Simulation Tool for the Analysis of LEO Satellite Doppler Positioning

Antoni Reus-Bergas, José A. López-Salcedo, Gonzalo Seco-Granados
Institut d'Estudis Espacials de Catalunya (IEEC), Universitat Autònoma de Barcelona (UAB) - CERES
{antoni.reus, jose.salcedo, gonzalo.seco}@uab.cat

***Abstract*—Positioning is a fundamental technology in modern society, however, the advancement of human life also brings new challenges, such as jamming attacks or indoor positioning, which positioning has to cope with. Due to this reason, it is necessary to study new approaches to positioning, to enhance its resilience and accessibility in all possible scenarios. In this work, a tool for the analysis of LEO-PNT constellations is proposed, as well as an algorithm that uses Doppler measurements of signals of opportunity and TLE files to obtain the position and velocity of the satellites, all combined with the least squares estimator to estimate the user's position.**

***Index Terms*—Doppler Positioning, LEO-PNT, TLE files, Least Squares, Signals-of-Opportunity, SoOp.**

I. Introduction

Over the past decade, the offer of satellite services, such as communications, earth observation, or positioning, has significantly increased. According to ESA [1], nowadays, more than 35000 satellites are orbiting the earth, using three main orbits: GEO, whose satellites are above 36000 km in height, MEO, with a height between 2000 km and 36000 km, and LEO, which goes up to 2000 km height. This latest will be the one used in this work.

The LEO orbit has more than 20000 LEO satellites, owned by public and private companies, dedicated to science, such as the Hubble Telescope and Sentinel satellites, or communications with constellations such as Starlink or OneWeb with more than 5000 and 500 satellites respectively. All these LEO satellites must transmit signals to the Earth, which can be received by everyone. These signals are known as opportunity signals and will be the signals of interest in this work.

Positioning using signals of opportunity, consists of estimating the user's position, using different types of signals received by the user, without regard to the purpose of those signals, even if they were not designed for positioning, and taking advantage of the effects suffered by the signals when traveling through space, particularly the Doppler shift. This technique allows position estimation without having to launch new satellites, thereby saving both physical and spectral space. Further advantages of this type of positioning will be discussed in the following section.

Catalan Government's New Space Strategy.

This technique can be used as a standalone solution as well as an augmentation system for GNSS positioning, to enhance performance and take advantage of the benefits of LEO satellites.

II. LEO Positioning

Positioning using low-earth orbit satellites can be accomplished by using different measurements, such as in the case of GNSS positioning. The main measurements used are code and phase measurements, but to apply these methods, we would need a LEO constellation specifically designed for positioning purposes.

The alternative is to estimate the user's position through the utilization of Doppler measurements, commonly referred to as Doppler Positioning. In this section, various advantages and disadvantages of LEO positioning using Doppler measurements will be presented, followed by a brief discussion of how the Doppler effect is produced, and its potential application for positioning. Finally, the procedure followed by this method to estimate the position will be discussed.

A. Advantages and Disadvantages

The main advantage of LEO-PNT, using Doppler measurements, is the usage of signals of opportunity, these signals can be taken from almost every satellite already deployed, more than 20000 bodies, without having to spend money or time, launching more satellites or designing a specific constellation.

Another advantage of using LEO satellites is the higher received power, due to the lower altitude of the orbits. This makes the signal significantly stronger than GNSS signals at the receiver. This will allow the user to receive those signals in indoor scenarios. The high dynamics of the LEO satellites are also advantageous for positioning in deep urban canyon environments, such as cities with a significant number of skyscrapers. [5].

On the contrary, some new challenges arise when employing this technique, for example, the ephemerids. In GNSS, this data is broadcast by the satellites, but when using signals of opportunity, the ephemerids are not necessarily broadcast, or, if there are, sometimes, we cannot access the data disseminated in the signals, so we need to figure out how to obtain the satellite position and velocity.

Another challenge that we have to face, is the heterogeneity of the signals received, owing to the variety of bands employed by these satellites, generally the L, S, C, K, Ku, and Ka bands, going from below 1 GHz, up to 40 GHz, forcing us to use a large variety of hardware, if we desire to receive signals from different constellations. This issue can be solved, in most cases, by using an SDR or Software Defined Radio, and different types of antennas adapted to each frequency band.

This last disadvantage can also be taken as an advantage, using the lower bands for indoor PNT, higher bands for smaller devices, and a wide range of frequencies to protect the system against jamming or spoofing attacks.

B. Doppler Effect

The Doppler effect is a phenomenon that happens when the transmitter, the receiver or both are moving, changing the frequency perceived by the receiver. In satellite communications, the case that fits better is the third, where the satellite is always moving due to its orbit, and the user is also moving due to its movement and the earth's rotation. This frequency shift perceived by the receiver is modeled by the equation 1, where f_o is the frequency observed by the receiver, c is the propagation velocity of the signal, the speed of light, f_c is the carrier frequency used by the transmitter, p_r and p_s represent the position of the receiver and satellite, and $v_{r,los}$ and $v_{s,los}$, are the receiver and satellite velocity projected on the LOS vector. These two last velocities are described in 2 and 3 respectively.

$$f_o = \frac{c - v_{r,los}}{c - v_{s,los}} \cdot f_c \tag{1}$$

$$v_{r,los} = \frac{\mathbf{v_r} \cdot (\mathbf{p_r} - \mathbf{p_s})}{\|\mathbf{p_r} - \mathbf{p_s}\|} \tag{2}$$

$$v_{s,los} = \frac{\mathbf{v_s} \cdot (\mathbf{p_r} - \mathbf{p_s})}{\|\mathbf{p_r} - \mathbf{p_s}\|} \tag{3}$$

Alternatively, we can describe the Doppler shift, as appears in the equation 4, where $v_{rel} = v_{s,los} - v_{r,los}$ is the relative velocity of both elements, due to their movement.

$$f_d = f_o - f_c = \frac{v_{rel}}{c - v_{s,los}} \cdot f_c \tag{4}$$

The Doppler shift f_d is directly related to the carrier frequency, but we can also express it as a normalized value, not frequency-dependent. This makes it easier to compare how it affects certain dynamics, regardless of the carrier frequencies. This normalized Doppler shift is denoted as $\frac{f_d}{f_c}$ in units of Hz/Hz. This normalized Doppler shift can be used to obtain the Doppler shift suffered by signals with different frequencies when those signals are emitted by the same satellite and received by the same receiver.

The most common LEO constellations orbit the earth at an altitude of between 200 km and 1500 km, and their speed range goes from 7.79 km/s to 7.12 km/s. Resulting in a normalized Doppler Shift of approximately 0.025 Hz/kHz, producing a shift of tens of kHz when our carrier frequency is in the order of GHz.

III. LEO Constellation Simulation Tool

To analyze the different LEO and GNSS constellations, the in-view satellites and the characteristics of the signals received, a simulator tool has been developed. This simulator uses TLE, or Two Line Elements Files, as input.

TLE files contain the orbital elements for each of the satellites of a constellation, which will be used as an input to an orbit propagator, in this case, the SGP4 propagator, to obtain the position and velocity of the satellites.

This tool has been developed using MATLAB and MATLAB's Satellite Communications Toolbox and allows the user to simulate all the LEO Constellation listed in Tab. I, plus the main GNSS constellations, but also any other constellation stored in a TLE file.

Tab. I: Main LEO Constellations

	# Satellites	Height [km]	Frequency [GHz]
Iridium	31	780	1.61
Iridium Next	80	780	30
Starlink	5424	550	10.7-12.7
OneWeb	636	1200	14
Globalstar	85	1400	2.4
Orbcomm	60	825	0.137

After the input of the TLE file into the tool, and setting all the configuration parameters, including the user position, minimum elevation mask, and frequency utilized by each satellite, the simulation can be run. Upon completion, a window depicted in Fig. 1 will be displayed, showing where each satellite in view is placed relative to the user, using a polar plot. Subsequently, the theoretical reception time instant is indicated, as well as a slicing bar to change that time, and a table where all numerical data is displayed, such as the Doppler Shift, Doppler rate, the time of flight of the signal broadcast by each satellite, the distance between the satellite and the user, and finally the C/N0 for each satellite.

The simulation tool also has a DOP analysis utility, which allows the user to select a group of vehicles from all the in-view satellites, and compute the DOP when using range measurements, Doppler measurements, or both.

This tool will be used to obtain the Doppler measurements, required by the algorithm discussed in section IV, to estimate the receiver position.

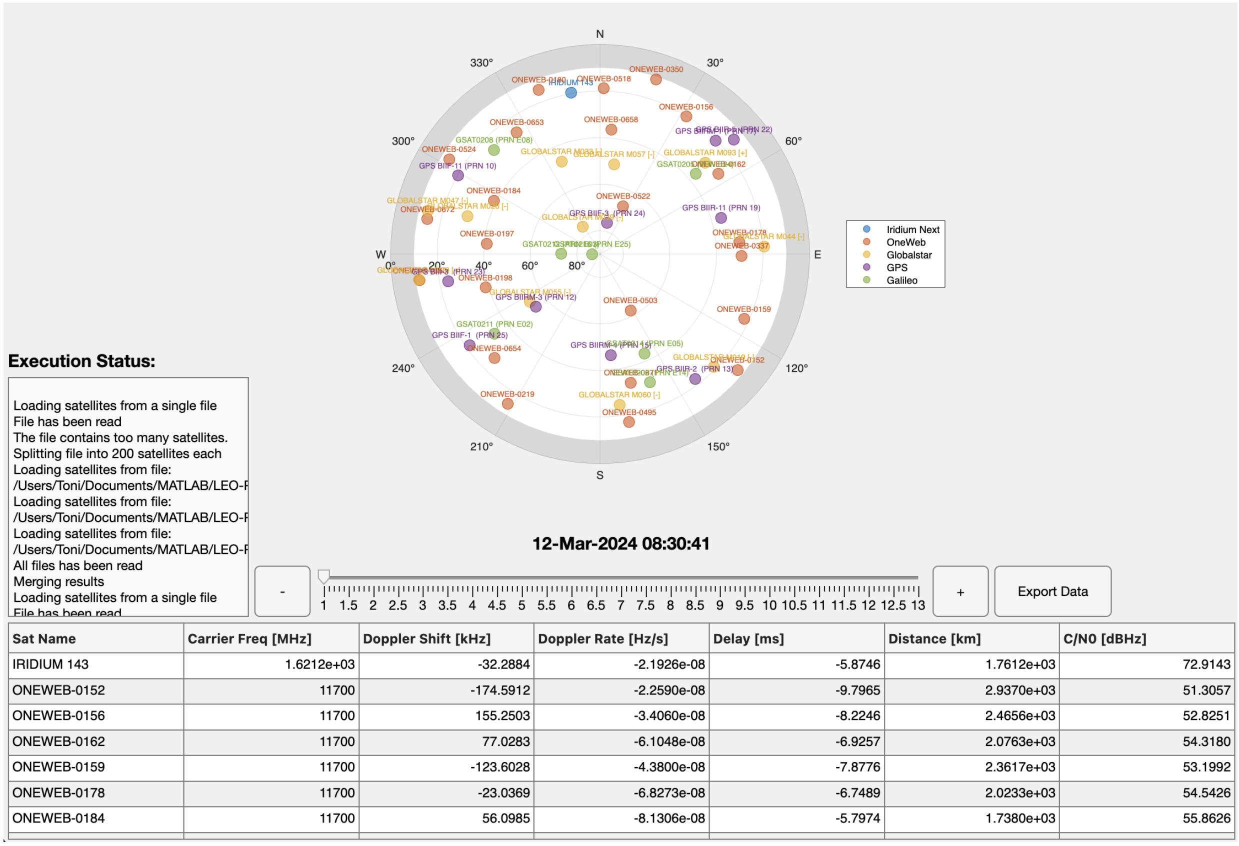

Fig. 1: Satellite Simulator Output

IV. Doppler Positioning

A. Main Considerations

This type of positioning is based on the nature of the Doppler effect, which depends on the position and velocity of both, the transmitter and the receiver, letting us obtain the position and velocity from the range rate $\dot{\rho}$, which is obtained by multiplying the Doppler shift measured, by the wavelength of the carrier frequency used to transmit the signal.

But the Doppler shift is not only affected by the relative movement between the satellite and the receiver, other effects have to be taken into account, to estimate an accurate position, like the satellite and receiver clock drift, which will produce an impact on the measurements, due to the use of clocks on signal generation, down-conversion and sampling. Other effects are the produced by the ionosphere and the troposphere when the signals travel through those atmospheric layers [6], as well as the Sagnac effect produced by the rotation of the earth, resulting in a variation on the receiver's position while the signal is traveling.

Taking into account all those effects, the resulting range rate equation is the following.

$$\dot{\rho} = f_d \cdot \lambda = v_{rel} + \frac{f_d}{f_c} v_{s,los} + c(\delta\bar{t_r} - \delta\bar{t_s}) + \delta\bar{T} - \delta\bar{I} + R + \nu \quad (5)$$

Where $\delta\bar{t_r}$ and $\delta\bar{t_s}$ represent the clock drift of the receiver and the satellite respectively, $\delta\bar{I}$ and $\delta\bar{T}$ are the ionosphere and troposphere effects on the Doppler shift, R models the Sagnac effect, and ν contains the remaining errors.

In this document, the effect produced by the ionosphere and troposphere will not be taken into account, and the Sagnac effect will be compensated by using an inertial reference coordinate frame, ECI, which is by default the coordinate frame used by orbit propagators, such as SGP4, instead of using a reference frame that rotates with the earth like the ECEF used by GNSS. We will also use a static receiver approach, but due to the use of an ECI coordinate frame, the receiver has an intrinsic velocity given by the rotation of the earth, this velocity is assumed to be known by the receiver. Considering all the previous assumptions, the equation used to estimate the position is the following.

$$\dot{\rho} = f_d \cdot \lambda = v_{rel} + \frac{f_d}{f_c} v_{s,dir} + c(\delta\bar{t_r} - \delta\bar{t_s}) + \nu \quad (6)$$

The main differences of this method, compared to other methods, are the usage of the precise Doppler shift expression, instead of using the equation 7, used by other authors, where the effect of the satellite velocity $v_{s,los}$, present in the equation 4 is neglected, but should be considered for future high-accuracy position estimation techniques. The second important difference is the use of the ECI coordinate frame, making it possible to remove the R parameter present in equation 5.

$$f_d = f_o - f_c = \frac{v_{rel}}{c} \cdot f_c \tag{7}$$

B. Algorithm

The algorithm used in this paper is described below.

Step 1: In this first step, we should take the measurements from each of the satellites that will be used to compute the position estimation, in our case, we need at least 4 satellites, one for each coordinate, and an extra satellite to estimate the receiver's clock drift, and all the Doppler measurements have to have a timestamp that will be used as the reception time of the signal. In this paper, the tool presented before in the section III, will be used to obtain those values.

Step 2: Once the reception time is obtained, we should estimate the transmission time of each signal received. To do that, we will use the geometrical method proposed in [4]. Once this transmission time is computed, we can proceed to estimate the position and velocity of each satellite at transmission time using the TLE files, and the orbit propagator.

Step 3: Having all the previous information, the following step is to linearize the equation, because the equation 6, contains non-linear elements, present in v_{rel} and $v_{s,los}$ due to the norm of the LOS vector used to compute those values. This linearization will be done through a first-order Taylor series expansion about an operative point, which will be a first guess of the receiver status, represented as x_o, y_o, z_o and $\delta \bar{t_{r,o}}$.

Step 4: When all the expressions have been linearized, the system is built and solved using an LS estimator. The solution is computed as follows.

$$\mathbf{b} = \mathbf{G} \cdot \mathbf{x} \tag{8}$$

$$\hat{\mathbf{x}} = (\mathbf{G}^\top \mathbf{G})^{-1} \mathbf{G}^\top \mathbf{b} \tag{9}$$

In the equation 9, $\mathbf{b}$ contains the difference between the range rate computed using the Doppler measurements, and the theoretical range rate computed at the operative point used to linearize the equation, as defined in 10. $\mathbf{G}$, also known as positioning matrix, is defined in the equation 11, and lastly $\mathbf{x}$ contains the difference between the operative point used and the optimal solution at the current iteration, as described in 12.

$$b = f_d \cdot \lambda - \dot{\rho}_o \tag{10}$$

$$\mathbf{G} = \begin{bmatrix} \frac{\partial \dot{\rho}_1}{\partial p_{r,x}} & \frac{\partial \dot{\rho}_1}{\partial p_{r,y}} & \frac{\partial \dot{\rho}_1}{\partial p_{r,z}} & 1 \\ \frac{\partial \dot{\rho}_2}{\partial p_{r,x}} & \frac{\partial \dot{\rho}_2}{\partial p_{r,x}} & \frac{\partial \dot{\rho}_2}{\partial p_{r,x}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \dot{\rho}_N}{\partial p_{r,x}} & \frac{\partial \dot{\rho}_N}{\partial p_{r,x}} & \frac{\partial \dot{\rho}_N}{\partial p_{r,x}} & 1 \end{bmatrix} \tag{11}$$

$$\hat{\mathbf{x}} = \begin{bmatrix} \Delta p_x & \Delta p_y & \Delta p_z & \Delta c\delta \bar{t}_r \end{bmatrix}^\top \tag{12}$$

Once all these steps have been completed, we should go back to step 2, but using as the new operative point the previous operative point plus $\mathbf{x}$, and all the procedure has to be repeated until the values of $\mathbf{x}$ don't change concerning the previous iteration, or the variations are lower than a certain threshold. A pseudocode presenting the complete algorithm is shown below.

Algorithm 1 Doppler positioning algorithm

```
[rxT, shift, fc, sat] = getMeas();
nIt = 0;
while x > threshold and nIt < maxIt do
    G = [];
    b = [];
    while i < length(sat) do
        txT = estTxTime(initPoint);
        [pSat, vSat] = SGP4(sat[i], tle);
        b(i) = buildB(shift, fc, pSat, vSat);
        G(i, :) = buildG(initPos, pSat, vSat);
        i = i + 1
    end while
    x = solveLS(G, b);
    initPos = initPos + x;
    nIt = nIt + 1;
end while
```

V. Results

To test how the noise affects the position estimation, we've added noise to the Doppler measurements with a certain standard deviation, σ, from 0.1 Hz to 5 Hz in steps of 0.1 Hz, performing 2000 experiments for each standard deviation, and then analyzing the vertical and horizontal RMSE.

This experiment will be performed in three cases, using four, five and nine satellites. The satellites used for these tests are shown in Fig. 2. For the four satellites case, the ONEWEB-0366, 0670, 0664, and 0644, have been used, for the five satellites case, the ONEWEB-0188, placed near the receiver zenith, has been added to the previous satellites, and finally, for the last case, all the SVs in Fig. 2 have been used, achieving a better geometry.

Before commenting on the results obtained in the previous experiments, we should know which values we expect to obtain, based on the geometry of the 4, 5 and 9 satellites used.

To compute these values, we should build the DOP matrix, using the previously defined $\mathbf{G}$ matrix in 11. This matrix has units of $\frac{1}{s}$ for the three first columns and non-dimensional values in the fourth column. To convert the three first columns to non-dimensional units, we should apply a rescaling factor to those three columns. As explained in [7], this rescaling can be done by dividing the first three columns by γ, where γ has units of $\frac{1}{s}$ and is the upper bound of those values. This rescaling factor is described in the equation 13, where R_E is the radius of the earth, a_{orb} is the altitude of the satellite orbit from the center of the earth, and the right-hand side term is the mean motion of the satellite in radians per second.

$$\gamma = \frac{1}{1 - \frac{R_E}{a_{orb}}} \sqrt{\frac{\mu}{a_{orb}^3}} \tag{13}$$

Once the matrix has been rescaled, we can compute the DOP matrix as in 14, where $\mathbf{G_R}$ is the rescaled matrix.

$$\mathbf{DOP} = (\mathbf{G_R}^\top \cdot \mathbf{G_R})^{-1} \tag{14}$$

Finally, having the DOP matrix with dimensionless values, we can obtain the HDOP and VDOP factors, and by multiplying them by the error introduced in the measurements, the wavelength used, and dividing the result by the γ factor, we can obtain the expected theoretical error in meters.

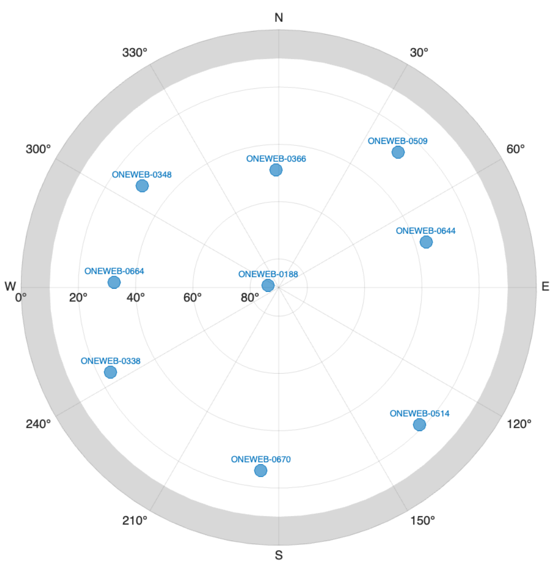

Fig. 2: Satellites used in experiment 1

In Fig. 3 can be seen the evolution of the horizontal RMSE and the vertical RMSE, with respect to the standard deviation of the noise added, as well as the theoretical error, obtained using the G_R matrix and the error applied.

For each of the three cases discussed above, two plots are shown, the experimental one, and the theoretical one. In some cases, the difference between the theoretical error and the experimental error is very small, but in other occasions, the difference is larger. This effect is due to the use of different matrices, since in the experimental results, 200 iterations for each sigma are performed, obtaining 200 different G_R matrices, but only one is used to obtain the theoretical error.

Regarding positioning errors, using four satellites, the minimum number of vehicles needed to estimate a solution, we can see how the evolution of the horizontal error goes from zero, when there is no error in the measurements, up to more than a thousand meters of error for 5 Hz standard deviation error, while in the vertical domain, the error goes from zero to around 350 meters.

By adding a fifth satellite, located at the zenith, we can appreciate that the impact on the error is very small. But using the total amount of vehicles, the improvement, especially in the horizontal error, is remarkable, passing from around 1000 meters, for a standard deviation of 5 Hz, down to about 400 meters.

The second test is intended to check how far the initial guess can be, for the algorithm to keep converging, taking into account the small footprint of a LEO satellite, compared to a GNSS satellite. The positions used are variations of the true position, where the measurements were taken, but shifted to the north. In the results of this test, we can check how many iterations it takes to converge into a reliable position estimation for each initial point, and from which distance the algorithm does not converge anymore.

Tab. II: Number of Iterations to Convergence

Distance	# Iterations	Distance	# Iterations
1 km	4	75 km	5
5 km	5	100 km	6
10 km	5	250 km	6
25 km	5	500 km	8
50 km	5	>500 km	Don't converge

As can be seen in Tab. II, the greater the initial guess distance to the true position, the more iteration we do need to converge, but this distance has a limit, which seems to be 500 km. In this last case, the lack of convergence is due to the method used to compute the transmission time, which, for great differences, diverges, getting out of the simulated time, and not providing a reliable transmission time.

VI. Conclusions

In this document, a constellation simulation tool is proposed, that can be useful for both, analyzing the characteristics of received signals, and gathering information to perform simulation-based experiments, allowing the user to experiment with both real and artificial constellations, thanks to the capability to introduce any TLE file.

A Doppler positioning technique has been also presented, which uses the data provided by the previous tool to estimate the user's position, taking advantage of the inertial coordinate frame to avoid having to take into account the Sagnac effect and using the complete Doppler shift expression, which will

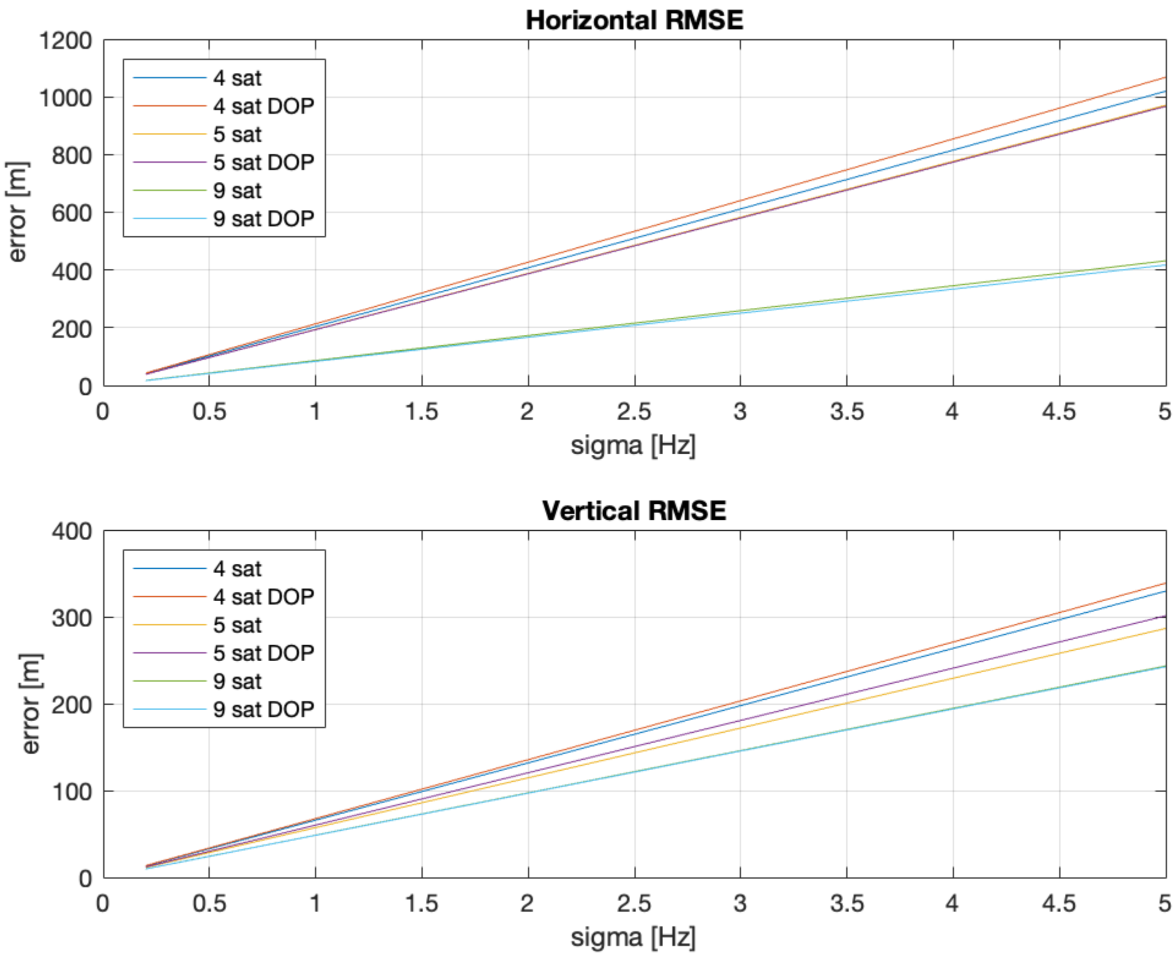

Fig. 3: Horizontal and vertical RMSE vs measurement noise standard deviation.

be necessary for future high-accuracy Doppler positioning methods.

As seen in previous sections, the performance of this method is very sensitive to Doppler measurement errors, but is also quite remarkable how the error is reduced drastically when more satellites are used. At this point, a wide variety of possible improvements can be applied like using weighted estimators, or more advanced techniques like Kalman filters. Another future work proposal is to estimate not only the user position and clock drift but also the user velocity, which in this work is assumed to be known.

Acknowledgments

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