$$

where $\mathbf{R}_i = E[\mathbf{h}_i(t)\mathbf{h}_i^H(t)]$ is the $M \times M$ spatial covariance matrix for $i-th$ user and $\mathbf{w}_i(t) \sim CN(0, \mathbf{I}_M)$. We assume a time-slot based transmission and a block-fading model so that fading coefficients $\mathbf{w}_i(t)$ are constant within each time-slot and vary independently across the time-slots and the different users (i.e., $E[\mathbf{w}_i(t)\mathbf{w}_j(k)] = \delta(i-j)\delta(t-k)$). Covariance matrix $\mathbf{R}_i$ is assumed constant over a large time-scale and it is modelled according to the Lee's model [7], which assumes a ring of uniformly distributed scatters around the mobile terminals. In this scenario the correlation between the antenna elements reduces to

$$\mathbf{R}_i(p,q) = E[h_i^p(t)h_i^q(t)^*] = J_0(2\pi(p-q)\Delta \cdot \gamma_{\max}\cos(\phi_i))e^{-j2\pi(p-q)\Delta\sin(\phi_i)}. \quad (3)$$

where $J_o(\cdot)$ is the Bessel function of the first kind of order zero, Δ is the antenna spacing in wavelength, ϕ_i is the angle of arrival relative to the $i-th$ user and $\gamma_{\max}$ is the maximum angular spread. We define the average system SNR as $\mu = E[trace(\mathbf{R}_i)/\sigma_i^2]$.

At the beginning of each time slot the base station constructs a beamforming vector $\mathbf{u}(t)$ [1] with $||\mathbf{u}(t)||^2 = 1$, so that the transmitted signal $\mathbf{x}(t)$ can be expressed from the information symbols streams $s(t)$ as $\mathbf{x}(t) = \mathbf{u}(t)s(t)$.The $i-th$ receiver estimates the value $\mathbf{h}_i^T(t)\mathbf{u}(t)$ (from training) and it feeds back to the BS the SNR

$$\gamma_i(t) = |\mathbf{h}_i^T(t)\mathbf{u}(t)|^2/\sigma_i^2 \quad (4)$$

Optimal strategy with the aim of maximizing the sum-rate capacity is to assign each stream to the user with the highest SNR. Nevertheless real systems are also concerned to guarantee fairness and latency requirements, thus a proportional fairness (PF) algorithm [8] is here considered. This keeps track of the average scheduled SNR of each user in a past window t_c and selects within each time slot the user that maximizes the ratio between the actual SNR $\gamma_i(t)$ and the average scheduled SNR. Specifically, the PF scheduler maximizes the metric

$$D(t) = \max_i \frac{\gamma_i(t)}{S_i(t)}, \quad (5)$$

where $S_i(t)$ is updated in each time slot as

$$S_i(t+1) = \begin{cases} (1-\frac{1}{t_c})S_i(t) + \gamma_i(t)/t_c & \text{user } i \text{ scheduled} \\ (1-\frac{1}{t_c})S_i(t) & \text{otherwise.} \end{cases} \quad (6)$$

3. Clustering

In this Section we briefly review (see [4] for details) the framework for user clustering used in the Cluster-EB technique. Clustering algorithm allocates the users set $\mathcal{C}$ into G clusters $\mathcal{C}_k$ for $k = 1 \ldots G$, each one containing $|\mathcal{C}_k|$ users, such that where $\mathcal{C}_k \cap \mathcal{C}_h = 0$ for $k \neq h$, $\bigcup_{k=1}^G \mathcal{C}_k = \mathcal{C}$ and $\sum_{k=i}^G |\mathcal{C}_k| = N$. The algorithm is based on a metric function $\eta_{i,j}$ that measures the efficiency of grouping two different users (or two groups of users). Since our goal is to design a general framework at moderate complexity, we propose to adopt the following metric

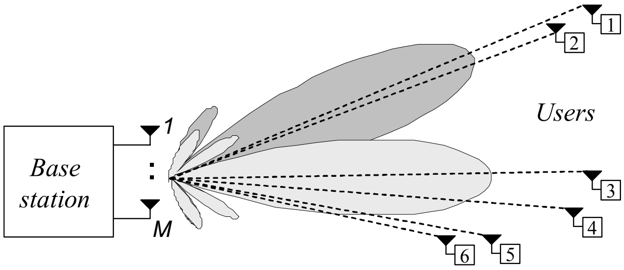

Figure 1: Broadcast system with M antennas at the BS and $N = 6$ users. Users position determines the correlation metric (directional model).

$$\eta_{i,j} = \frac{tr[\mathbf{R}_i\mathbf{R}_j^H]}{\sqrt{tr[\mathbf{R}_i\mathbf{R}_i^H]tr[\mathbf{R}_j\mathbf{R}_j^H]}}, \quad (7)$$

that provides a normalized measure of the average channel covariances correlation. Each covariance matrix can be decomposed as $\mathbf{R}_i = \sum_{p=1}^M \mathbf{u}_i^p \lambda_i^p \mathbf{u}_i^{pH}$, where λ_i^p and $\mathbf{u}_i^p$ stand for the eigenvalues and eigenvectors of $\mathbf{R}_i$, respectively. Thus, the metric $\eta_{i,j}$ can be rearranged as

$$\eta_{i,j} = \frac{\sum_{p,q} \lambda_i^{(p)}\lambda_j^{(q)}|\mathbf{u}_i^{(p)H}\mathbf{u}_j^{(q)}|^2}{\sqrt{\sum_p(\lambda_i^{(p)})^2\sum_q(\lambda_j^{(q)})^2}}. \quad (8)$$

Eq. (8) shows that the metric $\eta_{i,j}$ is an average of the square correlation of each pair of spatial modes $(\mathbf{u}_i^{(p)}, \mathbf{u}_j^{(q)})$ weighed by the corresponding modal amplitudes $(\lambda_i^{(p)}\lambda_j^{(q)})$. Metric maximum $(\eta_{i,j} = 1)$ is achieved when the users signal subspaces overlap completely, while minimum $(\eta_{i,j} = 0)$ is obtained in case of orthogonal users.

An intuitive insight on metric meaning is provided by Fig. 1 under assumption that $\gamma_{\max} = 0$ in (3) (i.e., the covariance matrix depends only on the direction of arrival). Users located along similar directions (1 and 2) have covariance matrices so that $range(\mathbf{R}_1) \simeq range(\mathbf{R}_2)$, thus it is $\eta_{1,2} \simeq 1$. The scheduler can achieve multiuser diversity gain by grouping the users 1 and 2 and by using the same spatial filtering (in figure the beamforming is focused on the middle so as it serves both users).

We compute the metric $\eta_{i,j}$ for each couple of users (i,j) with $i,j \in \{1,\ldots,N\}$. This requires the evaluation of the product $\mathbf{R}_i\mathbf{R}_j^H$ for each pair of users, thus leading to an overall computation complexity of $N^2/2$ matrix products. Then, we perform a low-complexity tree-based approach, that sequentially merges the users

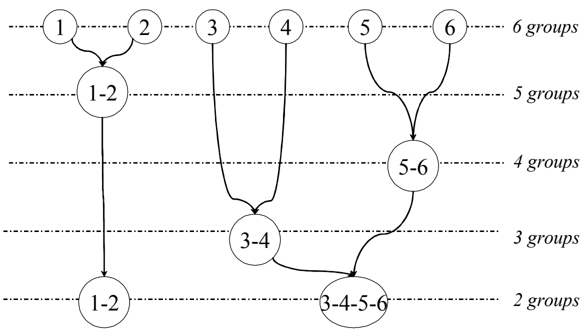

Figure 2: Tree-based clustering for $N = 6$ users (see geometrical view in Fig. 1)

into groups of high correlated elements as depicted in Fig. 2. In the exemplary tree we have the $N = 6$ users of Fig. 1 that are sequentially grouped in $G = 2$ clusters. At the first step, each users corresponds to a group. At each step we have one group less. This is achieved by merging two groups at the previous level into a single group. Between all the possible combinations, we select the pair (i, j) that maximizes the metric $n_{i,j}$. At each step the metrics of the group obtained by merging is computed as $\eta_{(ij),z} = (\eta_{i,z} + \eta_{j,z})/2$ where $\eta_{(ij),z}$ is the metric between the group containing users $i - th$ and $j - th$ and a different user $z - th$. Whenever the maximum of the metric $n_{i,j}$ between two group is lower than a given threshold η_{th}, we stop the merging procedure, thus the number of clusters G is adaptively selected.

4. Opportunistic schemes

4.1. Conventional (OB)

The opportunistic beamforming was first introduced in [1] with the goal to induce large and fast channel fluctuations and to exploit the MUD. The scheme is particularly effective for a large number N of users and for slowly-varying fading channels. Nevertheless, the OB is often employed in a fast-fading scenario as it does not require the knowledge of instantaneous $\mathbf{h}_i(t)$ at the BS as the beamforming vector $\mathbf{u}(t)$ is randomly generated. In this paper we generate the beam as a phased array

$$\mathbf{u}(t) = \frac{1}{\sqrt{M}}[1, e^{-j2\pi\Delta\sin\phi(t)} \cdots e^{-j2\pi\Delta\sin\phi(t)(M-1)}]^T, \quad (9)$$

where the angle $\phi(t)$ is random. This generation model is shown to be effective (see [3]) when assuming the propagation scenario depicted in Sect. 2..

4.2. Eigenbeamforming (EB)

Several improvement of the OB have been proposed in literature. If the BS is provided by the channel users covariances, the eigenbeamforming (EB) strategy [3] generates the beams from the set of the eigenvectors corresponding to the largest eigenvalues of the users covariance matrices $\mathbf{R}_i$. Each beamforming maximizes the average user SNR

$$\overline{\gamma}_i = \frac{E[|\mathbf{u}^H(t)\mathbf{h}_i|^2]}{\sigma_i^2} = \frac{\mathbf{u}^H(t)\mathbf{R}_i\mathbf{u}(t)}{\sigma_i^2}. \quad (10)$$

4.3. Cluster Eigenbeamforming (Cluster-EB)

The cluster Eigenbeamforming (Clust-EB) technique has been proposed in [4]. The main idea is to group the users into clusters of high-correlated elements using the tree-based algorithm sketched in Sect. (3.). Each cluster is associates a single beam $\mathbf{u}_k$ matched with the users spatial properties, while different clusters are served in different time-slots. Strong correlation of users sharing the same cluster guarantees high $\overline{\gamma}_i$ (comparable to that of EB) for each user i belonging to the selected cluster. At the same time, clustering enhances the diversity gain with respect to EB, where each beam is matched to a single user channel. Furthermore, Clust-EB can be exploited to reduce the feedback rate as only the users within the selected cluster $\mathcal{C}_k$ are matched with the transmitted beam $\mathbf{u}_k$ and are required to send back the SNR.

4.4. Performance analysis

We investigate the algorithm performance as the running average scheduled SNR $S_i(t)$ defined in eq. (6) for PF. We initially assume perfect and instantaneous feedback. It can be proved [8] that for PF scheduler with large t_c the scheduled SNR $S_i(t)$ is described by

$$\frac{\partial S_i(t)}{\partial t} = S_i(t) - E[\gamma_i | \frac{\gamma_i}{S_i} = \max(\frac{\gamma_i}{S_1}, \cdots, \frac{\gamma_i}{S_N})] \cdot$$
$$\cdot \Pr\text{ob}(\frac{\gamma_i(t)}{S_i} = \max(\frac{\gamma_1(t)}{S_1}, \cdots, \frac{\gamma_N(t)}{S_N}), \quad (11)$$

where γ_i is a random variable (RV) corresponding to the SNR of $i - th$ user. At steady state it is $S_i = lim_{t\to\infty} S_i(t)$ $1 \le i \le N$ and the performance metric can be obtained from

$$S_i = E[\gamma_i | \frac{\gamma_i}{S_i} = \max(\frac{\gamma_1}{S_1}, \cdots, \frac{\gamma_N}{S_N})] \cdot$$
$$\cdot \Pr\text{ob}(\frac{\gamma_i(t)}{S_i} = \max(\frac{\gamma_1(t)}{S_1}, \cdots, \frac{\gamma_N(t)}{S_N}). \quad (12)$$

It follows that each user is selected for equal fraction of time, defined as $\pi_i = 1/N$. Analytic derivation of S_i needs the knowledge of the probability density function of the instantaneous SNR $\gamma_i(t)$. This is far from being trivial for OB and EB techniques as the precoding at the BS is time-varying. Differently, in Cluster-EB the users belonging to cluster $\mathcal{C}_k$ are always served when transmitting the beam $\mathbf{u}_k$. Thus, the SNR $\gamma_i(t)$ of each users is distributed as χ_2^2 RV for Rayleigh fading. We can investigate the performance of each cluster separately. Let us consider the cluster $\mathcal{C}_k$, the PF converges to the solution of (12) for all the users i belonging to $\mathcal{C}_k$. Normalization of the users average SNR makes the statistic of the maximum equivalent to the statistic of the maximum of $|\mathcal{C}_k|$ i.i.d χ_2^2 distributed RV. Thus it holds

$$S_i = \pi_k \cdot \frac{\Gamma(|\mathcal{C}_k|)}{|\mathcal{C}_k|} \frac{\mathbf{u}_k^H \mathbf{R}_i \mathbf{u}_k}{\sigma_i^2} = \pi_k \frac{\Gamma(|\mathcal{C}_k|)}{|\mathcal{C}_k|} \overline{\gamma}_i, \quad (13)$$

where π_k is the fraction of time allocated to $k - th$ cluster, $\overline{\gamma}_i = \mathbf{u}_k^H \mathbf{R}_i \mathbf{u}_k / \sigma_i^2$ is the average user SNR, $|\mathcal{C}_k|$

is the number of users in cluster $\mathcal{C}_k$, $\mathbf{u}_k$ is the beam for cluster $k-th$ and $\Gamma(|\mathcal{C}_k|) = \sum_{k=1}^{|\mathcal{C}_k|} 1/k$ is the multiuser diversity gain for χ_2^2 RV (see [8]). To provide a fair resource allocation, time-slots are allocated proportionally to the number of elements of each cluster, thus it is $\pi_k = \frac{|\mathcal{C}_k|}{N}$. Consequently the scheduled SNR for cluster k is

$$\sum_{i\in\mathcal{C}_k} S_i = \frac{\Gamma(|\mathcal{C}_k|)}{N} \sum_{i\in\mathcal{C}_k} \frac{\mathbf{u}_k^H \mathbf{R}_i \mathbf{u}_k}{\sigma_i^2}. \tag{14}$$

According to (13), S_i is maximized when the beam $\mathbf{u}_k$ corresponds to the eigenvectors relative to the largest eigenvalue of the matrix $\mathbf{R}_{\mathcal{C}_k}$, which contains the users spatial subspaces scaled by the noise power:

$$\mathbf{R}_{\mathcal{C}_k} = [\frac{\mathbf{R}_1}{\sigma_1^2}, \cdots, \frac{\mathbf{R}_{|\mathcal{C}_k|}}{\sigma_{|\mathcal{C}_k|}^2}], \tag{15}$$

where we assume that users $\{1, \ldots, |\mathcal{C}_k|\}$ belong to cluster $\mathcal{C}_k$. Finally, the system scheduled SNR reads

$$S = \sum_{i=1}^{N} S_i = \frac{1}{N} \sum_{k=1}^{G} \Gamma(|\mathcal{C}_k|) \sum_{i\epsilon\mathcal{C}_k} \overline{\gamma}_i. \tag{16}$$

The performance metric S is affected by two main components. The user SNR $\overline{\gamma}_i$ depends on the cluster geometrical properties, while the multiuser diversity $\Gamma(|\mathcal{C}_k|)$ is related on the number of users for each cluster. When the clusters consist in a single element ($|\mathcal{C}_k| = 1$) the technique reduces to EB. On the other hand, large clusters enhance the MUD gain, but reduce the average users SNR. As a consequence system throughput can be maximized by selecting a proper number of clusters. We terminate the merging procedure when the maximum metric $\eta_{i,j}$ between two groups is lower than the threshold value η_{th}. From our simulation we experienced that $\eta_{th} = 0.75$ is a reasonable trade off.

5. Imperfect feedback

The assumption of error-free and instantaneous feedback channel is often optimistic in real systems. Herein we focus on the performance of the opportunistic schemes when the BS is provided by imperfect feedback. We initially analyze the effect of a feedback delay, next we consider the degradation introduced by the presence of additive noise on the estimation of the receiver channel. Conversely, the spatial covariance matrices are assumed always perfectly known at the BS as the long term channel properties are stationary over a large time-scale, thus estimation of the channel covariance matrices can be performed with arbitrary accuracy.

5.1. Outdated feedback

Let us assume that the BS is provided by the outdated knowledge of the instantaneous SNR $\widehat{\gamma}_i(t) = |\mathbf{h}_i^T(t-\tau)\mathbf{u}_k|^2$ when the channel is $\mathbf{h}_i^T(t)$ and beam is $\mathbf{u}_k$ as $k-th$ cluster is served. According to Jakes channel model, the channel $\mathbf{h}_i^T(t)$ is correlated with the delayed version $\mathbf{h}_i^T(t-\tau)$ by a correlation coefficient $\rho = J_0(2\pi f_D \tau)$ where $J_0(\cdot)$ is the zero-order Bessel function of the first kind and f_D is the Doppler spread of the $i-th$ user. For sake of clarity, we denote here the instantaneous SNR $\gamma_i(t)$ as γ_i and the outdated SNR at time t as $\widehat{\gamma}_i$. The conditional probability density function (pdf) of the instantaneous SNR conditioned to the delayed SNR can be derived as [9]

$$f(\gamma_i|\widehat{\gamma}_i) = \frac{1}{(1-\rho^2)\overline{\gamma_i}} \cdot$$
$$\cdot \exp(-\frac{\rho^2\widehat{\gamma}_i + \gamma_i}{(1-\rho^2)\overline{\gamma_i}}) I_0(\frac{2\rho\sqrt{\widehat{\gamma}_i\gamma_i}}{(1-\rho^2)\overline{\gamma_i}}), \tag{17}$$

where $\gamma, \widehat{\gamma} \geq 0$ and $I_0(\cdot)$is the zero-order modified Bessel function of the first kind. Using the same strategy as above, we initially consider only transmission of $k-th$ cluster. The PF schedules the users so that S_i converges to the solution of the following equation

$$\frac{\partial S_i}{\partial t} = S_i - E[\gamma_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1}, \cdots, \frac{\widehat{\gamma}_{|\mathcal{C}_k||}}{S_{|\mathcal{C}_k|}})], \tag{18}$$

thus at steady state we obtain

$$S_i = E[\gamma_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1}, \cdots, \frac{\widehat{\gamma}_{|\mathcal{C}_k||}}{S_{|\mathcal{C}_k|}})]. \tag{19}$$

The pdf $f(\gamma_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|\mathcal{C}_k|}}{S_{|\mathcal{C}_k|}}))$can be decomposed as

$$f(\gamma_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|\mathcal{C}_k|}}{S_{|\mathcal{C}_k|}})) =$$
$$f(\gamma_i|\widehat{\gamma}_i) g(\widehat{\gamma}_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|\mathcal{C}_k|}}{S_{|\mathcal{C}_k|}})), \tag{20}$$

where the first term is from (17) and $g(\widehat{\gamma}_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|\mathcal{C}_k|}}{S_{|\mathcal{C}_k|}}))$ is the pdf of the maximum of $|\mathcal{C}_k|$ χ_2^2 RV. The normalization permits to simplify the computation of the maximum distribution as the RV are scaled to the same average power $\overline{\gamma}_i$

$$g(\widehat{\gamma}_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|\mathcal{C}_k|}}{S_{|\mathcal{C}_k|}})) =$$
$$(1 - \exp(-\frac{\gamma_i}{\overline{\gamma}_i}))^{N-1} \frac{N}{\overline{\gamma}_i} \exp(-\frac{\gamma_i}{\overline{\gamma}_i}). \tag{21}$$

As a conclusion the long term S_i of user i within cluster k yields (see Appendix A for more details)

$$S_i = \frac{\overline{\gamma}_i}{N}[\Gamma(|\mathcal{C}_k|)\rho^2 + (1-\rho^2)]. \tag{22}$$

When the feedback channel is instantaneous ($\rho = 1$) equation (22) reduces to (13), while for completely decorrelated channel ($\rho = 0$) the scheduler can not exploit the MUD gain and it reduces to a simple round robin scheme with $S_i = \frac{\overline{\gamma}_i}{N}$. We remark that $\overline{\gamma}_i = (\mathbf{u}_k^H \mathbf{R}_i \mathbf{u}_k)/\sigma_i^2$ stands for the average SNR of users i when the beam $\mathbf{u}_k$ is transmitted (assume that user i belongs to cluster $\mathcal{C}_k$). Thus, the MUD gain depends on the reliability of partial CSI, while the spatial matching between scheduled user and precoding is always guaranteed in Cluster-EB.

5.2. Noisy feedback

The signal received at user i can be expressed from (1) as

$$y_i(t) = \mathbf{h}_i^T(t)\mathbf{u}_k s(t) + n_i(t) = r_i(t)s(t) + n_i(t), \quad (23)$$

where $r_i(t) = \mathbf{h}_i^T(t)\mathbf{u}_k$ is the equivalent scalar channel. By assuming the noise power σ_i^2 as known, each user has to estimate the complex value $r(t)$ in order to feed back the instantaneous SNR $\widehat{\gamma}_i(t) = |\mathbf{h}_i^T(t)\mathbf{u}_k|^2/\sigma_i^2 = |r_i(t)|^2/\sigma_i^2$ to the BS. Since the received signal $y(t)$ is noisy, the estimation can be corrupted by estimation error

$$\widehat{r}_i(t) = r_i(t) + p_i(t), \quad (24)$$

where $r_i(t) \sim CN(0, \overline{\gamma}_i)$ and we assume $p_i(t) \sim CN(0, \beta)$. Thus, the pdf of the estimate given the real channel value is $f(\widehat{r}_i(t)|r_i(t)) \sim CN(r_i(t), \beta)$. By applying Bayes approach [10], we obtain that $f(r_i(t)|\widehat{r}_i(t)) \sim CN(\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\beta} r_i(t), \frac{\beta\overline{\gamma}_i}{\overline{\gamma}_i+\beta})$. As a consequence, the pdf of instantaneous SNR $\gamma_i(t)$ conditioned to the noisy SNR $\widehat{\gamma}_i(t)$ can be derived as

$$f(\gamma_i|\widehat{\gamma}_i) = \frac{\overline{\gamma}_i + \beta}{\beta\overline{\gamma}_i} \exp(-\frac{\overline{\gamma}_i + \beta}{\beta\overline{\gamma}_i} \cdot ((\frac{\overline{\gamma}_i}{\overline{\gamma}_i + \beta})^2 \widehat{\gamma}_i + \gamma_i) I_0(2\sqrt{\frac{\widehat{\gamma}_i \gamma_i}{\beta^2}}), \quad (25)$$

where $\gamma, \widehat{\gamma} \geq 0$. For user i belonging to cluster $\mathcal{C}_k$, it is $S_i = E[\gamma_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|C_k|}}{S_{|C_k|}})]$ and the pdf of the maximum of the outdated SNRs can be decomposed as

$$f(\gamma_i|\widehat{\gamma}_i = \max(\widehat{\gamma}_1 \cdots \widehat{\gamma}_{|C_k|})) = f(\gamma_i|\widehat{\gamma}_i) g(\widehat{\gamma}_i|\widehat{\gamma}_i = \max(\frac{\widehat{\gamma}_1}{S_1} \cdots \frac{\widehat{\gamma}_{|C_k|}}{S_{|C_k|}})), \quad (26)$$

After some algebra (see Appendix A) we obtain that the channel estimation error affects S_i as [1]

$$S_i = \frac{\overline{\gamma}_i}{N}(\frac{\mu_i}{1+\mu_i}\Gamma(|\mathcal{C}_k|) + \frac{1}{1+\mu_i}), \quad (27)$$

where $\mu_i = \frac{\overline{\gamma}_i}{\beta}$. When the channel estimation is error free ($\beta = 0$, $\mu_i \to \infty$) expression (27) reduces to (13), while for very noisy estimation (say $\beta \to \infty$, $\mu_i \to 0$) MUD gain degrades and the scheduler reduces to a simple round robin scheme ($S_i = \frac{\overline{\gamma}_i}{N}$). As for delayed feedback, Cluster-EB preserves the spatial matching between the scheduled user $i-th$ belonging to cluster $\mathcal{C}_k$ and the BS precoding $\mathbf{u}_k$, thus leading to an high average SNR $\overline{\gamma}_i$ even when the partial CSI is strongly corrupted by noise.

6. Numerical results

We investigate the performance of the opportunistic schemes under assumption of imperfect feedback. In our simulation scenario, the base station is equipped by $M = 6$ antennas and there are N users in the cell. The channels correlation matrices $\mathbf{R}_i$ are drawn according to the model described in Sect. 2. for angles $\Phi_i \sim U[-\pi/2, \pi/2]$ and maximum angular spread $\gamma_{\max} \sim U[0, 0.5]$ rad. Although the users are moving, we assume that the velocity is slow enough to make the angles and the spread (thus the spatial covariances) to be stationary over several time-slots. All the users have equal average SNR $\mu = trace(\mathbf{R}_i)/\sigma_i^2 = M = 7.8\ dB$ for $1 \leq i \leq N$. Proportional fairness scheduling is based on a temporal window of $t_c = N$ time-slots.

Fig. 3 (a) shows the average SNR S_i vs. the channel decorrelation $\rho = J_0(2\pi f_D \tau)$ due to the delay τ between the channel estimation at the MB and the scheduling at the BS. We consider $N = 10, 50, 100$ users. It is easy to see that Cluster-EB has a linear slope for reduced feedback delay ($\rho \to 1$), while it saturates to $S = E_i[\overline{\gamma}_i]$ for decorrelated channel ($\rho \to 0$) as the BS has no instantaneous information on the channel fading. In this last case the performance of Cluster-EB reduces to that of the RR within each cluster and performance does not depend on the number of the users $|\mathcal{C}_k|$ (no MUD gain). Differently, in OB and EB the scheduled SNR still depends on N as the probability that the beam is spatially matched with the scheduled user increases with N. It can be noticed that Cluster-EB is less sensitive to the channel decorrelation as compared to EB and OB. The reason is that Cluster-EB forces the scheduler to select the mobile terminal among the set $\mathcal{C}_k$ of users whose channels are spatially matched with the transmitted beam $\mathbf{u}_k$. Differently, the OB and EB scheduler selects the users according only to the instantaneous information $\widehat{\gamma}_i(t)$, that can be outdated. This result is confirmed by Fig. 3 (b) which shows the cumulative density function (CDF) of the scheduled SNR $\gamma(t)$ corresponding to the served user when there are $N = 30$ users in the cell. It can be noticed that the performance of Cluster-EB is effective (as compared to EB and OB) also for large feedback delay ($\rho = 0.5$).

We investigate also the effect of estimation error at the mobile terminal. Fig. 4 (a) shows the system SNR vs. the noise power β for $N = 10, 50, 100$ users in the cell. We can notice that Cluster-EB outperforms the other techniques and the gain grows with the estimation noise power. The robustness of Cluster-EB with respect to the feedback imperfection is here even more evident than in case of delayed feedback. The rationale is that the additive noise can affect the partial CSI even more than the feedback delay (for high noise power β). Thus, the OB and EB performance degrades considerably and it is of crucial importance to group the users and to schedule only the cluster of spatially matched users. Finally, we show the CDF of the scheduled SNR for channel estimation error and $N = 30$ users Fig. 4 (b).

7. Conclusions

In this paper it has been investigated the sensitivity of different opportunistic schemes with respect to feedback imperfection. Two different models of feedback degradation have been assumed: feedback delay and estimation error at the mobile terminal. Analytical expressions

[1] Notice that Eq. (25) has the same structure as Eq. (17). Hence, the same procedure has been adopted for obtaining the scheduled SNR.

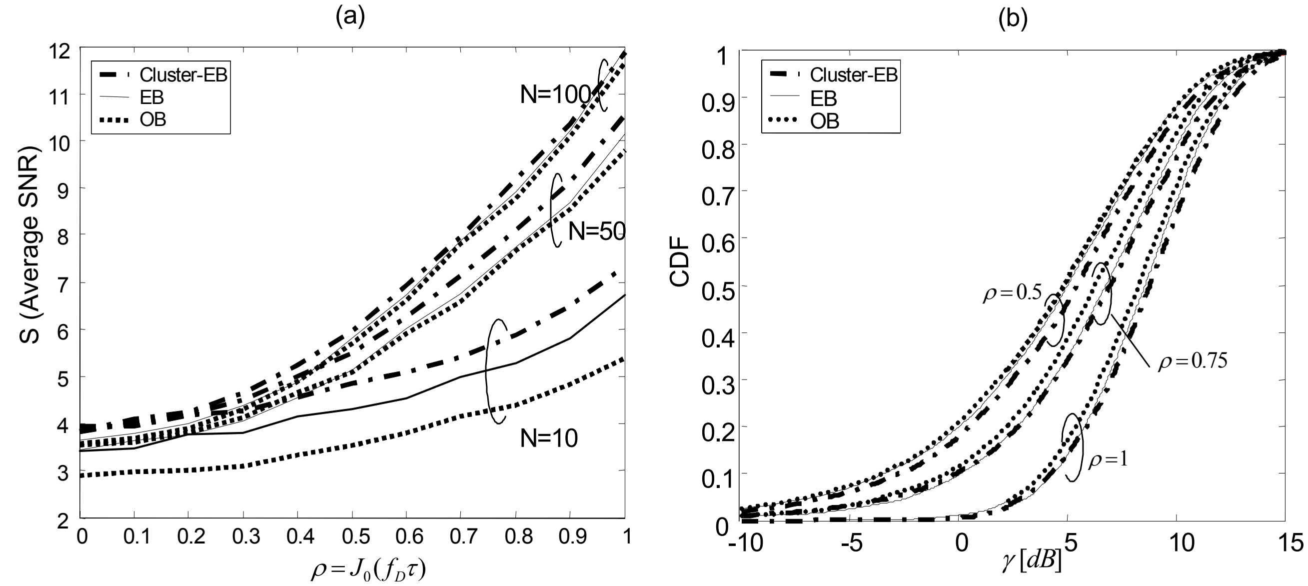

Figure 3: (a) Running average SNR vs. channel decorrelation for Cluster-EB, EB and OB schemes. (b) Cumulative density function (CDF) of the scheduled SNR. We consider M=6 antennas at the BS, system SNR=7.8 dB, $\gamma_{\max} \sim U[0, 0.5]$ rad N=10,50,100 users in (a) and N=30 users in (b).

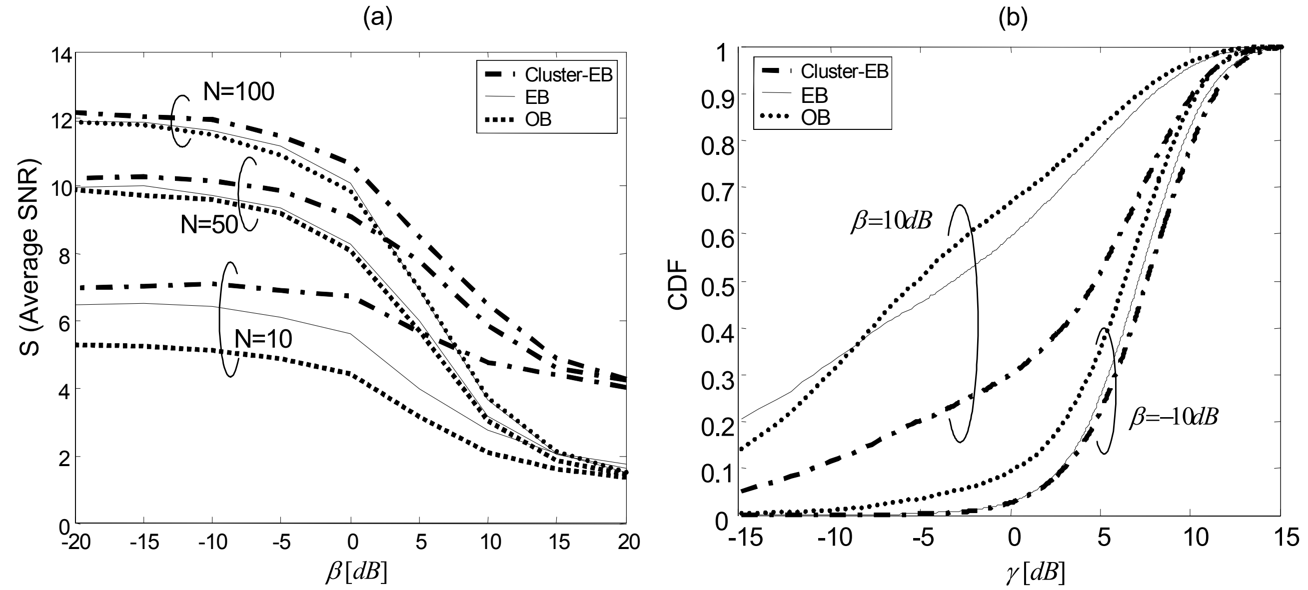

Figure 4: (a) Running average SNR vs. estimation noise power for Cluster-EB, EB and OB schemes. (b) Cumulative density function (CDF) of the scheduled SNR. We consider M=6 antennas at the BS, system SNR=7.8 dB, $\gamma_{\max} \sim U[0, 0.5]$ rad N=10,50,100 users in (a) and N=30 users in (b).

and simulation results have been provided to assess the performance degradation. Despite of its moderate complexity, Cluster-EB has been shown to be very effective against the feedback degradation as it exploits the long term CSI to guarantee the matching between user spatial pattern and transmitted precoding. The moderate complexity of clustering algorithm and the robustness against imperfections make Cluster-EB well suited to a practical implementation.

8. Acknowledgment

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A Proof of (22 and 27)

The long-term S_i of each user can be written as:

$$\begin{aligned} S_i &= E[\gamma_i] = \frac{1}{N}\int_{\gamma_i=0}^{\infty}\int_{\widehat{\gamma}_i=0}^{\infty}\gamma_i f(\gamma_i|\widehat{\gamma}_i)g(\widehat{\gamma}_i|\widehat{\gamma}_i \\ &= \max(\frac{\widehat{\gamma}_1}{S_1}\cdots\frac{\widehat{\gamma}_{|C_k|}}{S_{|C_k|}}))d\widehat{\gamma}_i d\gamma_i \end{aligned} \quad (28)$$

By substituting Eqs. (17) and (21) into the above expression, we obtain the following integral

$$\begin{aligned} S_i = \int_{\gamma_i=0}^{\infty}\int_{\widehat{\gamma}_i=0}^{\infty}\frac{\gamma_i}{(1-\rho^2)\overline{\gamma_i}^2}\exp(-\frac{\widehat{\gamma}_i+\gamma_i}{(1-\rho^2)\overline{\gamma_i}})\cdot \\ I_0(\frac{2\rho\sqrt{\widehat{\gamma}_i\gamma_i}}{(1-\rho^2)\overline{\gamma_i}})(1-\exp(-\frac{\widehat{\gamma}_i}{\overline{\gamma}_i}))^{N-1}d\widehat{\gamma}_i d\gamma_i \end{aligned} \quad (29)$$

By using the binomial expansion and identities [[11],Eq. 6.614.3], [[11],Eq. 9.220.2] and [[11],Eq. 9.215.1], the following intermediate expression results

$$\begin{aligned} S_i = \frac{1}{\overline{\gamma}_i}\sum_{z=0}^{z-1}\binom{N-1}{z}\frac{(-1)^z}{1+z(1-\rho^2)}\cdot \\ \int_{\gamma_i=0}^{\infty}\gamma_i\exp(-\frac{\gamma_i(z+1)}{\overline{\gamma}_i(1+z(1-\rho^2))})d\gamma_i \end{aligned} \quad (30)$$

Finally, by integrating by parts and after some manipulation one can easily achieve the following result

$$S_i = \frac{\overline{\gamma}_i}{N}[\Gamma(|\mathcal{C}_k|)\rho^2 + (1-\rho^2)]. \quad (31)$$

Following the same reasoning the long-term S_i of each user can be written from Eq. (25) as

$$\begin{aligned} S_i &= E[\gamma_i] = \frac{1}{N}\int_{\gamma_i=0}^{\infty}\int_{\widehat{\gamma}_i=0}^{\infty}\gamma_i f(\gamma_i|\widehat{\gamma}_i)g(\widehat{\gamma}_i|\widehat{\gamma}_i \\ &= \max(\frac{\widehat{\gamma}_1}{S_1}\cdots\frac{\widehat{\gamma}_{|C_k|}}{S_{|C_k|}}))d\widehat{\gamma}_i d\gamma_i. \end{aligned} \quad (32)$$

By substituting Eqs. (17) and (21) into the above expression, the following integral should be solved

$$\begin{aligned} S_i = \int_{\gamma_i=0}^{\infty}\int_{\widehat{\gamma}_i=0}^{\infty}\frac{\gamma_i}{\beta\overline{\gamma}_i}\exp(-\frac{\widehat{\gamma}_i+(\beta+\overline{\gamma_i})\gamma_i}{\beta\overline{\gamma_i}})\cdot \\ I_0(2\sqrt{\frac{\widehat{\gamma}_i\gamma_i}{\beta^2}})(1-\exp(-\frac{\widehat{\gamma}_i}{\overline{\gamma}_i+\beta}))^{N-1}d\widehat{\gamma}_i d\gamma_i \end{aligned} \quad (33)$$

By substituting Eqs. (17) and (21) into the above expression and after some algebra we obtain

$$S_i = \frac{1}{N}\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\beta}(\overline{\gamma}_i\Gamma(|\mathcal{C}_k|)+\beta), \quad (34)$$

that can be arranged as (27) in the main text.

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