

# PILOT OPTIMIZATION FOR TIME-DELAY AND CHANNEL ESTIMATION IN OFDM SYSTEMS

Michael D. Larsen\*, Gonzalo Seco-Granados\*\*, A. Lee Swindlehurst\*\*\*

\*Raytheon Company, Tucson, AZ 85756, USA, mdlarsen@ieee.com

\*\*Universitat Autònoma de Barcelona, 08193 Bellaterra, SPAIN, gonzalo.seco@uab.es

\*\*\*Univ. of California, Irvine, Irvine, CA 92697, USA, swindle@uci.edu

## ABSTRACT

Orthogonal frequency division multiplexing (OFDM) communication systems require accurate estimation of timing offset and channel impulse response in order to achieve desirable performance. In this paper, we consider the optimal placement of pilot symbols over the OFDM subcarriers in order to minimize a function of the Cramér-Rao bound on these parameters. Previous work has investigated this problem for channel estimation only, and found that equi-spaced, equi-powered pilots are optimal. We show that when the time-delay must be simultaneously estimated, the optimal pilot distribution is often quite different, with more pilot energy distributed to the edges of the signal bandwidth. Upper and lower bounds for the required number of optimal pilots are also presented for the case where the variance on the time-delay estimate is minimized.

**Index Terms**— OFDM, time-delay estimation, channel estimation, synchronization

## 1. INTRODUCTION

Multicarrier (MC) signals in various formats (OFDM, discrete multitone modulation, filter bank multicarrier modulation, etc.) have become the preferred option for all types of systems: wide-area networks (LTE, WiMAX), local-area networks (IEEE 802.11g), broadcasting (DVB-T/H, DRM, DAB, DMB) and short-range communications (UWB). On the other hand, satellite-based positioning systems still rely on code-division multiple access (CDMA) and direct-sequence spread-spectrum signals, like the previous generation of wireless communications systems did. This is indicative of the fact that wireless communications and positioning systems have essentially evolved independently of each other. For instance, the Global Positioning System (GPS) makes it possible to obtain user position information with accuracies on the order of one meter, but it transmits at an extremely low data rate (50 bits/s). On the other hand, WLAN systems based on the IEEE802.11x family of standards allow transmission rates above 54 Mbits/s, but attempts to use them for positioning have experienced serious limitations.

There is, however, increasing interest from users and service providers to design a single system that works well for both communications and positioning applications. This would have advantages in terms of user equipment cost (one receiver instead of two), coverage, improved possibilities for development of new value-added services, etc. As a result, public institutions and manufacturers have begun to consider the design of combined positioning/communication systems. For example, while the European Galileo system was designed as purely a positioning system conceptually very similar to

GPS, the next generation of Galileo may include broadband communications capability. Likewise, improvements in the ability of different wireless communications systems standards to perform localization are also being considered [1]. It is reasonable to think that MC modulation is a better choice than CDMA for a combined communications and positioning system. Besides the great reduction in equalization complexity which is useful in communications systems, MC signals provide other advantages such significant flexibility in shaping the signal spectrum, robustness against multipath and an inherent simplicity in its application to multiple-access schemes, which would be beneficial in the design of a combined system.

Optimal pilot designs for OFDM channel estimation have been derived by several researchers (e.g., [2–5]) whose results show that equi-spaced, equi-powered pilots are optimal in terms of mean-squared error. Others have considered designs for estimation of carrier frequency offset (CFO) [6, 7], or joint channel and CFO estimation [8, 9]. However, the study of MC pilot allocation for very precise timing estimation (e.g., as required in positioning systems) has received little attention. In [10], it was shown that to a first approximation, minimizing the variance of the time-delay estimate (TDE) requires an MC signal with maximum root-mean-square (RMS) bandwidth. This results in a pilot signal whose power is pushed to the subcarriers at the extreme edges of the frequency band, and is in clear contrast with the equi-spaced, equi-powered pilot designs for channel estimation.

In this paper, we investigate the trade-offs associated with allocating pilots for jointly estimating the channel and time-delay associated with an OFDM signal, and we propose a simple tuning approach to control the relative performance of these two competing objectives. The algorithm is based on minimizing a weighted trace of the Cramér-Rao bound (CRB) over the pilot allocation across the OFDM subcarriers. The minimization problem is shown to be convex, and results in a solution with only a relatively small number of non-zero pilot subcarriers. In particular, for the case where the pilots are chosen to optimize only the TDE, we demonstrate that no more than  $2L + 1$  subcarriers need be assigned pilot symbols, where  $L - 1$  is the channel duration in symbols.

## 2. MODEL AND ASSUMPTIONS

### 2.1. OFDM Signal Model

Consider the frequency-selective channel model given by

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - lT_s - \tau_d) \quad (1)$$

where  $L$  is a known upper bound on the number of discrete multipath components,  $T_s$  is the system sampling period,  $h_l$  is the complex channel coefficient for the  $l$ -th path, and  $\tau_d$  is the timing offset. Unlike other work on channel estimation, the delay is explicitly

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modeled inside the terms  $\delta(t - lT_s - \tau_d)$ , and therefore the channel coefficients are independent of the delay. Since this timing offset is equivalent to the primary channel time delay, *i.e.*, the time delay of the first path of the impulse response, we will refer to it simply as the time delay or delay in the remainder of this work. We assume that the channel coefficients  $\{h_l\}$  and the time delay  $\tau_d$  are unknown and must be estimated through the use of pilot tones transmitted as part of an OFDM symbol  $s(n)$ ,  $n = 0, \dots, N-1$ , which is constructed from a set of pilot symbols  $\{b_q\}$  by means of the length- $N$  inverse discrete Fourier transform (IDFT) as

$$s(n) = \frac{1}{\sqrt{N}} \sum_{q=-N/2+1}^{N/2} b_q e^{j\frac{2\pi}{N}nq} \quad (2)$$

$$= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} b_{m-N/2+1} e^{j\frac{2\pi}{N}(m-N/2+1)n}. \quad (3)$$

Note that we assume  $N$  here to be even, although this is not strictly necessary. Note also that we are ignoring the presence of data (non-pilot) symbols in  $s(n)$ . As we will see later, the required number of pilots is relatively small so there will be plenty of subcarriers available for data, but in this work we assume they are not used for channel or time-delay estimation.

We restrict our attention to the zero inter-carrier and inter-symbol interference case. Thus, we assume the carrier frequency is perfectly synchronized at the transmitter and receiver, and that a rough symbol synchronization has taken place so that the cyclic prefix  $T_G$  is larger than the delay spread, including the unknown delay:  $(L-1)T_s + \tau_d < T_G$ . With these assumptions, the sampled received signal after discarding the cyclic prefix is given by

$$y(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \sum_{L=0}^{L-1} h_l b_{m-N/2+1} e^{-j\frac{2\pi}{T}(m-N/2+1)\tau_d} \cdot e^{j\frac{2\pi}{N}(m-N/2+1)(k-l)} + \nu(kT_s), \quad (4)$$

where  $\nu(kT_s)$  is the noise contribution and  $T = NT_s$  is the OFDM symbol duration.

Taking the DFT of  $x(n)$ , we have

$$x(q) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nq} + n(q) \quad (6)$$

$$= \left( \sum_{l=0}^{L-1} h_l e^{-j\frac{2\pi}{N}ql} \right) b_q e^{-j\frac{2\pi}{T}q\tau_d} + n(q) \quad (7)$$

$$= g(q)b_q e^{-j\frac{2\pi}{T}q\tau_d} + n(q) \quad (8)$$

where  $g(q)$  and  $n(q)$  are the channel and noise frequency responses, respectively, at the  $q$ -th frequency bin and  $q \in [-N/2+1, -N/2+2, \dots, 0, 1, \dots, N/2]$ . In matrix notation, this may be written as

$$\mathbf{x} = \mathbf{B}\mathbf{\Gamma}\mathbf{F}_L\mathbf{h} + \mathbf{n} \quad (9)$$

where

$$\mathbf{x} = [x(-N/2+1) \ \dots \ x(N/2)]^T \quad (10)$$

$$\mathbf{\Gamma} = \text{diag}(w_T^{-(N/2+1)\tau_d}, \dots, w_T^{-(N/2)\tau_d}) \quad (11)$$

$$\mathbf{B} = \text{diag}(b_{-N/2+1}, \dots, b_{N/2}) \quad (12)$$

$$\mathbf{h} = [h_0 \ \dots \ h_{L-1}]^T \quad (13)$$

$$\mathbf{n} = [n(-N/2+1) \ \dots \ n(N/2)]^T \quad (14)$$

with  $w_x^y = e^{-j\frac{2\pi}{x}y}$ , and  $\mathbf{F}_L$  is composed of the first  $L$  columns of the zero-frequency-centered  $N \times N$  DFT matrix. The overall effect of the channel is accounted for by the term  $\mathbf{\Gamma}\mathbf{F}_L\mathbf{h}$ , where the contributions of the delay  $\tau_d$  and the channel taps  $\mathbf{h}$  appear separately. This differs from previous work where only the term  $\mathbf{F}_L\mathbf{h}$  is used, which implies that the effect of the delay appears in the coefficients  $\mathbf{h}$ . Although it may appear that this latter model reduces the number of unknown parameters by one, its effect is actually counter-productive since larger values of  $L$  are needed to accurately model the channel.

## 2.2. The Cramér-Rao Bound

The parameter vector  $\Theta$  associated with the OFDM signal model of (9) is represented as

$$\Theta \triangleq [\tau_d \ \text{Re}[\mathbf{h}^T] \ \text{Im}[\mathbf{h}^T]]^T \quad (15)$$

where  $\text{Re}[a]$  and  $\text{Im}[a]$  are the real and imaginary parts, respectively, of the complex variable  $a$ . Assuming that the additive noise is white in frequency, *i.e.*,  $E[\mathbf{nn}^H] = \sigma_n^2\mathbf{I}$ , the Fisher Information Matrix (FIM) for this problem is found to be

$$J(\Theta) = \frac{2}{\sigma_n^2} \begin{bmatrix} J_{\tau_d} & \mathbf{J}_{21}^T \\ \mathbf{J}_{21} & \mathbf{J}_h \end{bmatrix} \quad (16)$$

with

$$J_{\tau_d} = \mathbf{h}^H \mathbf{F}_L^H \mathbf{B}^H \mathbf{D}^2 \mathbf{B} \mathbf{F}_L \mathbf{h} \quad (17)$$

$$\mathbf{J}_{21} = \begin{bmatrix} \text{Im}[\mathbf{F}_L^H \mathbf{B}^H \mathbf{D} \mathbf{B} \mathbf{F}_L \mathbf{h}] \\ -\text{Re}[\mathbf{F}_L^H \mathbf{B}^H \mathbf{D} \mathbf{B} \mathbf{F}_L \mathbf{h}] \end{bmatrix} \quad (18)$$

$$\mathbf{J}_h = \begin{bmatrix} \text{Re}[\mathbf{F}_L^H \mathbf{B}^H \mathbf{B} \mathbf{F}_L] & -\text{Im}[\mathbf{F}_L^H \mathbf{B}^H \mathbf{B} \mathbf{F}_L] \\ \text{Im}[\mathbf{F}_L^H \mathbf{B}^H \mathbf{B} \mathbf{F}_L] & \text{Re}[\mathbf{F}_L^H \mathbf{B}^H \mathbf{B} \mathbf{F}_L] \end{bmatrix} \quad (19)$$

$$\mathbf{D} = \frac{2\pi}{T_s} \text{diag}\left(-\frac{N}{2} + 1, \dots, \frac{N}{2}\right). \quad (20)$$

Some straightforward calculations lead to the CRB:

$$\text{CRB}_{\tau_d, \mathbf{h}} = \frac{\sigma_n^2}{2} \begin{bmatrix} \gamma_{\tau_d}^{-1} & \text{CRB}_{21}^T \\ \text{CRB}_{21} & \text{CRB}_{22} \end{bmatrix} \quad (21)$$

where

$$\gamma_{\tau_d} = \mathbf{h}^H \mathbf{F}_L^H \mathbf{B}^H \mathbf{D} \mathbf{\Pi}_{\mathbf{B}\mathbf{F}_L}^\perp \mathbf{D} \mathbf{B} \mathbf{F}_L \mathbf{h} \quad (22)$$

$$\mathbf{\Pi}_{\mathbf{B}\mathbf{F}_L}^\perp = \mathbf{I} - \mathbf{B} \mathbf{F}_L (\mathbf{F}_L^H \mathbf{B}^H \mathbf{B} \mathbf{F}_L)^{-1} \mathbf{F}_L^H \mathbf{B}^H \quad (23)$$

$$\text{CRB}_{21} = \begin{bmatrix} -\gamma_{\tau_d}^{-1} \text{Im}[\mathbf{q}] \\ \gamma_{\tau_d}^{-1} \text{Re}[\mathbf{q}] \end{bmatrix} \quad (24)$$

$$\text{CRB}_{22} = \begin{bmatrix} \text{Re}[\mathbf{Q}^{-1}] + \gamma_{\tau_d}^{-1} \text{Im}[\mathbf{q}] \text{Im}[\mathbf{q}^T] \\ \text{Im}[\mathbf{Q}^{-1}] - \gamma_{\tau_d}^{-1} \text{Re}[\mathbf{q}] \text{Im}[\mathbf{q}^T] \end{bmatrix} \quad (25)$$

$$\begin{bmatrix} -\text{Im}[\mathbf{Q}^{-1}] - \gamma_{\tau_d}^{-1} \text{Im}[\mathbf{q}] \text{Re}[\mathbf{q}^T] \\ \text{Re}[\mathbf{Q}^{-1}] + \gamma_{\tau_d}^{-1} \text{Re}[\mathbf{q}] \text{Re}[\mathbf{q}^T] \end{bmatrix} \quad (26)$$

$$\mathbf{Q} = \mathbf{F}_L^H \mathbf{B}^H \mathbf{B} \mathbf{F}_L \quad (27)$$

$$\mathbf{q} = \mathbf{Q}^{-1} \mathbf{F}_L^H \mathbf{B}^H \mathbf{D} \mathbf{B} \mathbf{F}_L \mathbf{h}. \quad (28)$$

Note that, since diagonal matrices commute, the matrix  $\mathbf{B}$  always appears multiplied with its complex conjugate (transpose)  $\mathbf{B}^H \mathbf{B}$ . Thus, instead of parameterizing the CRB in terms of  $\mathbf{B}$ , we parameterize it instead using the diagonal elements  $\mathbf{p} = [p_1 \ \dots \ p_N]$  of  $\mathbf{P} = \mathbf{B}^H \mathbf{B}$ . The elements of  $\mathbf{p}$  must be non-negative, and we also assume a total power constraint  $\sum p_i \leq P_T$ . We let  $\mathcal{P}$  denote the set of all pilot vectors that satisfy these two conditions.

### 3. OPTIMAL PILOT SELECTION

Earlier work such as that in [3,4] assumed  $\tau_d$  was known and found the pilot structure that essentially maximized the trace of the resulting CRB relative to  $\mathbf{h}$  only. In this case, the CRB would be given by the inverse of  $\mathbf{J}_h$  in (19), and the optimal pilot allocation is found as

$$\mathbf{p}_h^* = \arg \min_{\mathbf{p} \in \mathcal{P}} \text{tr} \left[ (\mathbf{F}_L^H \mathbf{P} \mathbf{F}_L)^{-1} \right]. \quad (29)$$

It was found in [3,4] that (29) is minimized for allocations that possess equi-powered pilots evenly spaced in frequency. The solutions range from a minimum of  $L$  non-zero pilots (when  $L$  is a divisor of  $N$ ) to allocating equal pilot power over all  $N$  subcarriers, with each solution providing the same bound on the mean-squared error.

It is interesting to consider what occurs when we again attempt to minimize trace of the CRB associated with  $\mathbf{h}$ , but assuming that the time-delay  $\tau_d$  is unknown. In this situation, the problem would be posed as

$$\min_{\mathbf{p} \in \mathcal{P}} \text{tr}[\text{CRB}_{22}] = \min_{\mathbf{p} \in \mathcal{P}} \text{tr} \left[ (\mathbf{F}_L^H \mathbf{P} \mathbf{F}_L)^{-1} \right] + \frac{\|(\mathbf{B}\mathbf{F}_L)^T \mathbf{D}\mathbf{B}\mathbf{F}_L \mathbf{h}\|^2}{2\|\mathbf{I}_{\mathbf{B}\mathbf{F}_L} \mathbf{D}\mathbf{B}\mathbf{F}_L \mathbf{h}\|^2}. \quad (30)$$

The first part of this expression corresponds to the uncoupled case and, as discussed previously, is minimized by equi-spaced and equi-powered pilots. The second term in (30) is a penalty term resulting from the fact that  $\tau_d$  is unknown, and (as we will see in the numerical results) the presence of this penalty term significantly alters the equi-spaced, equi-powered pilot structure.

More generally, we will consider an optimization criterion that takes into account both the accuracy of the TDE and the channel estimate. Specifically, we wish to find the vector  $\mathbf{p} \in \mathcal{P}$  that minimizes a weighted sum of the diagonal entries of the CRB, *i.e.*,

$$\mathbf{p}^* = \arg \min_{\mathbf{p} \in \mathcal{P}} \mathbf{G}(\alpha, \mathbf{p}) \quad (31)$$

where the cost function  $\mathbf{G}(\alpha, \mathbf{p})$  is given by

$$\mathbf{G}(\alpha, \mathbf{p}) = \left[ \alpha \beta \gamma \tau_d^{-1} + (1 - \alpha) \left( \frac{1}{L} \right) \text{tr}(\text{CRB}_{22}) \right]. \quad (32)$$

The coefficient  $\beta$  is a normalization factor used to make the two terms the same order of magnitude, and will depend on the units used to represent the parameters. The parameter  $\alpha \in [0, 1]$  is used to control the relative importance of the estimation error for  $\tau_d$  and  $\mathbf{h}$  when selecting  $\mathbf{p}^*$ . Letting  $\alpha \rightarrow 1$  weights the error to favor accuracy in the TDE relative to the channel, and  $\alpha \rightarrow 0$  does the opposite. In either case, we will see that the optimal pilot allocation is considerably different than the equi-spaced, equi-powered allocation found in [2–4].

Note that the CRB, and hence the solution to (31), is not a function of the actual value of  $\tau_d$ . However, the CRB and  $\mathbf{p}^*$  will be a function of  $\mathbf{h}$ , which is not assumed to be known. In practice, there are various options available to address this issue. For example, an initial rough estimate of  $\mathbf{h}$  may be available from estimation in an earlier OFDM symbol, or one could average the FIM or CRB over some known distribution for  $\mathbf{h}$ . We will not address such approaches here, since our main objective is to examine the differences in the pilot allocation when  $\tau_d$  is unknown. Our primary result is summarized in the theorem below.

**Theorem:** The minimization in (31) is convex for any  $\alpha$ , and when  $\alpha = 1$ , there exists at least one optimal solution whose number<sup>1</sup> of non-zero pilots satisfies  $L + 1 \leq \|\mathbf{p}^*\|_0 \leq 2L + 1$ .

<sup>1</sup>The 0-norm of a vector  $\|\cdot\|_0$  is defined as the number of non-zero entries of the vector.

*Proof:* The FIM  $J(\Theta)$  in (16) is a linear function of  $\mathbf{p}$ , and its convexity in  $\mathbf{p}$  follows immediately from the convexity of the matrix fractional function [11] and the composition of convex and affine functions. The remainder of the proof can be found in [12].

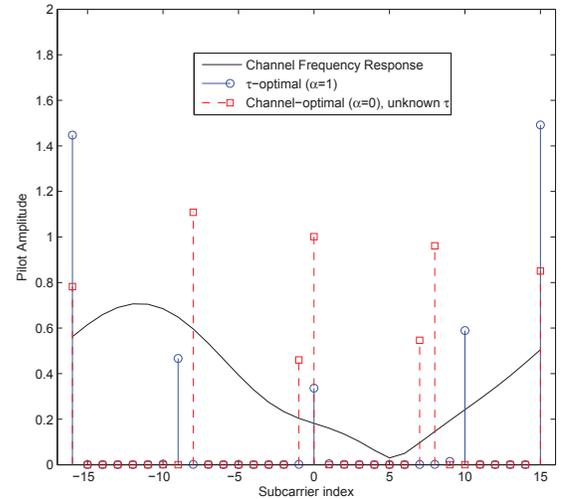
While there is no closed-form solution for  $\mathbf{p}^*$  in (31), a standard convex programming approach which is guaranteed to converge to the optimal solution can easily be found. The lower bound  $L + 1 \leq \|\mathbf{p}^*\|_0$  is required for the FIM to be full rank, and the problem to be identifiable. As we will see in the next section, for most channels the optimal number of pilots is closer to the lower bound than the upper bound. For typical values of  $N$  and  $L$ , the required number of pilots will be quite sparse, even when  $2L + 1$  are required.

### 4. SIMULATION RESULTS

For all of the results presented in this section, we assume a channel impulse response of length  $L = 4$  and an OFDM signal with  $N = 32$  subcarriers, a total pilot power of  $P_T = 5$ , and a noise variance of  $\sigma_n^2 = 10^{-4}$ . Figure 1 shows the pilot distributions for the case of a specific channel defined by

$$\mathbf{h} = \begin{bmatrix} 0.38 + j0.23 & 1.30 - j0.92 \\ -1.60 - j0.31 & 0.61 + j0.24 \end{bmatrix}.$$

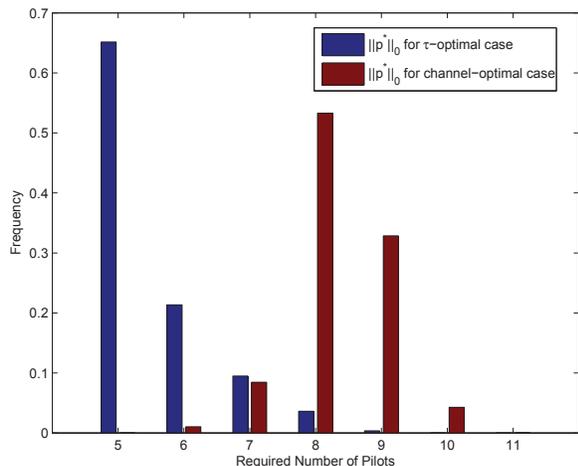
The optimal pilots are shown for the cases where  $\alpha = 0$  and  $\alpha = 1$ , together with the frequency response of the channel. In this case, the minimum of  $L + 1 = 5$  pilots was required for  $\alpha = 1$ , while 7 were necessary for  $\alpha = 0$ . We see that the equi-spaced, equi-powered pilot structure is lost when  $\tau_d$  is unknown. We also see that placing emphasis on accurately estimating  $\tau_d$  pushes the pilot power out towards the edges of the signal bandwidth.



**Fig. 1.** Demonstration of the influence of  $\alpha$  on the terms of  $\mathbf{G}(\alpha, \mathbf{b})$ .

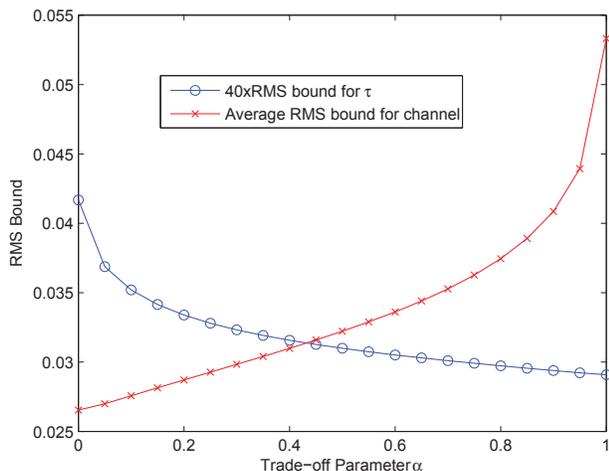
Figure 2 shows a histogram of the required number of pilots  $\|\mathbf{p}^*\|_0$  for the channel-optimal ( $\alpha = 0$ ) and  $\tau_d$ -optimal ( $\alpha = 1$ ) cases, where the results were calculated based on 8000 random channels whose coefficients were chosen independently from a zero-mean unit-variance complex Gaussian distribution. In over 65% of the trials, the  $\tau_d$ -optimal pilot distribution required the minimum number of pilots (*i.e.*,  $L + 1 = 5$ ), and in only one case was the upper bound of 9 pilots optimal. More pilots are seen to be required

for the channel-optimal case, with about 85% of the trials producing non-zero pilot symbols on either 8 or 9 subcarriers.



**Fig. 2.** Histogram of the required number of pilots for  $\alpha = 1$  ( $\tau$ -optimal) and  $\alpha = 0$  (channel optimal).

In Fig. 3, the CRB for  $\tau_d$  and the channel coefficients evaluated at  $\mathbf{p}^*$  is plotted versus  $\alpha$ , assuming  $\beta = 10000$ . To compare the two curves in a single figure, so that they are of the same order of magnitude, we plot  $\sigma_n \sqrt{\text{tr}[\text{CRB}_{22}]/(2L)}$ , which is proportional to the bound on the RMS channel estimation error averaged over the  $L$  coefficients, and  $20\sigma_n \sqrt{\gamma_{\tau_d}^{-1}}$ , which is proportional to the bound on the RMS error for  $\tau_d$ . As expected, the bound for  $\tau_d$  improves as  $\alpha \rightarrow 1$ , while that for the channel decreases. In this example, the variation in performance for  $\tau_d$  with changes in  $\alpha$  is less pronounced than that for the channel, especially at the limiting values of  $\alpha$ . Even if one is primarily interested in the accuracy of  $\tau_d$ , a choice of  $\alpha$  near its midpoint causes relatively little degradation for  $\tau_d$  compared with  $\alpha = 1$ , but substantially improves the bound for estimation of  $\mathbf{h}$ .



**Fig. 3.** Influence of  $\alpha$  on the CRB for  $\tau_d$  and  $\mathbf{h}$ .

## 5. CONCLUSIONS

We have considered the problem of optimally allocating the power of pilot symbols for joint estimation of the channel and time delay of an OFDM signal. We presented a cost function composed of a linear combination of the trace of the CRB for the channel and time-delay estimation error, and showed that the function was convex in the choice of the pilots. We showed that the optimal pilot distribution that results from minimizing this function is often considerably different from the optimal distribution when the delay is known, a case which had been studied in earlier work. In particular, when the time delay is unknown and minimizing the TDE is paramount, more pilot power is allocated towards the edges of the signal's bandwidth. For the case where the cost function focuses only on the TDE error, we presented bounds on the required number of optimal pilots. We also presented the results of several simulations to illustrate the contributions of the paper.

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