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NOVEL COLLABORATIVE SPECTRUM SENSING BASED ON SPATIAL COVARIANCE STRUCTURE

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ABSTRACT

In collaborative spectrum sensing, spatial correlation in the measurements obtained by sensors can be exploited by adopting Generalized Likelihood Ratio Test (GLRT). In this process the GLRT provides a test statistics that is normally based on the sample covariance matrix of the received signal samples. Unfortunately, problems arise when the dimensions of this matrix become excessively large, as it happens in the so-called large-scale wireless sensor networks. In these circumstances, a huge amount of samples are needed in order to avoid the ill-conditioning of the GLRT, which degenerates when the dimensionality of data is equal to the sample size or larger. To circumvent this problem, we modify the traditional GLRT detector by decomposing the large spatial covariance matrix into small covariance matrices by using properties of the Kronecker Product. The proposed detection scheme is robust in the case of high dimensionality and small sample size. Numerical results are drawn, which show that the proposed detection schemes indeed outperform the traditional approaches when the dimension of data is larger than the sample size.

Index Terms— Spatial Correlation, Kronecker Structure, GLRT, Wireless Sensor Network, Cognitive Radio.

1. INTRODUCTION

Spectrum sensing is a fundamental task for cognitive radio, where unlicensed (secondary) users try to detect the allocated vacant bands of licensed (primary) users, in order to take advantage of the free spectrum in an opportunistic manner. The reliability of a single sensor in detecting weak primary signals is very low due to the destructive channel conditions between the primary user and the secondary users [1]. To improve the reliability, collaborative sensing can be used, where measurements of multiple neighboring sensors are combined into one common decision. Therefore, this collaborative approach circumvents most of the propagation impairments due to the presence of diversity in the set of measurements being processed by different sensors. Collaborative spectrum sensing methods based on the statistical covariance matrix of the received signals have been of great interest in recent research [2]. The reason is that covariance-based techniques do not assume any information on signal, channel and noise level [1]. The statistical covariance matrices of signal-and-noise are generally different compared to the case when noise-only is present. Thus, this difference is used to discriminate the signal component from the background noise. There is a comprehensive work in [3, Ch. 9-10], which discusses in detail the multivariate detector for testing the independence of random observations with the help of the GLRT on the basis of covariance matrices. These GLRT-based detectors typically end up with a simple quotient between the determinant of the sample covariance matrix and the determinant of its diagonal version. Work in [3] is meant for multivariate statistics but recently, these tests have been widely applied to the detection of signals with distributed sensor nodes especially in the context of cognitive radios [1, 4].

The GLRT approach for detecting spatial correlation in the received signal vectors involves the estimation of the unknown covariance matrix. Therefore, it depends on the sample size (i.e. the number of samples provided by the sensors as a function of time) and the dimensionality (i.e. the number of sensors being processed at a time). In practice, the GLRT is used based on the assumption that the sample size is large while the sample dimension is small. In many applications, where the sample support available for estimating the covariance matrix is limited, the GLRT may degenerate due to a singular and ill-conditioned sample covariance matrix [5, 6].

In order to circumvent these ill-conditioning problems, one may assume an a-priori structure on the covariance matrices involved herein, based on the underlying layout of the sensor field. For instance, a convenient structure that can be assumed is the Kronecker product structure [5], which can model the lattice-type spatial structure that appears in networks whose sensors are grouped in clusters. By grouping in clusters and using Kronecker product structure, a non-singular estimate of the required covariance matrix can be more easily obtained [5], and this leads to more robust and stable detection tests. Recently, the concept of exploiting the Kronecker product structure of the covariance matrix has received a lot of interest in statistics [7, 8, 6] . Based on these studies of multivariate statistic, in [9] we proposed a GLRT detector that exploits the Kronecker product structure of the spatio-temporal correlation in the received observations of multiple sensors. In that work, we used the Kronecker prod-

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uct structure to take advantage of the inherent spatio-temporal structure of the received observations. Unlike previous work, in the proposed method we indeed take advantage of the Kronecker product structure but for the spatial dimension only, by using the novel concept of *inter-cluster* and *intra-cluster* relationships. Proceeding in this way we show that one can reduce the number of parameters to be estimated in the covariance matrix, which facilitates the estimation of large matrices in practice. Moreover, we also analyze herein the proposed scheme in the Least-Square (LS) paradigm and compare this approach with the proposed scheme based on maximum likelihood paradigm. Numerical results are drawn, which show that the proposed detection schemes indeed out perform the traditional approaches especially when the sample support is smaller.

The remaining paper is organized as follows. In Section 2, problem statement and details of signal models are presented. In Section 3, we present the proposed methodology and in Section 4 detection schemes are discussed. In Section 5 we present simulation results and Section 6 concludes the work.

2. PROBLEM FORMULATION AND ASSUMPTIONS

We consider a centrally coordinating wireless sensor network (WSN), where we assume that K sensors are placed in the form of Grid with uniform spacings. In this problem we also assume that the coverage of the signal emitted by the primary user (PU) is spread over all the sensor field. Consequently, we consider that the WSN exclusively operates over unlicensed bands. Hence, it needs to use the cognitive radio technology to arm the sensor nodes with opportunistic spectrum access capability. Towards this end, a cognitive radio sensor network is proposed that is a distributed network of wireless cognitive radio sensor nodes (CRNs), which collaboratively sense the PU's signal. In the process of collaborative spectrum sensing, the fusion center directs the sensors to perform spectrum sensing periodically in the region. All the sensors report their measurements to the fusion center, which makes the final decision about the presence of the PU. Once the final decision is made at the fusion center, it broadcasts the decision to the SUs(or CRNs) within the cell. We assume that the sensors are connected with the fusion center through dedicated channels[1]. Moreover, when the PU's signal appears, it results in change in the distribution of observations at the sensors and the observations will posses a spatial correlation structure due to PU's signal [1]. The degree of correlation in the observations of SUs increases with inter-nodes proximity. In other words, due to PU's signal the change will occur both in the energy level and the covariance structure of the received observations. Fusion center uses these changes as a detection metric to detect the PU signal.

We consider that the PU emits a Gaussian signal with zero mean and variance σ_s^2 as: $s \sim N\left(0, \sigma_s^2\right)$. The signal emitted by the PU decays isotropically as a function of distance and every sensor will receive attenuated version of the PU signal in the presence of additive Gaussian noise. Consequently, we consider that the sensors simply measure the PU signal in the target frequency band, and they report their sensing results to the fusion center. The task of the fusion center is to use the collected information, to determine whether or not a PU is present in the region. At time instant n, the observation vector received at the fusion center is $\mathbf{x}(n) \doteq [x_1(n), x_2(n), \cdots, x_K(n)]^T$, thus the signal model can be written as:

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{x}(n) = \mathbf{w}(n), \\ \mathcal{H}_1 &: \mathbf{x}(n) = \mathbf{s}(n) + \mathbf{w}(n), \end{aligned} \tag{1}$$

where $\mathbf{s}(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_s)$ with i,j-th element of $\mathbf{\Sigma}_s$ is $\sigma_{s_i}\sigma_{s_j}\rho_{i,j}$, $i,j = 1,2,\cdots,K$ and σ_{s_i} (σ_{s_j}) is the standard deviation of received signal at sensor $i\,(j)$. Noise at different sensors is considered independent Gaussian but non-identical as: $\mathbf{w}(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_0)$ with $\mathbf{\Sigma}_0 = \text{diag}\left\{\sigma_{n_1}^2, \sigma_{n_2}^2, \cdots, \sigma_{n_K}^2\right\}$, where $\sigma_{n_i}^2$ is noise variance at i-th sensor. Therefore, under $\mathcal{H}_0$, $\mathbf{x}(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_0)$ and under $\mathcal{H}_1$, $\mathbf{x}(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_1)$ with $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_s + \mathbf{\Sigma}_0$. In this work, we consider all of the parameters as unknown and these need to be estimated. We assume that the fusion center has available the measurements of K sensors for N consecutive samples of vector $\mathbf{x}$, which are stacked into the $(K \times N)$ matrix $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(N)] \in \mathbb{R}^{K \times N}$. To solve the detection problem (1) with unknown covariance matrices $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_0$, we need to have $K \ll N$, otherwise estimation of the these parameter will suffer due to the effects of ill-conditioned sample covariance matrix [6]. In the process of spectrum sensing, normally time is limited, thus, the sample support available for estimating the covariance matrix is limited. To circumvent this problem, in the following we propose modification to the traditional methodology.

3. PROPOSED METHODOLOGY

In order to avoid ill-conditioned estimate of the covariance matrix, we propose to impose some covariance structure on our data and a convenient structure that can be assumed is the Kronecker product structure [5]. It is because the Kronecker product structure considerably relax the condition on the sample size, thus it circumvents the need for a very large sample size required by the classical technique of unstructured case. In this method the single large covariance matrix is decomposed into two small covariance matrices based on Kronecker product. The Kronecker product between two arbitrary matrices $\mathbf{\Phi}$ and $\mathbf{\Psi}$ is expressed as: $\mathbf{\Phi} \otimes \mathbf{\Psi}$. Having said this, in the proposed method we consider to slice the whole field of K sensors equally, into L_c clusters and the number of clusters L_c should be sub-multiple of K. Consequently, the received vector $\mathbf{x}(n) \in \mathbb{R}^K$ is sliced into L_c sub-vectors as: $\mathbf{x}_l(n) \in \mathbb{R}^{K_c}$: $l = 1,2,\cdots,L_c$ in a way that the l-th sub-vectors correspond to l-th cluster. For further insight, the process is elaborated in Figure 1. Once we have these sub-vectors of the received observation $\mathbf{x}(n)$ then all of the sub-vectors are stacked into a $K_c \times L_c$ ma-

[1] As in the case of Femtocells, these channels can be established by using the existing wired links between nodes and the central entity.

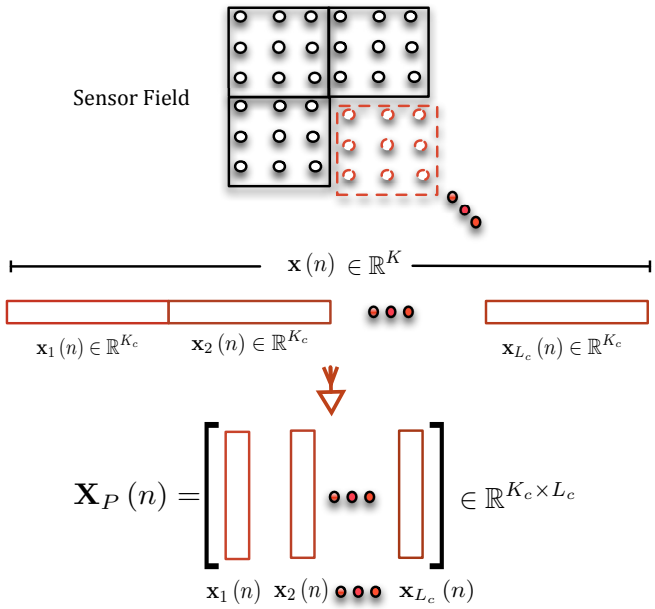

Fig. 1. Slicing process of sensor field and the observation vector.

trix $\mathbf{X}_p(n) = \{\mathbf{x}_1(n), \mathbf{x}_2(n), \cdots, \mathbf{x}_{L_c}(n)\} \in \mathbb{R}^{K_c \times L_c}$. The elements present in each column of the matrix $\mathbf{X}_p(n)$ are spatially correlated and this correlation is quantified by inter-cluster covariance matrix $\mathbf{\Sigma}_{K_c}$. On the other hand, the cross-correlation between the columns of the matrix $\mathbf{X}_p(n)$ is quantified by intra-cluster covariance matrix $\mathbf{\Sigma}_{L_c}$. Now considering the fact that the sensors are placed in a uniform grid, we can infer that the covariance structure of the columns of matrix $\mathbf{X}_p(n)$ remains the same as the topology of the K_c sensors in all of the L_c clusters is the same as if the clusters are uniform antenna arrays. Note that the Kronecker product based decomposition makes an implicit assumption that for multiple sensors in each cluster, the spatial correlation structure remains the same [10]. Taking into account these considerations the slicing method given in Figure 1 can leads us to say that $\mathbf{X}_p(n)$ is approximately comply with the matrix normal of the repeated data [6, 11]. It is widely reported in the literature that the covariance structure of the matrix normal of repeated measurements can be safely decomposed by using Kronecker product [6, 11, 10]. Hence, this provides us the required motivation to approximate our observations matrix $\mathbf{X}_p$ as having the separable structure based on the Kronecker product [6]. Therefore, we use the concept of the Kronecker product to decompose the large scale covariance matrix into two sub-matrices. To be more specific, the overall correlation structure is represented by $\mathbf{\Sigma}_{K_c} \otimes \mathbf{\Sigma}_{L_c}$. By using linear algebraic properties of the Kronecker product, the signal model under the hypothesis $\mathcal{H}_1$ can be represented as [6, 11]:

$$\mathbf{X}_p(n) = \mathcal{N}\left(\mathbf{0}_{\mathrm{K}_c \times \mathrm{L}_c}; \mathbf{\Sigma}_{\mathrm{K}_c}, \mathbf{\Sigma}_{\mathrm{L}_c}\right). \quad (2)$$

On the other hand as noise process has no structure, hence, under the hypothesis $\mathcal{H}_0$ we have $\mathbf{\Sigma}_0$ as a diagonal matrix. Herein, we remark that although the structure of separable covariances dramatically reduces the number of parameters in the covariance matrix to be estimated but it also imposes a number of constraints on the variances and correlations of the elements of the received vector $\mathbf{x}(n)$ [6]. In-spite of knowing about these minor constraints, the main motivation of exploiting the Kronecker product structure is to add robustness against the small sample support to the existing covariance based detection schemes. We further add that in the case where the Kronecker product structure is not fully present, the detection scheme with the Kronecker product structure can still provide advantages over those schemes that ignore the spatial correlation. Because instead of completely ignoring the spatial correlation, it slices the number of parameters in the covariance matrix to be estimated. However, this is a qualitative statement. At this point a more quantitative study is not available.

4. DETECTION ALGORITHMS

In our detection problem, the parameters under both hypotheses are unknown that prevent us from adopting the well-known Neyman-Pearson detector. Hence, we adopt the GLRT approach, since it usually results in simple detectors with good performance. Solution to the problem (1) based on the traditional GLRT approach ends up with a simple quotient between the determinant of the sample covariance matrix $\hat{\mathbf{\Sigma}}_1 = \frac{1}{N}\sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}^T(n)$ and the determinant of its diagonal version $\hat{\mathbf{\Sigma}}_0 = \mathrm{diag}(\hat{\mathbf{\Sigma}}_1)$. Interested readers can find details about traditional GLRT formulations in [3, Ch. 9-10]. Consequently, to avoid repercussions of the small sample support (i.e. $K \ll N$), in Subsections 4.1 and 4.2, we derive the improved detectors based on the proposed methodology in Section 3.

4.1. Proposed GLRT

The traditional GLRT approach for detection problem in (1) degenerates due to singularity issues that arises in detection problems with small samples sizes and large number of sensors. To cope with this problem, we propose a GLRT for the detection problem (1), that is expressed as:

$$\Lambda_{\text{Proposed}}(\mathbf{X}) = \frac{\max\limits_{\mathbf{\Sigma}_0} f(\mathbf{X}; \mathbf{\Sigma}_0)}{\max\limits_{\mathbf{\Sigma}_{1,\mathrm{K}_c}, \mathbf{\Sigma}_{\mathrm{L}_c}} f\left(\mathbf{X}_p^{(N)}; \mathbf{\Sigma}_{\mathrm{K}_c}, \mathbf{\Sigma}_{\mathrm{L}_c}\right)} \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \gamma, \quad (3)$$

where $f\left(\mathbf{X}_p^{(N)}; \mathbf{\Sigma}_{\mathrm{K}_c}, \mathbf{\Sigma}_{\mathrm{L}_c}\right)$ and $f(\mathbf{X}; \mathbf{\Sigma}_0)$ are the likelihood functions under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively. As we have mentioned in Section 2 that N samples of $\mathbf{x}(n)$ are available, hence, the fusion center has $\mathbf{X}_p^{(N)} = [\mathbf{X}_p(1), \mathbf{X}_p(2), \cdots, \mathbf{X}_p(N)] \in \mathbb{R}^{K_c \times NL_c}$. Solving (3), by using maximum likelihood estimation(MLE) paradigm, we can get the expression [9],

$$\Lambda_{\text{Proposed}}(\mathbf{X}) = \left[\frac{\left(\det\hat{\mathbf{\Sigma}}_{\mathrm{L}_c}\right)^{K_c}\left(\det\hat{\mathbf{\Sigma}}_{\mathrm{K}_c}\right)^{L_c}}{\left(\det\hat{\mathbf{\Sigma}}_0\right)^{K}}\right] \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \gamma. \quad (4)$$

Algorithm 1 ML based Non-Iterative Flip-Flop

1. Choose a starting value for $\hat{\mathbf{\Sigma}}^0_{\mathrm{K}_c}$ as: $\mathbf{I}_{K_c \times K_c}$
2. Estimate $\hat{\mathbf{\Sigma}}^1_{\mathrm{L}_c}$ from (5) with $\hat{\mathbf{\Sigma}}_{\mathrm{K}_c} = \hat{\mathbf{\Sigma}}^0_{\mathrm{K}_c}$.
3. Find the following
   - Estimate $\hat{\mathbf{\Sigma}}_{\mathrm{K}_c}$ from (6) with $\hat{\mathbf{\Sigma}}_{\mathrm{L}_c} = \hat{\mathbf{\Sigma}}^1_{\mathrm{L}_c}$.
   - Estimate $\hat{\mathbf{\Sigma}}_{\mathrm{L}_c}$ with $\hat{\mathbf{\Sigma}}_{\mathrm{K}_c}$ from the previous step.

The MLE of unknowns $\mathbf{\Sigma}_{\mathrm{L}_c}$ and $\mathbf{\Sigma}_{\mathrm{K}_c}$ is found by taking the derivative of $\log f\left(\mathbf{X}_p^{(N)}; \mathbf{\Sigma}_{\mathrm{K}_c}, \mathbf{\Sigma}_{\mathrm{L}_c}\right)$ with respect to the unknown $\mathbf{\Sigma}_{\mathrm{L}_c}$ and $\mathbf{\Sigma}_{\mathrm{K}_c}$ and then equating to 0. By doing so, the estimates under the hypothesis $\mathcal{H}_1$ can be written as [6]:

$$\hat{\mathbf{\Sigma}}_{\mathrm{L}_c} = \frac{1}{K_c N} \sum_{n=1}^{N} \mathbf{X}_p^T(n)\, \hat{\mathbf{\Sigma}}_{\mathrm{K}_c}^{-1} \mathbf{X}_p(n), \tag{5}$$

$$\hat{\mathbf{\Sigma}}_{\mathrm{K}_c} = \frac{1}{L_c N} \sum_{n=1}^{N} \mathbf{X}_p(n)\, \hat{\mathbf{\Sigma}}_{\mathrm{L}_c}^{-1} \mathbf{X}_p^T(n). \tag{6}$$

Expression (5) and (6) suggest that $\hat{\mathbf{\Sigma}}_{\mathrm{L}_c}$ and $\hat{\mathbf{\Sigma}}_{\mathrm{K}_c}$ can be estimated using an iterative method such as the Flip-Flop algorithm. The Flip-Flop algorithm is obtained by alternately maximizing $\log f\left(\mathbf{X}_p^{(N)}; \mathbf{\Sigma}_{\mathrm{K}_c}, \mathbf{\Sigma}_{\mathrm{L}_c}\right)$ w.r.t. $\mathbf{\Sigma}_{\mathrm{K}_c}$ keeping the last available estimate of $\mathbf{\Sigma}_{\mathrm{L}_c}$ fixed and vice versa. In [6], numerical experiments have been reported which indicate that the Flip-Flop algorithm performs very well and is much faster than a more general purpose optimization algorithm such as Newton–Raphson [6]. In [8], it has been discussed that for the case of large enough N, there is no need to iterate the algorithm. Taking into account this fact, we adopt non-iterative Flip-Flop approach and only perform the the steps given in Algorithm 1. To begin the process of algorithm, we use an initial value of $\hat{\mathbf{\Sigma}}^0_{\mathrm{K}_c} = \mathbf{I}_{K_c \times K_c}$. Under $\mathcal{H}_0$ we have to estimate the the diagonal matrix $\mathbf{\Sigma}_0$. This can be found by taking derivative of $\log f(\mathbf{X}; \mathbf{\Sigma}_0)$ with respect to the unknown $\mathbf{\Sigma}_0$ and solving the result of derivative by equating it to 0. Doing so the estimate of $\mathbf{\Sigma}_0$ can be written as: $\hat{\mathbf{\Sigma}}_0 = \mathrm{diag}\left(\frac{1}{N}\sum_{n=1}^{N} \mathbf{x}(n)\, \mathbf{x}^T(n)\right)$.

The main advantage of the proposed GLRT (4) over the traditional is that under $\mathcal{H}_1$ instead of $\frac{1}{2}K(K+1)$ parameters, it has only $\frac{1}{2}K_c(K_c+1) + \frac{1}{2}L_c(L_c+1)$ parameters to estimate. The dimensions of these two covariance matrices $\mathbf{\Sigma}_{\mathrm{L}_c}$ and $\mathbf{\Sigma}_{\mathrm{K}_c}$ are much smaller than the dimension of full covariance matrix $\mathbf{\Sigma}_1$, that is why the computations are much less demanding.

4.2. Flip-Flop Algorithm with Least Square Estimation

In every iteration of algorithm 1, to calculate (8) and (7), the inverse of covariance matrices is required. It means that these expressions demands increased computation burden. Keeping this in mind and using results of [7, Theorem 4.1], we present the Flip-Flop algorithm based on least square estimation to find $\hat{\mathbf{\Sigma}}_{\mathrm{L}_c}$ and $\hat{\mathbf{\Sigma}}_{\mathrm{K}_c}$. The expressions for $\hat{\mathbf{\Sigma}}_{\mathrm{L}_c}$ and $\hat{\mathbf{\Sigma}}_{\mathrm{K}_c}$ in Least square(LS) paradigm can be written as:

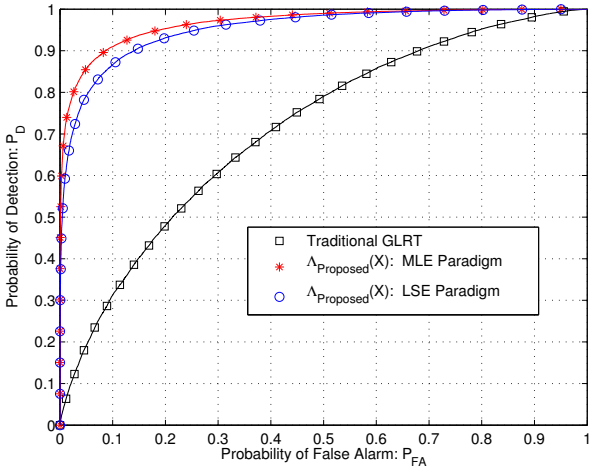

Fig. 2. Receiver Operating Characteristic (ROC) Curve: $N = 40$, $K = 36$, $\sigma_s^2 = 2$, $a_c = 0.3$, $\beta = 2$

$$\hat{\mathbf{\Sigma}}_{\mathrm{L}_c} = \frac{(N-1)^{-1}}{\mathrm{Tr}\left(\hat{\mathbf{\Sigma}}^2_{\mathrm{K}_c}\right)} \sum_{n=1}^{N} \mathbf{X}_p^T(n)\, \hat{\mathbf{\Sigma}}_{\mathrm{K}_c} \mathbf{X}_p(n), \tag{7}$$

$$\hat{\mathbf{\Sigma}}_{\mathrm{K}_c} = \frac{(N-1)^{-1}}{\mathrm{Tr}\left(\hat{\mathbf{\Sigma}}^2_{\mathrm{L}_c}\right)} \sum_{n=1}^{N} \mathbf{X}_p(n)\, \hat{\mathbf{\Sigma}}_{\mathrm{L}_c} \mathbf{X}_p^T(n), \tag{8}$$

where $\mathrm{Tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$. In the LS case, the Kronecker product of $\mathbf{\Sigma}_{\mathrm{L}_c}$ and $\mathbf{\Sigma}_{\mathrm{K}_c}$ is actually fitted to the sample covariance matrix by minimizing the difference in Frobenius norm with respect to $\mathbf{\Sigma}_{\mathrm{L}_c}$ and $\mathbf{\Sigma}_{\mathrm{K}_c}$ as:

$$\min_{\mathbf{\Sigma}_{\mathrm{L}_c}, \mathbf{\Sigma}_{\mathrm{K}_c}} \left\| \hat{\mathbf{\Sigma}}_1 - \mathbf{\Sigma}_{\mathrm{L}_c} \otimes \mathbf{\Sigma}_{\mathrm{K}_c} \right\|_F^2 \tag{9}$$

The derivation process of (7) and (8) based on solving (9) is given in Appendix A of [12]. Moreover, the steps in the algorithm based on the LS are the same as given in Algorithm 1. The only difference is that instead of (5) and (6), it uses (7) and (8). Furthermore, LS paradigm can be used without assuming any prior information about the probability distribution of the received signal should be known.

5. NUMERICAL RESULTS

For the purpose of simulation we consider a sensor network, where $K = 36$ sensors are placed in the uniform grid. For the simulations we consider $L_c = 4$ clusters, that are basically located in the form of four quadrants of a cartesian co-ordinate system and each cluster consists of $K_c = 9$ sensors. Furthermore, we assume that the event (PU) appears at the center $(x = 0, y = 0)$. We analyze the performance of both of the detection approaches based on the probability of

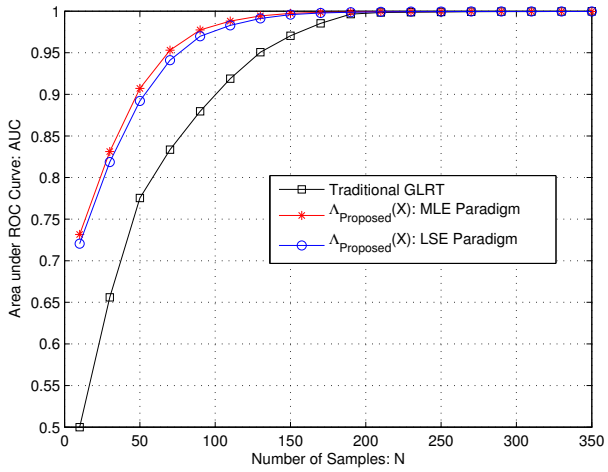

Fig. 3. Area under the ROC curve (AUC): K = 36, σ_s^2 = 3, $a_c = 0.3$, $\beta = 2$

detection (P_D) and the probability of false alarm (P_{FA}). In Figure 2 we simulate P_D vs P_{FA} for the case, where we have total $K = 36$ sensors and $N = 40$. The results clearly show that the detection performance of the traditional GLRT suffers at low level of sample supports. We can also see that the proposed detection schemes give good performance even at very small values of N. We have also observed from our experiments that most of time when $N < K$, the traditional GLRT completely collapse and the proposed schemes have reasonable good performance even under $N < K$ regime. To show the effects of changing samples size in a single picture, it is desirable to have a single and quantitative figure of merit in order to compare different detectors. This metric is typically the area under the ROC curve (AUC), which varies between 0.5 (poor performance) and 1 (good performance). In Figure 3 we analyze the detectors, for different values of the number of samples N used to estimate the covariance matrices. The results confirm that the performance of the proposed scheme is better even in the case of very small N. Another interesting conclusion can be drawn from the overall results is that the performance of proposed scheme with MLE paradigm is slightly better than the LSE paradigm.

6. CONCLUSION

In this paper a novel spectrum scheme has been proposed with the aim of detecting the signal of a licensed user by exploiting the spatial correlation present in the observations received from the sensors. The traditional detection approaches suffer due to singularity and ill-conditioned sample covariance matrix. To address this problem, we have proposed detection schemes by slicing the large covariance matrix into sub- covariance matrices based on the concept of Kronecker product. Simulation results obtained, have shown that the proposed detection schemes consistently have better detection performance in the case when the sample size is small.

7. REFERENCES

[1] E. Axell and E. Larsson, "A unified framework for GLRT-based spectrum sensing of signals with covariance matrices with known eigenvalue multiplicities," in *International Conference on Acoustics, Speech, and Signal Processing*, pp. 2956 –2959, May 2011.

[2] Y. Zeng, Y.-C. Liang, A. T. Hoang, and R. Zhang, "A review on spectrum sensing for cognitive radio: challenges and solutions," *EURASIP J. Adv. Signal Process*, vol. 2010, p. 15, January 2010.

[3] T. W. Anderson, *An Introduction to Multivariate Statistical Analysis*. Wiley-Interscience, 2003.

[4] Y. Zeng and Y.-C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE Transactions on Communications*, vol. 57, pp. 1784 – 1793, June 2009.

[5] D. Akdemir, "Slicing: Nonsingular estimation of high dimensional covariance matrices using multiway Kronecker delta covariance structures," *Submitted arXiv:1104.1767v2 [math.ST]*, 04 2011.

[6] N. Lu and D. L. Zimmerman, "The likelihood ratio test for a separable covariance matrix," *Statistics & Probability Letters*, vol. 73, pp. 449–457, July 2005.

[7] C. V. Loan and N. Pitsianis, "Approximation with Kronecker products," *Linear Algebra for Large Scale and Real Time Applications*, pp. 293–314, 1993.

[8] K. Werner, M. Jansson, and P. Stoica, "On estimation of covariance matrices with Kronecker product structure," *IEEE Transactions on Signal Processing*, vol. 56, pp. 478 –491, Feb. 2008.

[9] S.Ali, J. A. López-Salcedo, and G. Seco-Granados, "Exploiting structure of spatio-temporal correlation for detection in wireless sensor networks," in *20th European Signal Processing Conference*, pp. 774–778, Aug. 2012.

[10] A. Roy and R. Khattree, "Testing the hypothesis of a Kronecker product covariance matrix in multivariate repeated measures data," in *SUGI30*, no. 2, pp. 297–306, April 2005.

[11] P. Dutilleul, "The MLE algorithm for the matrix normal distribution.," *Journal of Statistical Computation and Simulation*, vol. 64, pp. 105–123, 1999.

[12] F. Bijma, J. C. de Munck, and R. M. Heethaar, "The spatiotemporal MEG covariance matrix modeled as a sum of Kronecker products," *Neuro-Image*, vol. 27, pp. 402–415, 2005.