Cross-layer Interaction Between Spatial and Multi-user Diversity in Selective Feedback Systems: Outage Capacity Analysis

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ABSTRACT

In this paper, we explore spatial vs. multi-user diversity tradeoffs in terms of outage capacity for a non-homogeneous cellular system with selective feedback. More precisely, we analytically derive an upper bound of the outage capacity of both a SISO and an OSTBC transmission scheme and, further, analyze some interesting trade-offs concerning the impact of terminal count and bandwidth restrictions. Such outage capacity analysis is particulary suited for the study of delay-constrained services. Numerical examples reveal that in many scenarios OSTBC is far less sensitive to impairments in the feedback channel.

Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Communications Applications

General Terms

Performance, Reliability, Theory

Keywords

Wireless systems, cross-layer designs, multi-antenna schemes, multi-user diversity, selective scheduling, spatial diversity

1. INTRODUCTION

Multi-user diversity (MUD) concepts, first introduced by Knopp and Humblet in [1], rely on the assumption that different users in a wireless multi-user system experience independent fading processes. In those circumstances, the aggregated cell throughput can be substantially increased by scheduling in each time slot the user with the most favorable channel conditions.

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Besides, in such fading environments, the exploitation of transmit spatial diversity (e.g., by means of orthogonal space-time block coding, OSTBC) makes transmission links more robust by using low-complexity receivers [2][3] and, for that reason, much attention has been recently paid to the cross-layer interaction between transmit spatial diversity (physical layer) and multi-user diversity (MAC layer). For instance, in [4] and [5], the inclusion of OSTBC in multiuser schemes was analyzed. It was shown that, in a multi-user context, Single-Input Single-Output (SISO) schemes outperform OSTBC-based ones in terms of aggregated cell capacity. Certainly, spatial diversity helps reduce the probability of deep fades but, by averaging over different diversity branches, SNR peaks (those that multi-user diversity can exploit) are suppressed as well. Consequently, the resulting ergodic capacity is lower. In [6], it was proven that with perfect Channel State Information (CSI) at the transmitter, spatial diversity can be efficiently exploited in a multiuser context by using optimal beamforming. Since *perfect* CSI is seldom available at the base station, a second scheme that concentrates transmit power in the antenna with the largest gain is considered as well. For this second approach only low-rate partial CSI is needed but its performance is considerably sensitive to a number of impairments in the feedback channel.

Recently, several studies have shown that the increased robustness of OSTBC against imperfect CSI provides significant capacity gains with respect to SISO schemes. For instance, in [7] and [8] the authors analyze the impact of *delays* in the feedback channel. The consequences of *bandwidth restrictions* are explored in [9] and [10] by Gesbert and Alouini where a bandwidth-efficient Selective-MUD (SMUD) scheduler is presented. In [11], the authors derive closed-form expressions of the ergodic system capacity for both SISO and OSTBC transmission schemes in order to *analytically* assess the impact of such bandwidth restrictions in SMUD.

However, the aforementioned studies analyze system performance in terms of ergodic capacity. In delay-limited systems where codewords length is potentially short, ergodic system capacity is not a valid measure since those codewords undergo a finite number of fading states only. In those circumstances, outage capacity seems to be more appropriate. In this paper, then, we focus on the interaction of spatial and multi-user diversity for a Selective MUD system in terms of outage capacity. The study is conducted in a non-homogeneous system, i.e., for a system where users in the cell experience different average SNRs but, still, the proposed scheduler guarantees fairness among users. In particular, we analytically derive an upper bound of the outage capacity of both a SISO and an OSTBC tranmission scheme and analyze some interesting trade-offs by means of computer simulations. This work extends that of [11] by providing an analysis in terms of outage capacity (in addition to the ergodic capacity results presented there). In addition, it complements [10] and derives simplified expressions of the post-scheduling SNR statistics (not only SISO as in [10] but also in the OSTBC case) by exploiting the fairness properties of the proposed scheduler. Such simplified expressions make the outage (or ergodic) capacity analysis much more tractable.

This paper is organized as follows. In Section 2, the corresponding signal model and scheduler are presented. Closed-form expressions for the density functions (pdf and CDF) of the post-scheduling SNRs are derived in Section 3. Next, analytical expressions for an upper bound for the outage capacity is obtained in Section 4. Finally, a number of trade-offs of the proposed transmission schemes are discussed in Section 5.

2. SIGNAL AND SYSTEM MODEL

2.1 Signal Model

Consider the downlink of a cellular system with one base station (BS) equipped with multiple antennas (N_{BS}), and K single-antenna terminals. For an arbitrary time instant, the received signal at the k-th terminal can be expressed as:

$$r_k = \mathbf{h}_k^T \mathbf{s} + n_k$$

where $\mathbf{h}_k \in \mathbb{C}^{N_{BS}}$ is the channel vector gain between the BS and the k-th terminal, for which each component is assumed to be independent and identically distributed, circularly symmetric Gaussian random variable with zero mean and user-dependent variance $\sigma^2_{h_k}$ ($\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2_{h_k}\mathbf{I}_{N_{BS}})$), $\mathbf{s} \in \mathbb{C}^{N_{BS}}$ is the symbol vector broadcasted from the BS and $n_k \in \mathbb{C}$ denotes additive Gaussian noise (AWGN) with zero mean and variance σ^2. The active users in the system are assumed to undergo independent Rayleigh fading processes and so does the signal being transmitted from different antennas in the BS. Further, we consider quasi-static fading, i.e, the channel response remains constant during one time-slot and, then, it abruptly changes to a new independent realization. Concerning CSI, we assume perfect knowledge for *each* user at the receive side, and the availability of a low-rate error-free feedback channel to convey partial CSI to the transmitter. Finally, We denote by $\gamma_k = \frac{P_t \|\mathbf{h}_k\|^2}{N_{BS}\sigma^2}$ the *instantaneous* signal-to-noise ratio experienced by user k in a given time-slot and by $\bar{\gamma}_k = \frac{P_t \mathrm{E}[\|\mathbf{h}_k\|^2]}{N_{BS}\sigma^2} = \frac{P_t \sigma^2_{h_k}}{\sigma^2}$ its long-term averaged SNR, with P_t standing for the total transmit power. Notice that the total transmitted power is constant and evenly distributed among transmit antennas.

At the BS, we will consider two transmission schemes: a SISO configuration ($N_{BS} = 1$) and an OSTBC scheme with $N_{BS} = 2$ transmit antennas[1], more precisely, the well-known Alamouti scheme [3]. For the SISO scheme, the pdf and CDF of the received SNR take the following expressions:

[1]For simplicity, we have assumed only two transmit antennas but the analysis can be easily extended to the general case.

$$f_{\gamma_k,SISO}(\gamma) = \frac{1}{\bar{\gamma}_k} e^{-\frac{\gamma}{\bar{\gamma}_k}}$$
$$F_{\gamma_k,SISO}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_k}} \quad (1)$$

respectively, whereas for the OSTBC case we can write:

$$f_{\gamma_k,OSTBC}(\gamma) = \frac{4\gamma}{\bar{\gamma}_k^2} e^{-\frac{2\gamma}{\bar{\gamma}_k}}$$
$$F_{\gamma_k,OSTBC}(\gamma) = 1 - e^{-\frac{2\gamma}{\bar{\gamma}_k}} \left(\frac{2\gamma}{\bar{\gamma}_k} + 1 \right) \quad (2)$$

For notational convenience, we define $F_x(x)$ as the corresponding CDF functions in the case of random variables with unit mean, that is,

$$F_{x,SISO}(x) = 1 - e^{-x}$$
$$F_{x,OSTBC}(x) = 1 - e^{-2x}(2x+1) \quad (3)$$

for the SISO and STBC cases, respectively.

2.2 Scheduler

The scheduler operates on a slot-by-slot basis according to a modified version of the Proportional Fair Scheduling rule [12]. In particular, in each time slot the user experiencing the maximum normalized SNR is selected for transmission. Therefore, a terminal is only scheduled when its instantaneous SNR is near to its own peak [12], that is with respect to its average SNR. Clearly, this scheduling rule is fair (every user is granted access with probability $1/K$ regardless of its average SNR) and, at the same time, some multi-user diversity can still be exploited. In order to reduce bandwidth requirements in the feedback channel, a *Selective* Multi-user Diversity (SMUD) approach is adopted: in a specific time slot, only terminals experiencing normalized-SNRs above a pre-defined threshold (ξ_{th}) are allowed to report their channel state information to the BS. Hence, the max normalized-SNR scheduler will conduct the search over such a subset of the active users only. In summary, the scheduler is driven according to the following rule:

$$k^* = \arg\max_k \left\{ \frac{\gamma_k}{\bar{\gamma}_k} \text{ s.t. } \frac{\gamma_k}{\bar{\gamma}_k} > \xi_{th} \right\}$$

when, at least, one user is reported. Conversely, when all the users remain silent (i.e. in the event of a *scheduling outage*) the scheduling rule amounts to:

$$k^* = \text{rand}\{1, ..., k, ..., K\}$$

where users are randomly selected, again with probability $1/K$.

To conclude this section, couple of comments on the selection of the normalized threshold ξ_{th}. First, notice this design parameter has a direct impact on users' individual contributions to the feedback load through the following expression:

$$\bar{F} = 1 - F_x(\xi_{th})$$

Clearly, the higher the threshold, the lower the percentage of times a user is allowed to report its SNR. We will refer to this ratio ($\bar{F}$) as the normalized average feedback load. Second, such normalized threshold must be identical for all the active users in the system since, otherwise, the user actually scheduled could not be the one with the highest SNR. As a result, fairness among users would not be preserved because of the introduction of SMUD mechanisms.

3. SNR STATISTICS IN A SELECTIVE-MUD SYSTEM

In this section, we derive the statistics of the *post-scheduling* SNRs, that is, the signal-to-noise ratio experienced by the scheduled user. Both pdf and CDF functions will be used later to derive closed-form expressions of the outage system capacity. We conduct the analysis for a generic transmission scheme (SISO or OSTBC, in this case) and, hence, we omit any explicit reference in the pdf and CDF expressions shown below.

First, by defining $\mathcal{A}_k$ as the event that user k actually scheduled and by applying Bayes theorem, one can readily obtain the CDF of the post-scheduling SNR, γ^*, as:

$$F_{\gamma^*}(\gamma) = \text{Prob}(\gamma^* \leq \gamma) = \sum_{k=1}^{K} \text{Prob}\left(\gamma_k \leq \gamma | \mathcal{A}_k\right) \text{Prob}\left(\mathcal{A}_k\right)$$

where $\text{Prob}(\mathcal{A}_k) = 1/K$, $k = 1..K$, due to the properties of the scheduler. Now, by focusing on user k, we can easily derive the probability $\text{Prob}\left(\gamma_k < \gamma | \mathcal{A}_k\right)$. In particular, the analysis must be conducted for two different SNR regions: (1) the random scheduling region $(\gamma_k/\bar{\gamma}_k \leq \xi_{th})$, where all users remain silent[2]; and (2) the max-normalized scheduling region $(\gamma_k/\bar{\gamma}_k > \xi_{th})$, where at least user k reports back its SNR.

For the $\gamma_k/\bar{\gamma}_k \leq \xi_{th}$ case and by recalling that all users experience independently distributed fading, we have:

$$\begin{aligned}
F_{\gamma_k|\mathcal{A}_k}\left(\gamma|\mathcal{A}_k\right) &= \text{Prob}\left(\gamma_k \leq \gamma|\mathcal{A}_k\right) \\
&= \text{Prob}(\gamma_k \leq \gamma)\text{Prob}\left(\frac{\gamma_i}{\bar{\gamma}_i} \leq \xi_{th} \text{ for all } i \neq k\right) \\
&= F_{\gamma_k}(\gamma) \prod_{\substack{i=1 \\ i\neq k}}^{K} F_{\gamma_i}\left(\xi_{th}\bar{\gamma}_i\right) = F_{\gamma_k}(\gamma)\left(F_x\left(\xi_{th}\right)\right)^{K-1}
\end{aligned}$$

and hence:

$$f_{\gamma_k|\mathcal{A}_k}\left(\gamma|\mathcal{A}_k\right) = f_{\gamma_k}(\gamma)\left(F_x\left(\xi_{th}\right)\right)^{K-1} \tag{4}$$

where $f_{\gamma_k}(\cdot)$ and $F_{\gamma_k}(\cdot)$ are the pdf and CDF of the SNR associated to user k (equations in (1) and (2)), whereas $F_x(\cdot)$ is defined in equations in (3).

On the other hand, for $\gamma_k/\bar{\gamma}_k > \xi_{th}$ the CDF/pdf functions are given by:

$$\begin{aligned}
F_{\gamma_k}\left(\gamma|\mathcal{A}_k\right) &= \text{Prob}\left(\gamma_k \leq \gamma|\mathcal{A}_k\right) \\
&= \text{Prob}(\gamma_k \leq \gamma)\text{Prob}\left(\frac{\gamma_i}{\bar{\gamma}_i} \leq \frac{\gamma}{\bar{\gamma}_k} \text{ for all } i \neq k\right) \\
&= F_{\gamma_k}(\gamma) \prod_{\substack{i=1 \\ i\neq k}}^{K} F_{\gamma_i}\left(\frac{\gamma}{\bar{\gamma}_k}\bar{\gamma}_i\right) = \left(F_{\gamma_k}(\gamma)\right)^K \\
f_{\gamma_k|\mathcal{A}_k}\left(\gamma|\mathcal{A}_k\right) &= K f_{\gamma_k}(\gamma)\left(F_{\gamma_k}(\gamma)\right)^{K-1}
\end{aligned} \tag{5}$$

Finally, the post-scheduling pdf can be written as follows:

$$\begin{aligned}
f_{\gamma^*}(\gamma) &= \sum_{k=1}^{K} f_{\gamma_k|\mathcal{A}_k}\left(\gamma|\mathcal{A}_k\right)\text{Prob}\left(\mathcal{A}_k\right) \\
&= \frac{1}{K}\sum_{k=1}^{K} f_{\gamma_k|\mathcal{A}_k}\left(\gamma|\mathcal{A}_k\right)
\end{aligned} \tag{6}$$

4. OUTAGE CAPACITY ANALYSIS

In a multi-user system, the instantaneous channel capacity achievable by the scheduled user k^* over the *equivalent* SISO channel is given by

$$C^* = \log_2\left(1 + \gamma^*\right)$$

In delay-limited services, channel coding is conducted over a finite (and potentially low) number of frames. In these circumstances, the resulting performance is strongly influenced by the short-term fluctuations in channel conditions [13]. This effect can be efficiently captured by outage capacity measures. As done in [14], we set a time horizon of T contiguous time slots and compute the probability for the average information rate sent to the arbitrary user k to be below a pre-defined threshold R. Finally, results are averaged for the whole set of K active users:

$$P_{COUT}(R) = \sum_{k=1}^{K}\Bigg[\text{Prob}\left(m_k = 0\right)\text{Prob}(\mathcal{A}_k) + \sum_{m_k=1}^{T}\text{Prob}(m_k) \times \text{Prob}\left(\frac{1}{m_k}\sum_{j=1}^{m_k}\log_2(1+\gamma_k(j)) < R\,\middle|\,\mathcal{A}_k\right)\text{Prob}(\mathcal{A}_k)\Bigg] \tag{7}$$

where m_k is the number of slots (out of T) granted by the scheduler in the BS to user k, and $\gamma_k(j)$ stands for the instantaneous SNR experienced by user k during slot $j = 1..m_k$. Notice that, for the case $m_k = 0$ slots, the probability for the average information rate to be below R is equal to 1 irrespective of R. As for the discrete random variable m_k, it follows a binomial distribution (i.e. set of independent trials) with individual probabilities $\text{Prob}(\mathcal{A}_k) = 1/K$. Then we have:

$$\text{Prob}(m_k) = \binom{T}{m_k}\left(\frac{1}{K}\right)^{m_k}\left(1 - \frac{1}{K}\right)^{T-m_k} \tag{8}$$

So, we focus now on in obtaining an analytical expression for:

$$\text{Prob}\left(\frac{1}{m_k}\sum_{j=1}^{m_k}\log_2(1+\gamma_k(j)) < R\,\middle|\,\mathcal{A}_k\right)$$

The case $m_k = 1$ poses no difficulty since it is straightforward to show that:

$$\text{Prob}\left(\log_2(1+\gamma_k) < R\,\middle|\,\mathcal{A}_k\right) = F_{\gamma_k}\left(2^R - 1|\mathcal{A}_k\right)$$

However, finding such analytical expression for the $m_k > 1$ case is barely tractable since, the computation of the pdf associated to the random variable $z = \frac{1}{m_k}\sum_{j=1}^{m_k}\log_2(1 + \gamma_k(j))$ involves an m_k-fold convolution of (log-) chi-square distributed random variables. Alternatively, we compute an upper bound of the outage capacity by resorting to the Chernoff bound (e.g. as in [15]), i.e.:

[2] In this region, user k is not allowed to report its SNR status to the BS. Therefore, this user can only be randomly selected for transmission when the scheduling is in random scheduling configuration, i.e., all the users are silent.

$$\text{Prob}\left(\frac{log_2(e)}{m_k}\sum_{j=1}^{m_k}\ln(1+\gamma_k(j)) < R \middle| \mathcal{A}_k\right) \leq P_{CFB}(m_k,\bar{\gamma}_k,R)$$
$$= \min_{s>0}\left\{ e^{\frac{sRm_k}{log_2(e)}} \mathrm{E}_{\gamma_k(1),\ldots,\gamma_k(m_k)|\mathcal{A}_k}\left[e^{-s\sum_{j=1}^{m_k}\ln(1+\gamma_k(j))}\right]\right\} \quad (9)$$

Due to the i.i.d nature of the channel fades associated to user i, the expectation inside Eq. (9) can be re-written as:

$$\mathrm{E}_{\gamma_k(1),\ldots,\gamma_k(m_k)|\mathcal{A}_k}\left[e^{-s\sum_{j=1}^{m_k}\ln(1+\gamma_k(j))}\right] =$$
$$= \left(\mathrm{E}_{\gamma_k|\mathcal{A}_k}\left[e^{-s\ln(1+\gamma_k)}\right]\right)^{m_k}$$
$$= \left(\mathrm{E}_{\gamma_k|\mathcal{A}_k}\left[(1+\gamma_k)^{-s}\right]\right)^{m_k}$$

Note that, the above equation can be solved by computing the following integral:

$$\mathrm{E}_{\gamma_k|\mathcal{A}_k}\left[(1+\gamma_k)^{-s}\right] = \int_{\gamma_k=0}^{\infty}(1+\gamma_k)^{-s} f_{\gamma_k|\mathcal{A}_k}(\gamma_k|\mathcal{A}_k)\,d\gamma_k$$
$$= (F_x(\xi_{th}))^{K-1}\int_{\gamma_k=0}^{\xi_{th}\bar{\gamma}_k}\frac{f_{\gamma_k}(\gamma_k)}{(1+\gamma_k)^s}d\gamma_k$$
$$+ K\int_{\gamma_k=\xi_{th}\bar{\gamma}_k}^{\infty}\frac{f_{\gamma_k}(\gamma_k)}{(1+\gamma_k)^s}(F_{\gamma_k}(\gamma_k))^{K-1}\,d\gamma_k \quad (10)$$

This can be easily done for the SISO case. In particular, by resorting to the binomial expansion, using the change of variables $t = 1+\gamma$ and with the help of [16, Eq. 3.381.3], one can readily prove the following result:

$$\mathrm{E}_{\gamma_{SISO,i}|\mathcal{A}_k}\left[(1+\gamma_k)^{-s}\right] = \frac{e^{\frac{1}{\bar{\gamma}_k}}}{\bar{\gamma}_k^s}\left[\left(1-e^{-\xi_{th}}\right)^{K-1}\right.$$
$$\times\left(\Gamma\left(1-s,\frac{1}{\bar{\gamma}_k}\right)-\Gamma\left(1-s,\frac{(1+\xi_{th}\bar{\gamma}_k)}{\bar{\gamma}_k}\right)\right)$$
$$+K\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i(i+1)^{s-1}e^{\frac{i}{\bar{\gamma}_k}}$$
$$\left.\times\Gamma\left(1-s,\frac{(1+\xi_{th}\bar{\gamma}_k)(i+1)}{\bar{\gamma}_k}\right)\right] \quad (11)$$

where $\Gamma(\alpha,x)$ stands for complementary incomplete gamma function ($\Gamma(\alpha,x) = \int_x^{\infty} e^{-t}t^{\alpha-1}dt$) [16, Eq. 8.350.2].

In the OSTBC case and after some algebraic manipulations, one can express eq. (10) as a function of $\mathcal{C}(a,m,s,\mu)$ and $\mathcal{D}(a,m,s,\mu)$:

$$\mathrm{E}_{\gamma_{OSTBC,i}|\mathcal{A}_k}\left[(1+\gamma_k)^{-s}\right] = \left[\frac{4}{\bar{\gamma}_k^2}\left(1-e^{-2\xi_{th}}(2\xi_{th}+1)\right)^{K-1}\right.$$
$$\times\mathcal{D}\left(\xi_{th}\bar{\gamma}_k,2,s,\frac{2}{\bar{\gamma}_k}\right)+K\sum_{i=0}^{K-1}\binom{K-1}{i}(-1)^i$$
$$\left.\times\sum_{n=0}^{i}\binom{i}{n}\left(\frac{2}{\bar{\gamma}_k}\right)^{n+2}\mathcal{C}\left(\xi_{th}\bar{\gamma}_k,n+2,s,\frac{2}{\bar{\gamma}_k}(i+1)\right)\right] \quad (12)$$

where the closed-form of the integrals:

$$\mathcal{C}(a,m,s,\mu) = \int_a^{\infty}\frac{t^{m-1}}{(1+t)^s}e^{-\mu t}dt$$
$$\mathcal{D}(a,m,s,\mu) = \int_0^{a}\frac{t^{m-1}}{(1+t)^s}e^{-\mu t}dt$$

are omitted here for brevity but can be found in Appendix. Last, by plugging (9) along with (8) into (7), the following upper bound for the outage capacity results:

$$P_{COUT}(R) \leq \frac{1}{K}\sum_{k=1}^{K}\left[\left(1-\frac{1}{K}\right)^T\right.$$
$$+\sum_{m_k=1}^{T}\binom{T}{m_k}\left(\frac{1}{K}\right)^{m_k}\left(1-\frac{1}{K}\right)^{T-m_k}$$
$$\left.\times\min_{s>0}\left\{e^{\frac{sRm_k}{log_2(e)}}\left(\mathrm{E}_{\gamma_k|\mathcal{A}_k}\left[(1+\gamma_k)^{-s}\right]\right)^{m_k}\right\}\right]$$

which can be simplified after some algebraic manipulation as follows:

$$P_{COUT}(R) \leq \frac{1}{K}\sum_{k=1}^{K}\left(1-\frac{1-P_{CFB}(1,\bar{\gamma}_k,R)}{K}\right)^T$$

Note that, the above expression can be particularized for the different transmission schemes with the help of (11) and (12).

5. NUMERICAL RESULTS AND DISCUSSION

In this section, we present some computer simulation results and assess some spatial vs. multi-user diversity trade-offs in situation where the partial CSI to be conveyed to the BS is subject to bandwidth restrictions in the feedback channel. Since we already proved the proposed scheduler is fair irrespective of the average SNRs, we will restrict ourselves to the homogeneous case, i.e. $\bar{\gamma}_k = \bar{\gamma}$.

In Fig. 1, we depict the outage capacity as a function of the requested rate R for both transmission schemes (SISO and OSTBC), different feedback loads ($\bar{F} = 0.01..1$) and a scenario with $K = 10$ active users. For a total number of $T = 80$ slots, this means that each user is scheduled *in average* $T/K = 80/10 = 8$ times. In the case of full feedback load ($\bar{F} = 1$) one can observe that the SISO scheme provides lower outage rates (for a given requested rate R) than its OSTBC counterpart. In other words, in a multi-user scenario where multi-user diversity can be effectively exploited, the suppression of SNR peaks due to the SNR-stabilizing effect associated to OSTBC penalizes system performance. Conversely, when the average feedback load per user is reduced, the degradation experienced by the SISO-based schemes is larger than that exhibited by the OSTBC ones and, eventually, the relative ordering of both curves changes. This follows from the fact that OSTBC approaches provide additional robustness against unfavorable fading conditions resulting from random user selection. However, one can still appreciate how the introduction of random scheduling makes links less stable since the slope of the curves decreases for low values of $\bar{F}$, being this effect more pronounced in the SISO case, due to the lack of spatial diversity.

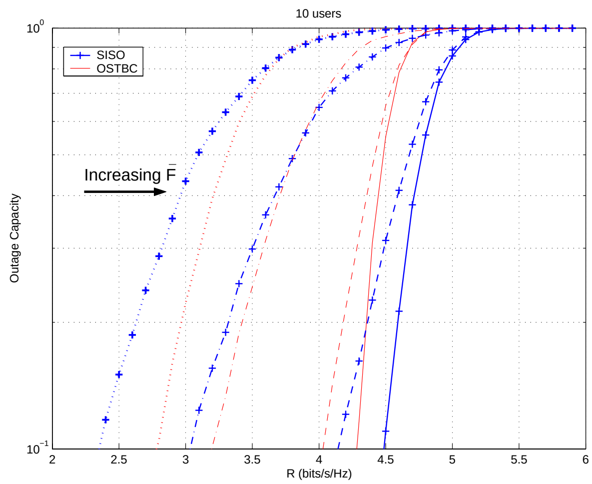

Figure 1: Outage capacity vs. R for the different transmission schemes and feedback loads ($\bar{F} = 0.01, 0.05, 0.2, 1$). $\bar{\gamma}$=10dB, T=80 slots, K=10 users.

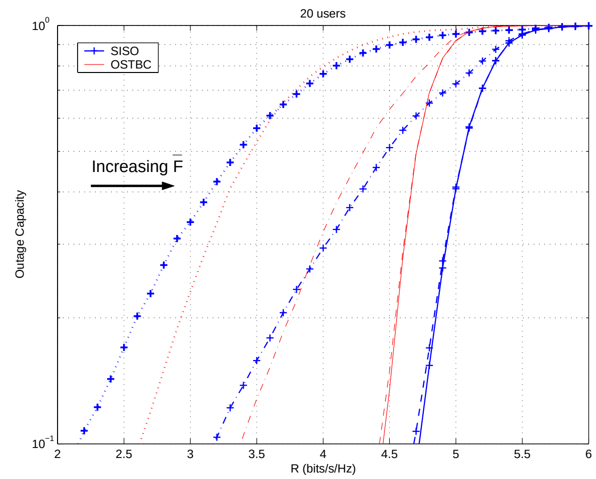

Figure 2: Outage capacity vs. R for the different transmission schemes and feedback loads ($\bar{F} = 0.01, 0.05, 0.2, 1$). $\bar{\gamma}$=10dB, T=80 slots, K=20 users.

When the number of active users is increased to $K = 20$ (Fig. 2), a substantial improvement in terms of outage capacity results for the whole range of requested rates (all curves shifted to the right). In addition, the impact of bandwidth savings in the feedback channel is negligible for both transmission schemes even for normalized loads as low as $\bar{F} = 0.2$. However, outage capacity curves are less steep now, this revealing that individual user links are less stable. There is a simple explanation for that: since T is kept constant but the number of active users is increased, the average number of times a user is scheduled ($T/K = 80/20 = 4$) decreases. Clearly, the variance of the average information rate over those time slots is higher now. This effect is even clearer for the lower values of $\bar{F}$ (random scheduling often takes place) where a higher value of m_k is key in order to average out fading effects. As an example, for 10 users and $\bar{F} = 0.01$ the achievable rate with 10% outage capacity is 2.37 (SISO) and 2.8 bits/s/Hz (OSTBC). When the number of users is doubled, these rates decrease to 2.13 and 2.62 bits/s/Hz, respectively.

In summary, a number of non-trivial trade-offs arise when considering different transmission schemes, feedback loads and terminal count. As usual, design decisions at the cell level will be closely linked to the QoS requirements of the services under consideration.

6. APPENDIX

In order to derive a closed-from expression of eq. (10) for the OSTBC case, one should solve the following two integrals:

$$\mathcal{C}(a, m, s, \mu) = \int_a^\infty \frac{t^{m-1}}{(1+t)^s} e^{-\mu t} dt \qquad (13)$$

$$\mathcal{D}(a, m, s, \mu) = \int_0^a \frac{t^{m-1}}{(1+t)^s} e^{-\mu t} dt$$

$$\mu > 0; m = 1, 2, \ldots$$

We start by deriving $\mathcal{C}(a, m, s, \mu)$ and, then, we will compute $\mathcal{D}(a, m, s, \mu)$ as $\mathcal{D}(a, m, s, \mu) = \mathcal{C}(0, m, s, \mu) - \mathcal{C}(a, m, s, \mu)$. By using the change of variables $x = 1 + t$ and the binomial expansion, equation (13) can be expressed in closed-form as:

$$\mathcal{C}(a, m, s, \mu) = e^\mu \int_{1+a}^\infty \frac{(x-1)^{m-1}}{x^s} e^{-\mu x} dt$$

$$= e^\mu \sum_{p=0}^{m-1} \binom{m-1}{p} (-1)^{m-1+p} \int_{1+a}^\infty x^{p-s} e^{-\mu x} dt$$

Next, with the help of [16, Eq. 3.381.3], $\mathcal{C}(a, m, s, \mu)$ can be expressed in terms of the complementary incomplete gamma function ($\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt$) [16, Eq. 8.350.2]:

$$\mathcal{C}(a, m, s, \mu) = e^\mu \sum_{p=0}^{m-1} \binom{m-1}{p} (-1)^{m-1+p} \mu^{s-p-1} \times \Gamma\left(1 - s + p, \mu(1+a)\right)$$

Finally, with the help of the above expression we can easily write $\mathcal{D}(a, m, s, \mu)$ as:

$$\mathcal{D}(a, m, s, \mu) = e^\mu \sum_{p=0}^{m-1} \binom{m-1}{p} (-1)^{m-1+p} \mu^{s-p-1} \times \Big(\Gamma\left(1 - s + p, \mu\right) - \Gamma\left(1 - s + p, \mu(1+a)\right)\Big)$$

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