IMPROVED GLRT BASED ON THE EXPLOITATION OF SPATIAL CORRELATION BETWEEN NEIGHBORING SENSORS

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ABSTRACT

On the basis of the use of the generalized likelihood ratio test (GLRT) and model order selection techniques, an improved event detector is proposed herein by incorporating the spatial correlation existing in measurements coming from neighboring wireless sensors. Such a spatial dependence is proposed to be extracted through the use of the so-called *signature vectors*. These vectors incorporate the a-priori knowledge of the effect that the event will have on the sensor field. For the proposed detector, analytical and empirical results are provided, showing a significant gain in performance compared to traditional approaches.

1. INTRODUCTION

A wireless sensor network (WSN) typically consists of a large number of inexpensive sensor nodes that are distributed over a large area. For the management of these large number of sensors, *centralized*, *decentralized* or *distributed* strategies can be devised [1], [2]. Among these three approaches, *centralized detection* involves placing all the intelligence of the network on a single point. Then, based on all the received data from sensor nodes, the fusion center makes a global decision on whether the event of interest is present or not. Although centralized detection systems have been deeply studied in the past decades, there are still some problems to be solved in the context of WSN. One of these problems appears in dense and largely deployed WSN, where observations appear to be highly correlated in the space domain due to spatial proximity among sensors [3]. In contrast, mostly current detection techniques are based on energy detection thus ignoring the important cross-sensor correlation information between these closely located sensors [4]. Moreover, the region where the event happens within a dense WSN usually spans across an area which includes just a subset of all the sensor nodes. Those sensors far away from the event are typically unable to receive the signal emitted from the event due to limited sensing ranges. On the other hand, sensors closer to the event will often be closely spaced, thus forming a cluster with highly correlated observations [3]. The presence of this structure within the received samples at the fusion center may be used to further improve the event detection performance.

There have been some attempts to incorporate correlated measurements into the formulation of signal detection problems. However, many of these studies consider the presence of correlation as a deleterious effect, or they focus on the discrimination between correlated and independent observations. This is the case of [5, Ch. 9-10] which discusses in detail the multivariate detector for testing the independence of random observations with the help of the GLRT on the basis of covariance matrices. These GLRT-based detectors typically end up with a simple quotient between the determinant of the sample covariance matrix and the determinant of its diagonal version. Recently, these covariance-based tests have been widely applied to the detection of signals in distributed sensor nodes, especially in the context of cognitive radios. In [6], the authors sense primary signals in cognitive radios by using the ratio of maximum to minimum eigenvalues of sample covariance matrices. A similar approach is also discussed in [7], where the authors use the ratio between the sum of elements of the sample covariance matrix and the sum of diagonal elements of that matrix. However, these detectors typically focus on detecting the presence of correlated data, as a possible indication that an event may be present within the data. They do not focus, instead, on exploiting the actual correlation structure that impinges onto the sensor field when an emitting target is present.

Another problem of interest is that in practice, not all the measurements that arrive at the fusion center do contain useful signal contribution. Due to limitations in the sensing range, some sensors observe strong signals and the rest receive small signal or no signal at all. This problem has already been addressed in [8], where a signal detection technique is proposed in order to identify the useful observations (i.e. those containing signal plus noise) among the set of all received measurements. The problem is formulated in terms of the Multifamily Likelihood Ratio Test (MFLRT), a joint detection and model order criterion which selects the correct subset of samples for detecting the presence of unknown nonzero signal samples [8].

Inspired by the approach in [8], a novel detector is proposed in this paper. Not only it exploits the selection of the useful set of samples in order to reject noise, as in [8], but it also takes advantage of the signal correlation occurring with an emitting target is present, based on the concept of *signatures*. The concept of signatures would be somehow equivalent to *steering vectors* in the field of array signal processing. Signatures are adopted herein as a way to capture the structure of spatially correlated measurements between neighboring sensor nodes. Each sensor node will have a signature vector representing its physical-layer connectivity with the rest of the sensors in the field. As it will be shown later on, this approach is found to significantly improve the detection performance compared to traditional approaches.

The paper is organized as follows. In Section 2, signal models and details of the problem are presented. Section 3

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introduces a modification of the GLRT and proposes a novel detector by using the signature matrix. Section 4 discuses the performances of the detector while simulation results are presented in 5 and Section 6 concludes the work.

2. PROBLEM FORMULATION

We consider a sensor network consisting of K deployed sensor nodes in a large sensor field. The task of these sensor nodes is to act as intercept receivers and observe the field for detecting the presence of an emitting source (i.e. also referred herein as an *event* or a *target*). To do so, the sensors report their observations to the fusion center where the signal processing and the final decision is drawn. In the sequel, a large scale WSN is assumed so that when the event happens, some nodes receive a large signal amplitude and others receive very small or zero amplitudes. The amplitude A_i received at the i-th sensor node is considered as a unknown deterministic parameter since both the amplitude of the emitting source A_0 and its distance to the sensor are unknown. The emitted signal power A_0^2 is assumed to decay isotropically as a function of the distance $d_{i,t}$ between the emitting source and the i-th sensor (i.e. $A_0^2/d_{i,t}^{\beta}$, β is the known path loss exponent). At the fusion center, the signal model for the received measurements can be formulated as:

$$\mathbf{x} = \mathbf{s} + \mathbf{w} \tag{1}$$

where $\mathbf{s}$ is a $(K \times 1)$ vector containing samples of the unknown deterministic received signal at each sensor, and $\mathbf{w}$ is a $(K \times 1)$ vector containing AWGN samples, with $\mathbf{w} \sim \mathscr{N}(\mathbf{0}, \sigma_w^2 \mathbf{I}_K)$ and $\mathbf{I}_K$ the $(K \times K)$ identity matrix. We assume that the noise variance σ_w^2 is known.

2.1 Signal model without spatial information

We consider very large sensor network that is why even though we have K non-zero received samples in $\mathbf{x}$ but the number (i.e. the length) of non-zero samples in vector $\mathbf{s}$ may be less than K. The sensor nodes corresponding to these non-zero samples are named as *active sensors* whose number is unknown to the fusion center. If we assume that there are L non-zero samples in $\mathbf{s}$ then the model order of (1) is L. Whereas L can be any number in the range $1 \leq L \leq K$, and it can be found by using model order selection technique [9]. Once we know L, then the signal model can be expressed as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{I}_L \\ 0_{(K-L)\times L} \end{bmatrix} \mathbf{s}_L + \mathbf{w} \tag{2}$$

where $\mathbf{s}_L$ is a vector containing only the L non-zero signals. It is important to mention that formulation in (2) assumes that the non-zero signal samples have been ordered. It is clear by observing signal model (2) that it resembles the well known classical linear model. It can also be inferred from (2) that it is a problem of rank-reduction because the number of useful signal samples is smaller than the total number of samples. Then for the case where we do not use spatial information, the rank-reduced version (2) of signal model (1) can be written as:

$$\mathbf{x} = \mathbf{T}_L \mathbf{s}_L + \mathbf{w} \tag{3}$$

where $\mathbf{T}_L = \begin{bmatrix} \mathbf{I}_L & 0_{(K-L)\times L} \end{bmatrix}^T$.

2.2 Signal model with spatial information

The active sensors are not only located close to the target but also located close to each other and thus result in a spatial cluster. Due to this neigbouring or proximity, there will exist some correlation structure. The main focus of this paper is to exploit this correlation structure and design a signal detector at the fusion center based on the principle of GLRT. In order to do so, a structure signal model is proposed on the concept of *signatures*. For the case of the i-th sensor, its signature is a vector that contains the attenuation terms to all the K sensors, as if signal source is located at the i-th sensor position. Thus, i-th signature is a $(K \times 1)$ vector $\mathbf{h}_i$ as follows,

$$\mathbf{h}_i \doteq [h(d_{1,i}), \ldots, h(d_{i-1,i}), 1, h(d_{i+1,i}), \ldots, h(d_{K,i})]^T . \tag{4}$$

where $h(d_{i,j}) = e^{-\beta d_{i,j}}$, takes into account the attenuation loss due to the distance between i-th and the j-th sensor locations. We assume herein that the fusion center has complete knowledge of the positions of the sensors in the network. Therefore all of the K signatures present in the matrix $\mathbf{H} \doteq [\mathbf{h}_1, \mathbf{h}_2 \ldots, \mathbf{h}_K]$ are known to the fusion center. Matrix $\mathbf{H}$ is assumed as a full-rank $K \times K$ matrix with K signatures as columns and we call it "*signature matrix*". If the spatial information is available then this means that the signal $\mathbf{s}$ is not completely unknown rather we know that it has some spatial structure. This spatial structure is reflected in the signatures matrix $\mathbf{H}$, and that the signal vector $\mathbf{s}$ must be a linear combination of these signatures as $\mathbf{s} = \mathbf{Ha}$. With such a spatial knowledge the following signal model applies.

$$\mathbf{x} = \mathbf{Ha} + \mathbf{w} \tag{5}$$

where $\mathbf{a}$ is the $K \times 1$ vector containing weights of each signature onto the received signal (i.e the virtual amplitudes of the sensors, when they act as emitters). In other words, these weights a_i , $i = 1, 2, \cdots\cdots, K$ can be understood as a kind of virtual amplitudes that when linearly combined with the sensors signatures, they reproduce the signal strength field captured by the whole WSN in the presence of an emitting source.

In practice the target appears at a random position within the sensor field, and normally it will be surrounded by L active sensors (i.e. the ones which receive non-zero amplitudes). It means that the important signatures are those which are related to the signals from active sensors and rest of the $K-L$ signatures could be ignored. Thus we are in-front of detection problem where it is convenient to use rank-reduced version of the signal model (5). As in section 2.1, we need to select the L signatures of active sensors with their corresponding unknown weights for the design of the detector and for that L need to be estimated by model order selection. The L signatures are stacked in matrix $\mathbf{H}_L$, which is the truncated version of $\mathbf{H}$. Similarly the unknown weights are stacked in $\mathbf{a}_L$, the reduced version of vector $\mathbf{a}$.

3. GLRT WITH PROXIMITY INFORMATION

Before introducing the proposed GLRT, we present here the test statistic based on the classical linear model, $\mathbf{y} = \mathbf{B}\theta + \mathbf{v}$ with θ as a $p \times 1$ (i.e. $p \leq K$) vector of unknown parameters, $\mathbf{v}$ as a vector that contains errors with PDF $\mathscr{N}(\mathbf{0}, \sigma_v^2 \mathbf{I}_K)$ and $\mathbf{B}$ as a known $K \times p$ observation matrix. Using this model for detecting the presence of $\theta \neq 0\,(\mathscr{H}_1)$ against the case $\theta = 0\,(\mathscr{H}_0)$ results in the test statistic as [10, Theoram 7.1,p. 274]:

$$L_G(\mathbf{y}) = \frac{\hat{\theta}^H \mathbf{B}^H \mathbf{B} \left(\mathbf{B}^H \mathbf{B}\right)^{-1} \mathbf{B}^H \mathbf{B} \hat{\theta}}{\sigma_v^2} \tag{6}$$

where $\hat{\theta} = \left(\mathbf{B}^H\mathbf{B}\right)^{-1}\mathbf{B}^H\mathbf{y}$ is the ML-estimate of θ. We can further simplify (6) to get:

$$L_G(\mathbf{y}) = \frac{1}{\sigma_v^2}\mathbf{y}^H\mathbf{P}_B\mathbf{y} = \frac{1}{\sigma_v^2}\|\mathbf{P}_B\mathbf{y}\|^2 \tag{7}$$

with $\mathbf{P}_B = \mathbf{B}\left(\mathbf{B}^H\mathbf{B}\right)^{-1}\mathbf{B}^T$ as the projection matrix. In subsections 3.2 and 3.3 we develope test statistics for different signal models on the basis of (7).

3.1 GLRT with Model Order Selection

The detection problem to be solved must cope with the presence of a set of unknown parameters of unknown dimension L (i.e. unknown length). The traditional Generalized Likelihood Ratio Test (GLRT) will always implement the test statistic based on the maximum order K. It is because to implement the test statistic, it always includes observation samples that contain only noise [8]. That is why L should first be estimated and then GLRT is conducted based on the estimated model $\hat{L}$. Estimation of the true model order L and GLRT can be found jointly by using multifamily likelihood ratio test (MFLRT). MFLRT is proposed in [8] and is given by:

$$T_{MFLRT}(x) = \max_{1\le i\le K} f\left(\bar{L}_{G_i}(x)\right) \gtrless_{\mathscr{H}_0}^{\mathscr{H}_1} \gamma. \tag{8}$$

where $f\left(\bar{L}_{G_i}(x)\right)$ is a key transforming function used to accommodate different signal models, with $\bar{L}_{G_i}(\mathbf{x}) = 2\ln L_{G_i}(\mathbf{x})$ and $L_{G_i}(\mathbf{x})$ as generalized likelihood ratio (GLR) while considering i as the size of the vector of true signals. The GLR $L_{G_i}(\mathbf{x})$ is found in sections 3.2 and 3.3 by using (7) for different signal models. In (8) γ is the threshold. Function $f\left(\bar{L}_{G_i}(x)\right)$ is given as [8]:

$$f\left(\bar{L}_{G_i}(x)\right) = \left[\bar{L}_{G_i}(x) - i\left(\ln\left(G_i(x)\right)+1\right)\right]u\left(G_i(x)-1\right) \tag{9}$$

where $G_i(x) = \frac{\bar{L}_{G_i}(x)}{i}$ and $u(x)$ is the unit step function. It is interesting to note that the MFLRT extends the GLRT to allow testing with multiple alternative model orders.

3.2 GLR without Spatial Information

For the signal model without spatial information (3) the expression for generalized likelihood ratio (GLR), $L_{G_i}(\mathbf{x})$ to be used in (8), can be written as:

$$L_{G_i}(\mathbf{x}) = \frac{\mathbf{x}_{sor}^H\mathbf{P}_{T_i}\mathbf{x}_{sor}}{\sigma_w^2} = \frac{\mathbf{x}_i^H\mathbf{x}_i}{\sigma_w^2} \tag{10}$$

where we used the fact that

$$\mathbf{P}_{T_i} = \mathbf{T}_i\left(\mathbf{T}_i^H\mathbf{T}_i\right)^{-1}\mathbf{T}_i^H = \begin{bmatrix} \mathbf{I}_i & \mathbf{0}_{i\times(K-i)} \\ \mathbf{0}_{(K-i\times i)} & \mathbf{0}_{(K-i)\times(K-i)} \end{bmatrix}.$$

$\mathbf{x}_i$ is the vector containing first i samples of the sorted observation vector $\mathbf{x}_{sor}$ (i.e. $\mathbf{x}_{sor}$=sort($\mathbf{x}$)). The reason for this sorting is that MFLRT assumes ordered true signal vector $\mathbf{s}$. In practice we do not have knowledge of the ordering and thus first $\mathbf{s}$ is estimated, then the magnitudes of the elements in $\hat{\mathbf{s}}$ are sorted in descending order and after that $\mathbf{x}$ is ordered according to ordered $\hat{\mathbf{s}}$. It is also necessary to mention that single snapshot from the sensor field is available and thus in the absence of spatial information, $\hat{\mathbf{s}}$ is just the observation vector $\mathbf{x}$. We denote the estimated L as L_{ns}, where subscript "ns" is used to indicate the fact that no spatial information is being used.

3.3 GLR with Spatial Information

In the case of spatial signal model (5), assuming model order $L = K$, the GLR (7) can be written as:

$$L_G(\mathbf{x}) = \frac{1}{\sigma_w^2}\mathbf{x}^H\mathbf{P}_H\mathbf{x} = \frac{1}{\sigma_w^2}\|\mathbf{P}_H\mathbf{x}\|^2 \tag{11}$$

where $\mathbf{P}_H = \mathbf{H}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^T$ is the projection matrix onto the space spanned by all of the K signatures. The correlation-aware detector (11) matches the received measurements with the expected correlation pattern implicitly contained within the projection matrix onto the signatures space, $\mathbf{P}_H$. In the problem under study, only the signatures related to the L active sensors are of importance and signatures of sensors that receive only noise should be discarded. That is why we use reduced-rank version of signal model (5) to find GLR and we use MFLRT to cope with issue of unknown L. To do so we first estimate the $K\times 1$ vector of signature weights $\mathbf{a}$ as $\hat{\mathbf{a}} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{x}$ then we sort the magnitudes of elements in $\hat{\mathbf{a}}$ in descending order. After that signatures in $\mathbf{H}$ are ordered according to the sorted $\hat{\mathbf{a}}$. On the basis of ordered $\mathbf{H}$ and ordered $\hat{\mathbf{a}}$, MFLRT is thus implemented. For MFLRT the GLR, after assuming order size equal to i, can be formulated as [10]:

$$L_{G_i}(\mathbf{x}) = \frac{1}{\sigma_w^2}\mathbf{x}^H\mathbf{P}_{H_i}\mathbf{x} = \frac{1}{\sigma_w^2}\|\mathbf{P}_{H_i}\mathbf{x}\|^2 \tag{12}$$

where $\mathbf{P}_{\mathbf{H}_i} = \mathbf{H}_i\left(\mathbf{H}_i^H\mathbf{H}_i\right)^{-1}\mathbf{H}_i^T$ is the reduced-rank projection matrix onto the subspace spanned by the i selected signatures. Here the resulting detector matches the received measurements with the expected correlation pattern implicitly contained within the projection matrix onto the subspace of active signatures, $\mathbf{P}_{\mathbf{H}_i}$. The energy of the resulting projection is the one which is then compared to a threshold for determining whether the expected signal structure was contained on the data or not. Indeed, in the case of using no spatial information rank-reduction (i.e. $L \le K$) with help of MFLRT shows a significant performance improvement compared to traditional (i.e. $L = K$) GLRT approaches [8]. The proposed detector with the reduced-rank spatial signal model aims further improvement in the detection performance of the GLRT by including the spatial information in the form of signatures. The implementation process of the proposed detector is summarized in Algorithm 1

4. PERFORMANCE ANALYSIS

We find the performance of the proposed detector in terms of probability of detection (P_D) and probability of false alarm (P_{FA}). For the model order L and using properties of the projection matrix $\mathbf{P}_{H_L}$, we can write the proposed detector (12) as: [10, Ch. 13],

$$T_L(\mathbf{x}) = \frac{1}{\sigma_w^2}\mathbf{x}^H\left(\mathbf{H}_L^{\#}\right)^H\mathbf{H}_L^H\mathbf{H}_L\mathbf{H}_L^{\#}\mathbf{x} \tag{13}$$

where $\mathbf{H}_L^{\#} = \left(\mathbf{H}_L^H\mathbf{H}_L\right)^{-1}\mathbf{H}_L^H$ is a $(L\times K)$ matrix corresponding to the Moore-Penrose pseudoinverse matrix of $\mathbf{H}_L$. The expression in (13) can be further rearranged by defining the $(L\times 1)$ vector, $\mathbf{z} = \mathbf{H}_L^{\#}\mathbf{x}$. In this way, the detector can be expressed as:

$$T_L(\mathbf{x}) = \frac{1}{\sigma_w^2}\mathbf{z}^H\mathbf{H}_L^H\mathbf{H}_L\mathbf{z} \tag{14}$$

Now $\mathbf{z}$ is Gaussian random vector with distribution $\mathbf{z} \sim \mathcal{N}\left(\mathbf{H}_L^{\#}\mathbf{H}\mathbf{a}, \sigma_w^2\left(\mathbf{H}_L^H\mathbf{H}_L\right)^{-1}\right)$. Let's define $\mu_z = \mathbf{H}_L^{\#}\mathbf{H}\mathbf{a}$ and

Algorithm 1 Implementation process of the proposed detector

1. The received observations from the sensor field are stacked in vector $\mathbf{x}$.
2. Using already known full signature matrix $\mathbf{H}$, we define $\mathbf{x} = \mathbf{Ha} + \mathbf{w}$.
3. Estimate the the full virtual weights vector $\mathbf{a}$ as $\hat{\mathbf{a}} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H\mathbf{x}$
4. Take the absolute values$|\hat{\mathbf{a}}|$ and sort $|\hat{\mathbf{a}}|$ in descending order resulting in $\tilde{\mathbf{a}} = sort\left(|\hat{\mathbf{a}}|\right)$
5. Shuffle the signature vectors in $\mathbf{H}$ according to the sorting of $\tilde{\mathbf{a}}$ as $\tilde{\mathbf{H}} = sort\left(\mathbf{H}\right)$.
6. Implement (8) by using (12) as:
   - $\mathbf{t} \leftarrow \langle\rangle$
   - while $i \leqq K$ do
     - Find $\mathbf{H}_i$=$\tilde{\mathbf{H}}(1:i)$
     - Calculate $\mathbf{P}_{\mathbf{H}_i} = \mathbf{H}_i\left(\mathbf{H}_i^H\mathbf{H}_i\right)^{-1}\mathbf{H}_i^H$
     - Calculate $\ln L_{G_i}(\mathbf{x}) = \frac{\mathbf{x}^H\mathbf{P}_{\mathbf{H}_i}\mathbf{x}}{\sigma_w^2}$
     - Obtain $f_i = f\left(\bar{L}_{G_i}(x)\right)$ as given in (9)
     - Push f_i onto $K\times 1$ vector $\mathbf{t}$
     - $i = i+1$
     - end while
   - $\mathbf{t} = [\ f_1,\quad f_2,\quad \cdots,\quad f_K\ ]$
   - $T_{MFLRT}(x) = \max\left(\mathbf{t}\right) \gtrless_{\mathscr{H}_0}^{\mathscr{H}_1} \gamma$ as in (8) and the index corresponding to the maximum is L_{sp}, the estimated L.

$\mathbf{C}_z = \sigma_w^2\left(\mathbf{H}_L^H\mathbf{H}_L\right)^{-1}$ then the statistics of the proposed detector are:

$$T_L(\mathbf{x}) = \mathbf{z}^H\mathbf{C}_z^{-1}\mathbf{z} \sim \begin{cases} \chi^2_{L_{sp}} & \text{for } \mathscr{H}_0 \\ \chi'^2_{L_{sp}}\left(\lambda_{sp}\right) & \text{for } \mathscr{H}_1 \end{cases} \tag{15}$$

with corresponding non-centrality parameter being given by $\lambda_{sp} = \left(1/\sigma_w^2\right)\mathbf{a}^H\mathbf{H}^H\mathbf{P}_{\mathbf{H}_{L_{sp}}}\mathbf{Ha} = \left(1/\sigma_w^2\right)\mathbf{s}^H\mathbf{P}_{\mathbf{H}_{L_{sp}}}\mathbf{s}$. We have clearly indicated the estimated order by L_{sp}, indicating that rank-reduced dimension L has been estimated by performing a model order selection technique onto the full estimate of $\mathbf{a}$. Indeed, L_{sp} with the subindex "sp" is related to the fact that spatial information is being exploited. With the estimated model order L_{sp} the expression for the probability of false alarm and probability of detection becomes:

$$\begin{aligned} P_{FA}(L_{sp}) &= \mathsf{Q}_{\chi^2_{L_{sp}}}(\gamma) \\ P_D(L_{sp}) &= \mathsf{Q}_{\chi'^2_{L_{sp}}(\lambda_{sp})}(\gamma) \end{aligned} \tag{16}$$

One of the problems that appears in the calculation of the performance is that it depends upon model order L_{sp}, which is estimated by model order selection techniques based on noisy input measurements. As a result, the estimated model order becomes a random variable, which makes it very difficult to analyze the performance in close-form. This problem is also aggravated by the fact that the distribution of this random variable is unknown and that is why it is not so trivial to find close-form analytical expression for the performance. In [11], the asymptotic distribution of the model order for Akaike's information criterion (AIC) is obtained but the paper does not demonstrate the statistical optimality for practical cases. Whereas, in the case of MFLRT there are no such results available for the distribution of the estimated model order. The only way to proceed is to obtain this empirically, using histograms, which can be used to obtain the estimated distribution of L_{sp}, herein referred as $f_{L_{sp}}(l)$. Using $f_{L_{sp}}(l)$ the performance of (8) with (12) in term of average probability of false alarm $\overline{P}_{FA}$ and average probability of detection $\overline{P}_D$ can be expressed as:

$$\begin{aligned} \overline{P}_{FA} &= E_{L_{sp}}\left[P_{FA}(L_{sp})\right] = \sum_{l=1}^{L_{max}} P_{FA}(l)f_{L_{sp}}(l) \\ \overline{P}_D &= E_{L_{sp}}\left[P_D(L_{sp})\right] = \sum_{l=1}^{L_{max}} P_D(l)f_{L_{sp}}(l) \end{aligned} \tag{17}$$

The performance of the proposed detector is also compared with the detector that uses no spatial information (10). The PDF of the detector without spatial information is also chi-square and the expression for the probability of false alarm and probability of detection is given as:

$$\begin{aligned} P_{FA}(L_{ns}) &= \mathsf{Q}_{\chi^2_{L_{ns}}}(\gamma) \\ P_D(L_{ns}) &= \mathsf{Q}_{\chi'^2_{L_{ns}}(\lambda_{ns})}(\gamma) \end{aligned} \tag{18}$$

where the non-centrality parameter $\lambda_{ns} = \left(1/\sigma_w^2\right)\mathbf{s}_{L_{ns}}^H\mathbf{s}_{L_{ns}}$. In the same way as explained before, the performance in term of average probability of false alarm $\overline{P}_{FA}$ and average probability of detection $\overline{P}_D$ is:

$$\begin{aligned} \overline{P}_{FA} &= E_{L_{ns}}\left[P_{FA}(L_{ns})\right] = \sum_{l=1}^{L_{max}} P_{FA}(l)f_{L_{ns}}(l) \\ \overline{P}_D &= E_{L_{ns}}\left[P_D(L_{ns})\right] = \sum_{l=1}^{L_{max}} P_D(l)f_{L_{ns}}(l) \end{aligned} \tag{19}$$

where $f_{L_{ns}}(l)$ is the empirical probability distribution of estimated model order for a fixed value of signal to noise ratio.

5. SIMULATION RESULTS

In this section, we present results of computer simulations to illustrate the performance of our proposed detector. The performance of the proposed detector is also compared with the detection techniques based on the signal model with no spatial information [8]. Experiments are performed with a total number of $K = 20$ sensor nodes, which are randomly distributed in the sensor field of size 12 sq-meters. This simplified scenario is depicted in Fig.1. To make the scenario resemble to more practical cases, we also fix the sensing range for the sensor nodes to $R = 3$ meters. Looking at Fig.1, the target is represented with the circle. The sensors that find the event within their sensing ranges are denoted by "▽" , while "□" indicate those sensors that receive zero amplitude from the target. On the basis of this scenario, we analyze the proposed detector with the help of receiver operating characteristics (ROC) curves and the curves showing probability of detection (PD) vs SNR(dB). To simulate these curves, we set the amplitude emitted from the event as $A = 3$. The results are obtained with single snapshot ($N = 1$). In Fig.2 as well in Fig.3 subscript "ns" indicates the detector that uses no spatial information and subscript "sp" represents the proposed detector which uses spatial information. It can be seen in Fig.2 that the proposed detector has better ROC characteristic. In Fig.3, the proposed detector is further analyzed with the help of average probability of detection ($\overline{P}_D$) plotted against different SNR values in dB. These plots also show that detector with spatial information is superior to the detector which is using no spatial information.

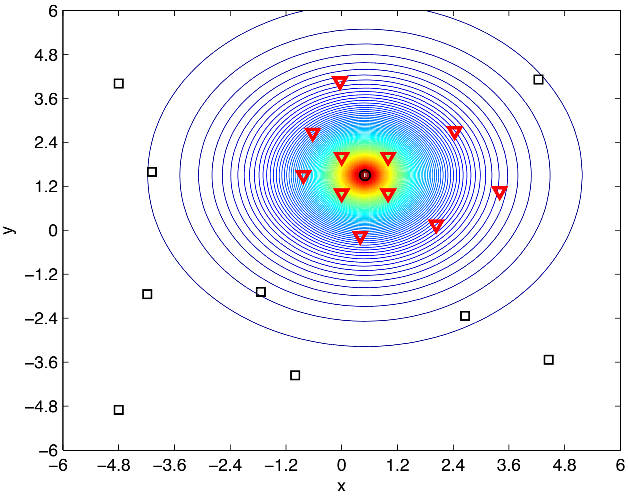

Figure 1: Field scenario

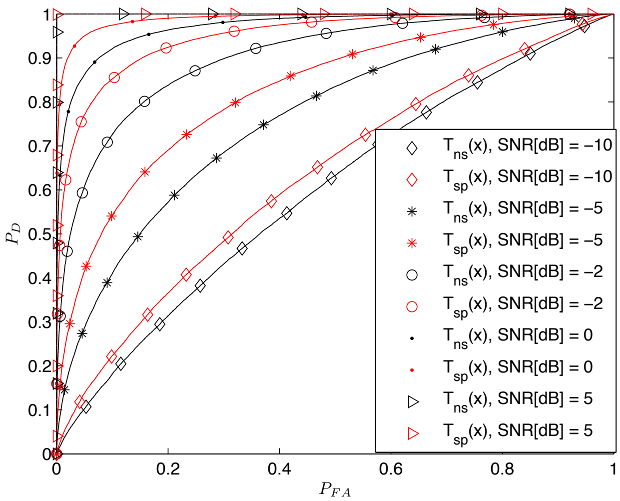

Figure 2: **[ROC Curve]**: Probability of detection vs probability of false alarm.

6. CONCLUSIONS

In this paper a new Generalized likelihood ratio test (GLRT) based detector for centralized detection system has been proposed. The aim is to achieve an improvement in the detection performance by exploiting correlation among neighboring sensor nodes. Prior information on basis of sensor's positions has been incorporated through a novel signal model based on signature matrix, which captures the correlation among different sensors. Simulation results obtained, have shown that the proposed detector is superior in performance as compare to the detectors which do not use the correlation information.

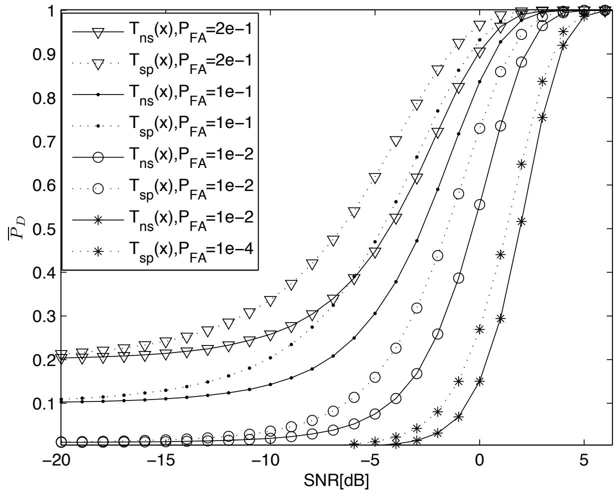

Figure 3: $\overline{P}_D$ vs $SNR\,[dB]$

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