

# AN INTERACTIVE MULTIPLE MODEL APPROACH FOR ROBUST GNSS CARRIER PHASE TRACKING UNDER SCINTILLATION CONDITIONS

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## ABSTRACT

This contribution deals with robust carrier phase tracking in Global Navigation Satellite Systems, where the ultimate goal is to obtain accurate and robust phase estimates under non-nominal conditions, such as high dynamics, strong fading and ionospheric scintillation. Within this framework, an Interacting Multiple Model approach, using a bank of parallel Kalman-based filters with different dynamic state models, is proposed to cope with signals corrupted by severe ionospheric scintillation. In the proposed formulation, the time-varying and correlated scintillation phase is introduced into the dynamic system using an  $AR(1)$  model. Simulation results are provided to show the enhanced robustness and improved accuracy of the proposed approach, with respect to state-of-the-art carrier phase tracking techniques.

**Index Terms**— GNSS, robust tracking, ionospheric scintillation, carrier phase, interactive multiple models.

## 1. INTRODUCTION

The problem under study concerns the derivation of efficient and robust methods for carrier phase tracking in Global Navigation Satellite Systems (GNSS), considering harsh propagation environments (i.e., non-nominal propagation conditions), where the signal may be affected by high dynamics, shadowing, strong fadings, multipath effects or ionospheric scintillation. Among these propagation conditions, the ionospheric scintillation is certainly the most challenging one due to the combination of both fading and rapid phase changes, in a simultaneous and random manner [1]. Moreover, the fact that scintillation effects are typically unnoticed for mass-market GNSS receivers has led this effect to receive rather little attention in the signal processing literature. These are the reasons why this contribution mainly focuses on the robust carrier phase estimation problem under scintillation conditions.

The synchronization in GNSS receivers is typically carried out following a two-steps approach: acquisition and tracking. The first stage gives a coarse estimate of the synchronization parameters, and the second one refines these estimates, filtering out noise and tracking any possible time-variation. The carrier phase tracking techniques implemented in conventional GNSS receivers rely on well known phase-locked loop (PLL) architectures [2]. The problem of standard PLLs is the existing tradeoff between noise reduction and dynamic range, which is driven by the bandwidth and order of the loop. These two parameters are the ultimate factors that control the phase tracking jitter and stress error: a small bandwidth is needed

to be able to filter out the noise and track signals with low carrier to noise ratios ( $C/N_0$ ), and a large bandwidth has to be used to cope with high dynamics (i.e. fast variations on the parameters of interest). These techniques have been shown to deliver poor estimates or even fail under harsh propagation conditions [3, 4].

On the basis of conventional PLL architectures, some improvements have been proposed in the literature: hybrid architectures coupling the PLL with a frequency-locked loop (FLL) to reduce the dynamics of the signal to be tracked [5], then being able to use a smaller bandwidth; wavelet denoising techniques to reduce the noise affecting the system [3], then being able to use large bandwidths to track high dynamics; adaptive methods that sequentially adapt the bandwidth of the system according to the actual working conditions, based on the estimation of some performance metrics [6, 7], or Kalman filter (KF)-based tracking techniques [8], where the filter is automatically adjusted so as to minimize the mean square error.

Standard KFs, which are formulated from an optimal filtering approach, have been shown to improve all the methods based on PLL architectures previously mentioned [3, 4, 9, 10], and that is why these methods are in the core of all the advanced carrier phase tracking techniques. The problem of KFs and other Bayesian filters, such as Particle Filters (PFs) [11, 12] and Sigma-point KFs (SPKFs) [13], is that the performance obtained depends on the accuracy of the dynamic model being assumed and the a priori fixed system parameters, so these methods are not able to adapt to time-varying conditions. To overcome the latter, a further improvement of KF-based methods are the so-called Adaptive KFs (AKFs) [14], which sequentially adapt the filter parameters (e.g., the covariance matrix of the measurement noises) to the actual working conditions [15, 16]. These methods still rely on a specific dynamic model, which defines the evolution of the parameters of interest (e.g., the carrier phase), and thus, they do not provide a robust solution to time-varying scenarios.

In this contribution, to overcome the problems previously discussed, and inspired by the use of Interacting Multiple Models (IMMs) in target tracking, navigation and high dynamics applications [17–19], an IMM approach is proposed to deal with robust carrier phase tracking under scintillation conditions. Standard IMM applications use a bank of KFs, each one designed to track a given dynamics. In the problem at hand and depending on the scintillation intensity, the scintillation effect may be characterized by different operation regions. For instance, one may consider 3 regions: low, moderate and severe scintillation. In the proposed formulation, the time-varying and correlated scintillation phase is introduced into the dynamic system using an  $AR(1)$  model, and then, several models are considered, each one designed to cope with a different scintillation region and the corresponding different effects on the incoming signal. The different filters implemented (one for each branch of the IMM) are able to track the carrier phase corrupted by

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a given level of scintillation (i.e., lying in a specific region). The IMM merges the results delivered by the different filters to obtain an improved estimate, and to correctly deal with the changing working conditions.

The paper is organized as follows: Section 2 introduces the ionospheric scintillation and its effects on carrier phase. The GNSS carrier signal model is given in Section 3. Section 4 proposes a robust IMM solution for GNSS carrier phase tracking, and an example of the proposed method to deal with scintillation corrupted scenarios is given in Section 5.

## 2. IONOSPHERIC SCINTILLATION

Ionospheric scintillation is the name given to the disturbance caused by electron density irregularities along the propagation path through the ionosphere. These irregularities affect the GNSS signals with amplitude fades and phase variations. An important feature of the scintillation effect is the existing correlation between deep amplitude fades and phase variations, the so-called canonical fades. That is, the largest amplitude fades are usually associated with half-cycle phase jumps [1].

The fact that the scintillation effects are typically unnoticed for mass-market GNSS receivers, whose dominant degradations are caused by thermal noise and multipath reflections, has led this effect to receive little attention in the signal processing literature. For instance, most of the existing contributions related to scintillation are focused on very specific scientific applications and are often of limited applicability in GNSS signal processing applications. The purpose of this section is to introduce a simple effective signal model to represent the behavior of scintillation onto the GNSS received signal samples, which is based on the Cornell Scintillation Model [20].

### 2.1. Scintillation model

In terms of the complex-valued baseband received signal,  $x(t)$ , the presence of scintillation can be modeled as a complex-valued multiplicative channel,

$$x(t) = \xi_s(t)s(t), \quad (1)$$

where,  $s(t)$  is the complex-valued baseband equivalent of the transmitted signal (i.e., the signal of interest), and the complex-valued stochastic process representing the presence of scintillation (i.e., scintillation complex gain) is defined as,

$$\xi_s(t) = \rho_s(t)e^{j\theta_s(t)}, \quad (2)$$

with the corresponding envelope and phase components,  $\rho_s(t)$  and  $\theta_s(t)$ , respectively. Because of the interdependence that exists between both magnitudes (i.e., amplitude and phase) which is apparent in the presence of canonical fades [21], the characterization of their individual distributions, widely used in the literature, is not a valid approach. Some recent contributions [20, 22] have introduced a method called the Cornell Scintillation Model (CSM) to synthesize realistic scintillation, based on a statistical model and the proper shaping of the spectrum of the entire complex scintillation signal.

The strength of amplitude scintillation is described by the so-called scintillation index  $S_4$ , and is usually considered within three main regions: weak, moderate and strong/severe scintillation [20];

and is defined as

$$S_4 = \sqrt{\frac{\mathbb{E}(\rho_s^4) - (\mathbb{E}(\rho_s^2))^2}{(\mathbb{E}(\rho_s^2))^2}} \begin{cases} S_4 \leq 0.3 & (\text{weak}) \\ 0.3 < S_4 \leq 0.6 & (\text{moderate}) \\ 0.6 < S_4 & (\text{severe}) \end{cases} \quad (3)$$

The envelope of this amplitude scintillation is typically modeled as a Nakagami-m random variable [23], but the experimental results in [20] show that a simpler Ricean distribution can also be used, while preserving a close fit with empirical data. This is a very convenient approach for simulation purposes, because it means that the complex-valued scintillation  $\xi_s(t)$  can easily be modeled as a Gaussian random variable with a given autocorrelation  $R_{\xi_s}(\tau)$ ,

$$\rho_s \sim \text{Ricean}(K_{\rho_s}, \Omega_{\rho_s}) \rightarrow \xi_s \sim \mathcal{N}(\mu_{\rho_s}, \sigma_{\rho_s}^2), \quad (4)$$

with the following parameters:  $K_{\rho_s} = \frac{\mu_{\rho_s}^2}{2\sigma_{\rho_s}^2}$ ,  $\Omega_{\rho_s} = \mu_{\rho_s}^2 + \sigma_{\rho_s}^2$ ,  $\mu_{\rho_s} = \mathbb{E}(\rho_s)$  and  $\sigma_{\rho_s}^2 = \mathbb{E}(\rho_s^2) - (\mathbb{E}(\rho_s))^2$ . Notice that for the simulation of a scintillation data set only two parameters must be specified,  $\{S_4, \tau_0\}$ , which determine, respectively, the intensity and correlation of the scintillation complex gain components. In general, higher  $S_4$  and lower  $\tau_0$  lead to more severe scintillation, where the ranges of possible values are, respectively,  $0 < S_4 \leq 1$  and  $0.1 \leq \tau_0 < 2$  (seconds) [20].

The CSM has been embedded in the so-called Cornell Scintillation Simulation Matlab toolkit, which is available at <http://gps.ece.cornell.edu/tools.php>. This software will be used in the computer simulations to generate the desired scintillation effect and then assess the performance of the proposed method.

### 2.2. Scintillation phase modeling using an $AR(1)$ process

In order to embed into the state-space model the scintillation effect on the carrier phase, a proper model for this phase evolution is needed. In the CSM model previously discussed the components of the scintillation complex gain are correlated, and obtained in the simulator via a second-order low-pass Butterworth filter [20].

For severe scintillation, which is the most challenging scenario for current state-of-the-art carrier phase tracking techniques, the correlation in the scintillation phase can be fairly modeled using an  $AR(1)$  process. The general  $AR(1)$  model for a discrete sequence  $z_k$  is specified by the following recursion,

$$z_k = \beta z_{k-1} + \eta_k, \quad (5)$$

where  $\eta_k$  is a white Gaussian noise sequence with variance  $\sigma_\eta^2$ . The coefficient  $\beta$  and the variance  $\sigma_\eta^2$  can be obtained through the Yule-Walker equations [24], using the autocorrelation function of the sequence  $z_k$ , which is denoted, for an autocorrelation lag  $m$ ,  $R_{zz}(m)$ . For an  $AR(1)$  process, the required parameters can easily be obtained as,

$$\beta = \frac{R_{zz}(1)}{R_{zz}(0)} ; \quad \sigma_\eta^2 = R_{zz}(0) - \beta R_{zz}(1). \quad (6)$$

Using the scintillation time histories generated with the CSM, these parameters have been computed and the  $AR(1)$  model has been validated. For instance, considering a severe scintillation scenario and a sampling rate  $f_s = 0.1$  kHz (integration time  $T_s = 10$  ms), the following parameters were obtained:

$$\begin{cases} S_4 = 0.8 \\ \tau_0 = 0.1 \end{cases} \rightarrow \begin{cases} \beta = 0.914 \\ \sigma_\eta^2 = 0.08 \end{cases} . \quad (7)$$

The state-space formulation including the  $AR(1)$  model is detailed in Section 3.2.

### 3. SIGNAL MODEL

#### 3.1. GNSS signal model

The baseband analytic representation of a signal received from a generic GNSS satellite can be expressed as

$$x(t) = P_x(t)d(t - \tau(t))c(t - \tau(t))e^{j(2\pi f_d(t) + \theta_e(t))} + w(t), \quad (8)$$

where  $P_x(t)$ ,  $d(t)$ ,  $c(t)$  and  $w(t)$ , stand respectively for the signal amplitude, the navigation message, the spreading code and the noise term, which may include the thermal noise, signals from the same or other satellites, replicas of the transmitted signal due to multipath, and any other interference. The synchronization parameters are the code delay,  $\tau(t)$ , the carrier Doppler frequency shift,  $f_d(t)$ , and the carrier phase,  $\theta_e(t)$ . The digitized signal (sampling period  $T_s$ ) at the output of the radio frequency (RF) front-end feeds the digital receiver  $M$  channels. The goal of each channel is to acquire and track the signal of a single satellite.

After the acquisition stage, which provides the first code delay and Doppler shift estimates, respectively,  $\hat{\tau}(t)$  and  $\hat{f}_d$ , the sampled signal is correlated with a locally-generated replica and then accumulated over the integration period  $T_s$ . The samples at the output of the correlators are usually expressed as

$$y_k = A_k d_k R(\Delta\tau_k) \frac{\sin(\pi \Delta f_{d,k} T_s)}{\pi \Delta f_{d,k} T_s} e^{j(2\pi \Delta f_{d,k} T_s + \Delta\theta_{e,k})} + n_k, \quad (9)$$

where  $k$  stands for the discrete time  $t_k = kT_s$ ,  $A_k$  is the signal amplitude at the output of the correlators after accumulation over  $T_s$ ,  $d_k$  is the data bit,  $R(\cdot)$  is the code correlation function and  $\{\Delta\tau_k, \Delta f_{d,k}, \Delta\theta_{e,k}\}$  are, respectively, the code delay, Doppler shift and carrier phase errors.

Taking into account the problem at hand, which is to focus on the study of the carrier phase estimation problem under scintillation conditions, a simplified signal model can be considered to take into account only the carrier phase tracking stage. The simplified signal model do not include the spreading code and a perfect code delay estimation is assumed,  $\Delta\tau_k = 0$ . Moreover, it is also considered that no data bits are present in the received samples (i.e., this is the case when using pilot signals or a data wipe-off technique has already been implemented), and that the attenuation factor is negligible,  $\text{sinc}(\Delta f_{d,k} T_s) \approx 1$ . Under these assumptions, the samples at the input of the carrier phase tracking stage are given by

$$r_k = \alpha_k e^{j\theta_k} + n_k, \quad (10)$$

where the amplitude,  $\alpha_k$ , may include the scintillation amplitude effects,  $\alpha_k = A_k \rho_{s,k}$ ; and the carrier phase,  $\theta_k$ , includes the phase variations due to the receiver's dynamics,  $\theta_{d,k}$ , and the scintillation phase variation,  $\theta_{s,k}$ . In the following section, a state-space formulation is proposed.

#### 3.2. State-space model

In the carrier phase tracking problem, considering that the input to the tracking block is given by eq.(10), the parameter of interest is the phase  $\theta_k$ , which includes the time-varying evolution caused by the receiver dynamics and the possible scintillation effect.

- Concerning the phase evolution due to the receiver dynamics, the following model is considered,

$$\theta_{d,k} = \theta_0 + 2\pi \left( f_{d,k} k T_s + \frac{1}{2} \dot{f}_{d,k} k^2 T_s^2 \right), \quad (11)$$

where  $\theta_0$  (rad) is a random constant phase value,  $f_{d,k}$  (Hz) is the carrier Doppler frequency shift and  $\dot{f}_{d,k}$  (Hz/s) the Doppler frequency rate (i.e., the Doppler frequency shift dynamics).

- The model for the phase fluctuations due to the scintillation has already been introduced in Section 2.2 and reads

$$\theta_{s,k} = \beta \theta_{s,k-1} + \eta_k \quad (12)$$

where  $\eta_k$  is a white Gaussian noise with variance  $\sigma_\eta^2$ . Both, the parameter of the  $AR(1)$  process,  $\beta$ , and the variance of the process driving noise are determined via simulation from scintillation time histories obtained with the CSM.

From the previous results, a state evolution model considering both effects can be defined. Considering that the state to be tracked is  $\mathbf{x}_k \doteq [\theta_{d,k} \ f_{d,k} \ \dot{f}_{d,k} \ \theta_{s,k}]^T$ , where the phase is expressed in cycles, the process equation is

$$\mathbf{x}_k = \begin{pmatrix} 1 & T_s & T_s^2/2 & 0 \\ 0 & 1 & T_s & 0 \\ 0 & 0 & T_s & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix} \mathbf{x}_{k-1} + \mathbf{v}_k, \quad (13)$$

where the transition matrix is denoted  $\mathbf{F}$  and the process noise,  $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q})$ , stands for possible uncertainties or errors on the state transition model. The process noise covariance matrix is designed according to the problem at hand and depending on the system working conditions. The general formulation is

$$\mathbf{Q} = \begin{pmatrix} \mathbf{G}\mathbf{G}^T \sigma_j^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}, \quad (14)$$

where  $\mathbf{G} = [T_s^3/3, T_s^2/2, T_s]^T$  and  $\sigma_j^2$  is the Doppler frequency rate error variance. Equations (10) and (13) define the state-space formulation of the problem.

### 4. INTERACTIVE MULTIPLE MODELS FOR ROBUST CARRIER PHASE TRACKING

Using the state-space formulation given in the previous section, it is easy to construct a KF to solve the carrier phase tracking (i.e., estimation) problem. The KF gives the optimal estimate of the posterior distribution in linear Gaussian systems, which is computed following a two-step approach: time update (i.e., prediction) and measurement update (i.e., correction). Due to the lack of space the well-known KF equations [25] are not detailed, but some comments on the derivation are given in the sequel.

First, note that measurements  $r_k$  (eq.(10)) are not used directly in the filter implementation, but instead go through a discriminator to obtain the noisy phase measurements, similar to a conventional PLL-like architecture. In our case, the four quadrant arctangent was used, which is the Maximum Likelihood estimator in the absence of data bits. This allows to use a traditional KF and avoids the derivation of suboptimal nonlinear solutions. Then, the Kalman gain,  $K_k$ , is computed using the variance of the measurement noise, which must be specified. An expression for the approximated variance of the phase noise at the output of the discriminator is

$$\sigma_{n_\theta}^2 = \frac{1}{8\pi^2 C/N0T_s} \left( 1 + \frac{1}{2C/N0T_s} \right). \quad (15)$$

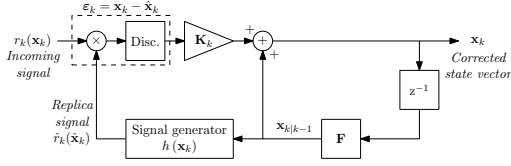


Fig. 1. Block diagram of the KF-based phase tracking method [26].

And finally, the Doppler frequency rate error variance was set to  $\sigma_j^2 = 1.579 \cdot 10^{-10} ((rad/sample)^2)^2$ . The block diagram of this KF-based method is sketched in Figure 1 [26].

The general IMM structure and formulation is not detailed due to the lack of space, see [17] and references therein for a complete discussion on IMMs. The main idea behind the IMMs is to overcome the main problem of stand-alone KFs, which is the a priori fixed process equation (i.e., fixed dynamic model) and fixed system parameters, following a divide and conquer strategy to deal with changing scenarios using several easier fixed operation KFs. The IMM is compound of four main parts: first, a prediction of the model probabilities is computed. Then, each filter runs taking into account these mixing probabilities. When the new state estimates are available, the model probabilities are updated using the measurement prediction error and the innovations covariance matrix. And finally, the filter outputs are combined to obtain the overall estimate. In our solution, two models are considered: dynamics only, which is the standard KF-based carrier tracking model, and dynamics plus scintillation,

- Model 1 - Dynamics:  $\mathbf{x}_k^{(1)} = [\theta_{d,k} \ f_{d,k} \ \dot{f}_{d,k}]^T$ .
- Model 2 - Dynamics + scintillation:  $\mathbf{x}_k^{(2)} = \mathbf{x}_k$ .

Note that within this approach, one may include a higher number of scintillation regions just adding more models.

## 5. COMPUTER SIMULATIONS

In this section, in order to provide illustrative numerical results, the performance of the proposed method (i.e., the 2-model IMM presented in previous section) is shown in a carrier phase tracking example where the signal of interest is corrupted by severe scintillation.

To assess the performance of the proposed method, a simulated trajectory with different dynamics was defined. In the sequel, the three different regions considered are summarized:

- Region I - No motion without scintillation:  $\theta_k = \theta_0$ ; for  $0 \leq kT_s \leq 10s$ .
- Region II - Constant acceleration without scintillation:  $\theta_k = \theta_0 + 2\pi (f_{d,k}kT_s + 1/2\dot{f}_{d,k}k^2T_s^2)$ ; for  $10s < kT_s \leq 20s$ .
- Region III - Constant acceleration with severe scintillation:  $\theta_k = \theta_{d,k} + \theta_{s,k}$ ; for  $20s < kT_s \leq 50s$ .

The following parameters were used: simulation time  $T_{sim} = 50s$ , integration time  $T_s = 10ms$ ,  $C/N0 = 45$  dB-Hz,  $f_{d,0} = 2$  Hz,  $\dot{f}_{d,0} = 0.5$  Hz/s,  $S_4 = 0.8$  and  $\tau_0 = 0.1$ .

Figure 2 (top) plots one realization of the tracking solution for the trajectory defined by regions I - III, to show the good behavior of the proposed method in different dynamic scenarios. For comparison, the solution obtained with a standard KF-based tracking technique (running on Model 1) is also plotted. In the case of severe scintillation it is clear that the proposed approach outperforms the

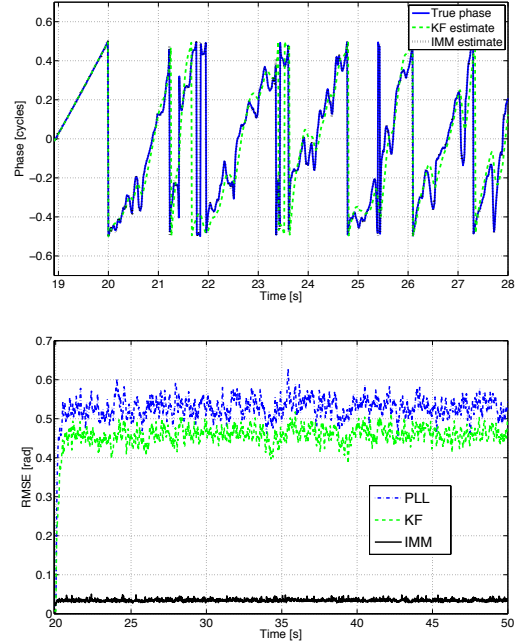


Fig. 2. One realization of the algorithm (top) and the RMSE obtained with different methods for the carrier phase tracking under severe scintillation conditions (bottom).

standard KF, which is not able to correctly track the fast variation on the signal and suffers several cycle slips. In this challenging case, for this given realization, the filter perfectly (the IMM estimates are on the true phase line in the plot) follows the fast phase variations.

To obtain a statistically more significant result, the root mean square error (RMSE) was used as a measure of performance, and obtained from 500 Monte Carlo runs. The results obtained are given in Figure 2 (bottom), where the IMM is compared with a 3rd order PLL (PLL bandwidth set to 10 Hz) and a KF-based solution. From 0 to 20s (regions I and II), the RMSE is really low because there is no scintillation and the 3 methods correctly deal with the estimation of the dynamic parameters, that is why this regions are not plotted. After time step 20s, the performance decreases because of the severe scintillation effect. From the results, it is clear to see the performance improvement obtained with the IMM-based solution with respect to the other methods, providing also a validation of the  $AR(1)$ .

## 6. CONCLUSIONS

This paper presented a solution to robust carrier phase tracking under severe scintillation conditions. The proposed solution considered an IMM approach with two coupled KF-based tracking methods, one dealing with dynamics only, and the other taking into account the severe scintillation effects. The scintillation phase was modeled and introduced into the state-space formulation using an  $AR(1)$  process model. The performance was validated by computer simulation in a GNSS carrier phase tracking application, using a simulated trajectory with different scenarios. In this example, we saw that the proposed method attains good performance results dealing correctly with severe ionospheric scintillation and outperforming current state-of-the-art techniques.

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