SUBOPTIMAL METHOD FOR PILOT AND DATA POWER ALLOCATION IN COMBINED POSITIONING AND COMMUNICATIONS OFDM SYSTEMS

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ABSTRACT

The use of multicarrier signals for combined positioning and high data rate communications systems requires accurate estimation of timing offset and channel impulse response in order to achieve desirable performance. Previous work has investigated optimal pilot structures for joint timing offset and channel impulse response estimation, and the capacity maximizing pilot and data power allocation when taking only channel estimation into account. We study the problem of capacity maximization in an OFDM system when a certain time-delay estimation accuracy is required, thus taking into account the capabilities of the designed signal for positioning.

I. INTRODUCTION

As evidenced by the efforts to use or adapt present multicarrier communications signals for their use in positioning [1], [2], there exists an increasing interest in combined communications and positioning systems. The latest generation communications standards include signal configurations specifically targeted at positioning, such as LTE's positioning reference signal (PRS) [3]. However, the signals designed for communications can achieve very high data rates (tens to hundreds of Mbits per second), but are not adequate for accurate positioning. Therefore, the design of combined positioning and communications systems that can perform well in both domains is a topic that deserves further study, and requires the problem to be formulated taking into account all the parameters that affect both the system's attainable data rate and positioning accuracy.

Wireless multicarrier systems already rely on pilot symbols for channel estimation and, thus, it seems reasonable to apply these pilot symbols for time-delay estimation as well (which is key for positioning). Optimal pilot design for channel estimation has been studied extensively in the literature [4], [5], [6], and results show that equi-spaced and equi-powered pilots are optimal in terms of mean-square error. In [7], [8], it was shown that for a pilot-only multicarrier signal (i.e. when any subcarrier can be used to transmit pilot symbols), minimizing the variance of the time-delay estimate requires maximization of the root-mean-square (or Gabor) bandwidth of the signal. In [9], the problem of pilot design is considered for combined time-delay and channel estimation in OFDM signals. The design requires a trade-off between both estimation accuracies, but in general the obtained optimal pilot structures require the subcarriers at the edges of the bandwidth to be used for pilot transmission as well as somewhat equi-spaced pilot subcarriers in between.

This paper deals with the problem of finding the capacity maximizing pilot and data power allocation for OFDM systems, while taking into account the capabilities of the designed signal for positioning. This corresponds to a scenario where a certain positioning capability is desired (i.e., we fix a desired time delay estimation accuracy), and the system must be designed to achieve the highest possible capacity, while guaranteeing this positioning accuracy. Capacity maximization for ODFM signals using optimal pilot and data power allocations was also discussed in [4], [5], but in these papers only the use of the pilots subcarriers for channel estimation was considered. As our results show, the optimal allocations for capacity maximization greatly change when time-delay estimation accuracy is taken into account.

The paper is organized a follows. Section II presents the scenario and system model. Section III analyzes the problem of capacity maximization under a constraint on the time-delay estimation accuracy, formulated as a maximization of a lower bound on capacity when the effects of channel estimation accuracy are taken into account. Simulations and numerical examples are presented in Section IV, before drawing the conclusions in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

II-A. OFDM Signal Model

Consider the following frequency selective channel model,

$$h\left(t\right)=\sum_{l=0}^{L-1}h_l\delta\left(t-lT_s-\tau_d\right) \qquad (1)$$

where L is an upper bound on the number of discrete multipath components, h_l is the complex channel gain for the l-th path, T_s is the sampling period and τ_d is the time-delay between source and receiver. Note that since the delay is explicitly modeled inside the terms $\delta\left(t - lT_s - \tau_d\right)$, the channel coefficients $\{h_l\}$ are independent of tau_d.

In our case, both the channel coefficients $\{h_l\}$ and the channel delay τ_d need to be estimated through the use of pilot tones transmitted as part of the N subcarriers in an OFDM symbol. For the scope of this paper, we restrict our problem to the zero inter-carrier and inter-symbol interference case. This means that the carrier frequency must be perfectly synchronized at both the transmitter and receiver, and that the duration of the cyclic prefix (CP) T_G is larger than the delay spread plus the time uncertainty (i.e. $(L-1)\,T_s + \tau_d < T_G$).

Consider now the vector containing the DFT of N samples of the received signal collected during an OFDM symbol,

$$\mathbf{y} = \mathbf{\Gamma}\left(\tau\right)\mathbf{S}\left(\mathbf{s}\right)\mathbf{W}_L\mathbf{h} + \mathbf{n}, \tag{2}$$

where [9]

$$\begin{aligned}\mathbf{\Gamma}\left(\tau_d\right) &= \operatorname{diag}\left(\left[e^{-j2\pi\frac{-N/2+1}{N}\tau_d},\, e^{-j2\pi\frac{-N/2+2}{N}\tau_d}\right.\right. \\ &\qquad \left.\left.\ldots,\, e^{-j2\pi\frac{N/2}{N}\tau_d}\right]\right) & (3)\\ \mathbf{s} &= \left[s_{-N/2+1},\, s_{-N/2+2},\, \ldots,\, s_{N/2}\right]^T & (4)\\ \mathbf{S}\left(\mathbf{s}\right) &= \operatorname{diag}\left(\mathbf{s}\right) & (5)\\ \mathbf{h} &= \left[h_0,\, h_1,\, \ldots,\, h_{L-1}\right]^T. & (6)\end{aligned}$$

$\mathbf{W}_L$ is composed of the first L columns of the zero-frequency centered $N \times N$ Fourier matrix, $\mathbf{n}$ is a vector of additive Gaussian noise, and $\mathbf{s}$ contains the symbols being transmitted.

II-B. The Cramér-Rao Bound

In this section we describe the signal model and the performance metrics used for the design of the OFDM power distributions. Namely, we use two performance metrics: a lower bound on the channel capacity when there is uncertainty in the channel state information, and the Cramér-Rao Bound (CRB) of the joint time-delay and channel estimation.

II-C. The Cramér-Rao Bound

We define the following parameter vector,

$$\mathbf{\Theta} := \left[\tau_d,\, \Re\left\{\mathbf{h}^T\right\},\, \Im\left\{\mathbf{h}^T\right\}\right] \in \mathbb{R}^{(2L+1)\times 1}, \tag{7}$$

which contains the time-delay information and the channel response.

The CRB matrix with respect to the estimation of $\mathbf{\Theta}$ from (2) can be obtained from the Fisher Information Matrix (FMI) by using Bang's formula. As shown in [9], the expression is:

$$\mathbf{CRB}_{\tau,\mathbf{h}} = \frac{\sigma_n^2}{2}\begin{bmatrix} \mathrm{CRB}_{11} & \mathbf{CRB}_{21}^T \\ \mathbf{CRB}_{21} & \mathbf{CRB}_{22}\end{bmatrix} \tag{8}$$

where

$$\begin{aligned}\mathrm{CRB}_{11}^{-1} &= \mathbf{h}^H\mathbf{W}_L^H\mathbf{P}\mathbf{D}^2\mathbf{W}_L\mathbf{h} \\ &\quad -\mathbf{h}^H\mathbf{W}_L^H\mathbf{P}\mathbf{D}\mathbf{W}_\mathbf{L}\left(\mathbf{Q}\right)^{-1} \\ &\quad \cdot\mathbf{W}_L^H\mathbf{D}\mathbf{P}\mathbf{W}_L\mathbf{h} & (9)\\ \mathbf{CRB}_{21} &= \begin{bmatrix} -\gamma_\tau^{-1}\Im\left\{\mathbf{q}\right\} \\ \gamma_\tau^{-1}\Re\left\{\mathbf{q}\right\}\end{bmatrix} & (10)\\ \mathbf{CRB}_{22} &= \left[\begin{matrix} \Re\left\{\mathbf{Q}^{-1}\right\} + \gamma_\tau^{-1}\Im\left\{\mathbf{q}\right\}\Im\left\{\mathbf{q}^T\right\} \\ \Im\left\{\mathbf{Q}^{-1}\right\} - \gamma_\tau^{-1}\Re\left\{\mathbf{q}\right\}\Im\left\{\mathbf{q}^T\right\}\end{matrix}\right. \\ &\qquad \left.\begin{matrix} -\Im\left\{\mathbf{Q}^{-1}\right\} - \gamma_\tau^{-1}\Im\left\{\mathbf{q}\right\}\Re\left\{\mathbf{q}^T\right\} \\ \Re\left\{\mathbf{Q}^{-1}\right\} + \gamma_\tau^{-1}\Re\left\{\mathbf{q}\right\}\Re\left\{\mathbf{q}^T\right\}\end{matrix}\right] & (11)\\ \mathbf{q} &= \mathbf{Q}^{-1}\mathbf{W}_L^H\mathbf{P}\mathbf{D}\mathbf{W}_L\mathbf{h} & (12)\\ \mathbf{Q} &= \mathbf{W}_L^H\mathbf{P}\mathbf{W}_L & (13)\\ \mathbf{D} &= \frac{2\pi}{T_s}\operatorname{diag}\left(\left[-\frac{N}{2}+1,\, \ldots,\, \frac{N}{2}\right]\right) & (14)\\ \mathbf{P} &= \operatorname{diag}\left(\mathbf{p}_p\right) = \mathbf{S}^H\mathbf{S}. & (15)\end{aligned}$$

For the derivation of (8) we have assumed that the vector $\mathbf{s}$ only contains pilot symbols, and thus vector $\mathbf{p}_p$ contains the power of the pilots assigned to each subcarrier. Note that the CRB depends only on the power assigned to each of the pilot subcarriers.

II-D. Capacity

In [10] a lower bound on the channel capacity for serial transmissions over flat-fading channels was derived. We apply this lower bound on a per-subcarrier basis, and summing across the data subcarriers, we have that the lower bound on the multicarrier signal channel capacity is

$$C \geq C_{\mathrm{lb}} = \frac{1}{N}\sum_{i\epsilon\Omega}\log\left(1 + \mathrm{SNReq}_i\right), \tag{16}$$

where Ω contains the indices of the subcarriers allocated for data transmission, and where an equivalent signal to noise ratio for subcarrier i is defined as

$$\mathrm{SNReq}_i = \frac{g_i p_{d,i}}{\sigma_{g,i}^2 p_{d,i} + \sigma_n^2}, \tag{17}$$

where g_i is the channel response estimate for subcarrier i, $p_{d,i}$ is the power allocated to data transmission in subcarrier i, $\sigma_{g,i}^2$ is the variance of the carrier response estimate and σ_n^2 is the Gaussian noise power. The inequality in (16) becomes an equality only in the worst case, in which channel estimation uncertainty has the worst possible effect [10].

Note that for perfect channel state information (CSI) we have $\sigma_{g,i}^2 = 0$, and the expression (17) coincides with the signal to noise ratio when only AWGN is present. We can define the capacity for perfect CSI as

$$C_{\mathrm{csi}} = \frac{1}{N}\sum_{i\epsilon\Omega}\log\left(1 + \mathrm{SNR_i}\right) \tag{18}$$

where $SNR_i = {g_i p_{d_i}}/{\sigma_n^2}$.

For an unbiased estimator, the value of $\sigma_{g_i}^2$ is lower bounded by the Crámer Rao Bound of the channel frequency response estimate for subcarrier i, which can be computed as a function of parameters from (11):

$$\mathbf{CRB_g} = \mathbf{T}\,\mathbf{CRB_h}\mathbf{T}^H, \tag{19}$$

where

$$\mathbf{T} = \begin{bmatrix} \Re\{\mathbf{W}_L\} & -\Im\{\mathbf{W}_L\} \\ \Im\{\mathbf{W}_L\} & \Re\{\mathbf{W}_L\} \end{bmatrix}. \tag{20}$$

In this paper, we consider the maximization of the lower bound on the capacity assuming the lowest possible uncertainty in the channel estimates is achieved. Thus, we are going to work with

$$C'_{\mathrm{lb}} = \frac{1}{N}\sum_{i\epsilon\Omega}\log\left(1+\frac{g_i p_{d,i}}{[\mathbf{CRB}_g]_{ii}p_{d,i}+\sigma_n^2}\right). \tag{21}$$

III. POWER ALLOCATION BASED ON THE OPTIMIZATION OF THE LOWER BOUND ON THE CAPACITY

In order to maximize the lower bound on the capacity (21) while ensuring a certain level of positioning capabilities (i.e. a certain level of time-delay estimation accuracy), we need to optimally distribute pilot and data power across the subcarriers.

One possible formulation of the problem is

$$\begin{cases} \max\limits_{\mathbf{p},\mathbf{b}} & C'_{\mathrm{lb}}(\mathbf{p},\mathbf{b}) \\ \text{s.t.} & \mathrm{CRB}_{11}(\mathbf{p},\mathbf{b}) \le \beta \\ & \mathbf{p}^T\cdot\mathbf{1} \le P_T \\ & \mathbf{p} \ge \mathbf{0} \\ & b_i^2 - b_i = 0 \end{cases} \tag{22}$$

where vector $\mathbf{p}$ contains the powers assigned to each of the subcarriers ($\mathbf{p} = \mathbf{p}_p + \mathbf{p}_d$), $\mathbf{b}$ is binary vector of length N than contains a '1' in the position corresponding to pilot subcarriers and a '0' in the positions corresponding to data subcarriers, and P_T is the total available power. Note that $\mathbf{p}_p$ is a vector that contains the powers assigned to pilot symbols in each subcarrier ($p_{p,i} = b_i p_i$) and $\mathbf{p}_d$ contains the power of the subcarriers devoted to data transmission ($p_{p,i} = p_i(1-b_i)$).

For a given subcarrier allocation (i.e. for fixed $\mathbf{b}$), the function C'_{lb} we are trying to maximize appears to be concave in $\mathbf{p}$ through our extensive experimental test campaign (an analytical proof of its concavity remains as future work). That means that given $\mathbf{b}$, the optimization problem is convex and it is easily solvable using standard numerical methods. However, in order to find the global optimum power distribution, one must test all the possible pilot and data subcarrier assignments, which is only feasible for few subcarriers, although it is possible to apply some combinatorial optimization algorithms in order to reduce the amount of computation involved.

Note that the optimization problem (22) could be easily modified to account for the case where we do not take the effect of channel estimation uncertainty into account. In that case, the cost function would be C_{csi} instead of C'_{lb}. We justify our decision to use a lower bound formulation in Section IV.

III-A. Relaxed Problem Solution

As explained in above, the problem of power distribution for capacity optimization can be numerically solved when the assignment of the subcarriers to data or pilot symbols is fixed. However, there is no simple way to find the optimal subcarrier assignment.

One possible way to tackle the problem is to solve a relaxed version of the problem that eliminates its combinatorial nature. If we relax the fouth restriction in (22), a given subcarrier could be shared by data and pilot symbols. This relaxation is equivalent to allowing each of the components of vector $\mathbf{b}$ to take values between 0 and 1 instead of a binary value.

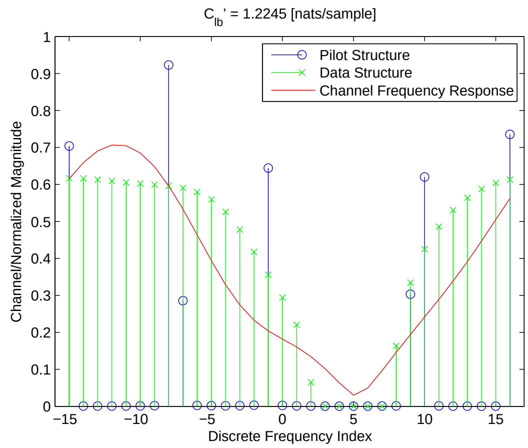
The solution to this relaxed problem can be found numerically by using a a standard convex programming approach which is guaranteed to converge to an optimal solution. As we will see in Section IV, simulations results show that the number of subcarriers that contain pilot power is very low, generally equal to or very close to $L+1$, which is the lower bound required for the FMI to be full rank. This was an expected result as it corresponds with the results obtained in [9], where the pilots were designed under a CRB minimization criteria. On the other hand, the data power distribution that is obtained closely resembles that of a traditional water-filling.

III-B. Approximation to a fixed assignment solution

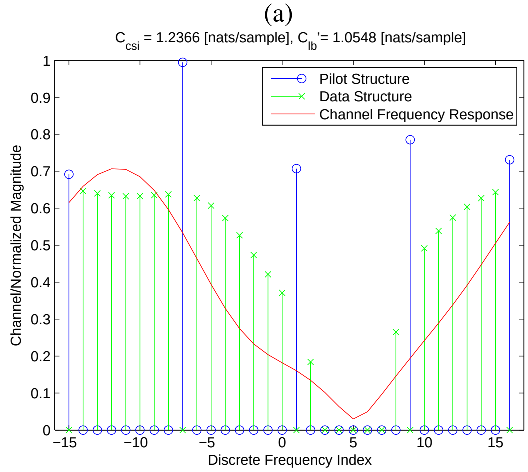
Taking into account the results obtained for the relaxed problem, if we go back to the formulation presented in (22), we can see that optimal solutions for the relaxed problem consist of a vector $\mathbf{b}$ with most of its entries set to '0' and just a few of them are set to values greater than '0'. Based on this, we propose the following method to find a solution close or equal to the global optimum. After solving the relaxed problem, one could approximate the almost binary vector $\mathbf{b}$ to a completely binary vector. As previously discussed, the maximization problem in (22) can be solved easily if the subcarrier assignment is fixed. A gap in the optimum value obtained will exist between the relaxed solution and the solution with a fixed subcarrier assignment.

IV. SIMULATION RESULTS

In this section we compute several power distributions obtained as a solution to the optimization problems discussed in Section III. For all the results presented, we assume a channel impulse response of length $L=4$ and an OFDM signal of $N=32$ subcarriers. Total power for both pilots and data has been fixed to $P_T=5$ and noise variance to $\sigma_n^2 = 10^{-2}$. The maximum value of CRB_{11} has been constrained to $\beta = \cdot 10^{-3}$ and channel impulse response $\mathbf{h} = [0.3802 +$



(a)



(b)

Fig. 1. Pilot and data power distributions under different conditions: (a) C'_{lb} maximizing distribution (relaxed problem). (b) C'_{lb} maximizing distribution (fixed subcarrier assignment).

$j0.2254,\ 1.2968 - j0.9247,\ -1.5972 - j0.3066,\ 0.6096 + j0.2423]^T$ has been used.

IV-A. Power Allocation

Figure 1 shows the optimal data and power distribution that achieves the maximum possible lower bound on capacity C'_{lb} under different constraints. Figure 1(a) shows an example of the optimal solution for the relaxed case. Note how the subcarriers allocate both pilot and data powers. However, most of the subcarriers are just allocated to data transmission and only a few of them are allocated to pilot power. If we fix the subcarriers with more power allocated to pilot transmission to transmit only pilots and leave the rest of the subcarriers free for data transmission, the power allocation obtained is the one shown in Figure 1(b). Note that in both the cases the pilot power seems to be more or less evenly distributed among the pilot subcarriers, while data power assignment is a water-filling type distribution. As expected, in terms of the lower bound achieved, the solution to the relaxed problem outperforms the solution to the problem with a fixed assignment.

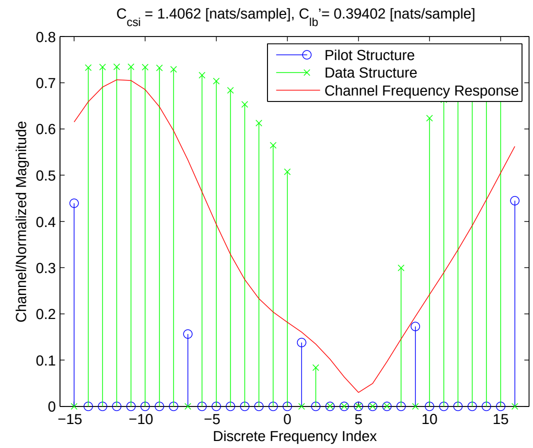


Fig. 2. C_{csi} maximizing pilot and data power distributions (fixed subcarrier assignment).

One might wonder how necessary it is to take channel state information uncertainty into account when designing the power distribution of an OFDM symbol. Figure 2 shows the power distribution obtained for the problem of maximizing capacity for a fixed subcarrier assignment when perfect knowledge of the CSI is available (C_{csi} is considered the cost function for this optimization problem). The pilot distributions obtained concentrate most of their power in the edges of the bandwidth, a result that corresponds with the time-delay estimation optimization as seen in [9]. The pilot power is being used just to fulfill the restriction on the time-delay estimation accuracy, while the rest of the power is used for data transmission. However, in most wireless systems CSI has to be obtained from the pilot structure. If we compute the lower bound on capacity taking into account the effect channel estimation uncertainty, the value obtained (0.39 nats/sample) is noticeably lower than in lower-bound-based design (Figure 1 (b), 1.05 nats/sample).

IV-B. Channel Capacity

For the channel described above, Figure 3 studies the trade-off of capacity versus time-delay estimation uncertainty under different assumptions. For the relaxed problem formulation we have:

- The dash-dot curve with point markers shows the lower bound on capacity, C'_{lb}, achieved by distributions obtained through the maximization of C'_{lb}.
- The dash-dot curve with cross markers corresponds to the capacity computed considering perfect CSI, C_{csi}, achieved by distributions obtained as a solution to the maximization of C_{csi}, i.e., without taking into account the effect of channel estimation uncertainty on capacity.
- The dash-dot curve with square markers represents the values of C'_{lb} achieved by power distributions designed to maximize C_{csi}.

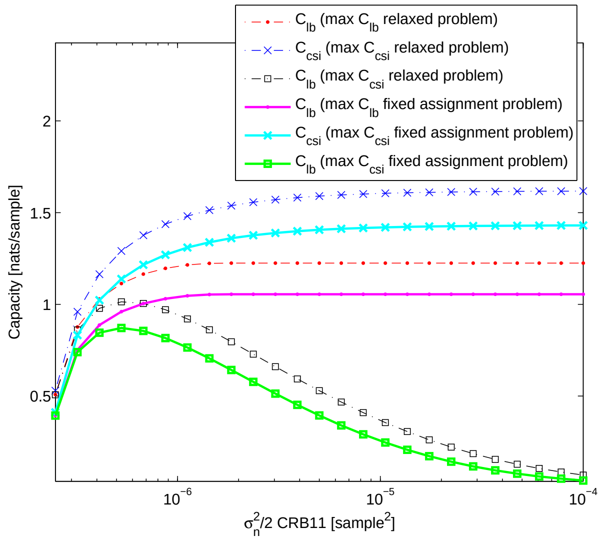


Fig. 3. Channel capacity versus time-delay estimation accuracy.

Similarly, for the case with fixed subcarriers assignment, we have:

- The solid line with point markers represents the C'_{lb} achieved by distributions obtained through the maximization of C'_{lb}.
- The solid line with cross markers shows the C_{csi} achieved by power distributions designed to maximize C_{csi}.
- The solid line with square markers shows the values of C'_{lb} achieved by power distributions designed to maximize C_{csi}.

The results show the importance of taking channel estimation uncertainty into account, as the power distributions designed without taking it into account (the ones designed using C_{csi} as a cost function) perform poorly in terms of the lower bound they achieve. This is due to the fact that pilot structures are designed only to fulfill the restriction on the time-delay, and don't work well for channel estimation, which in turn has a great impact in capacity.

It is also interesting to note the gap existing between the values achieved by the relaxed problem solutions and the solutions obtained for a fixed assignment. The gap produced by constraining the subcarriers to only allocate one type of symbol (pilot or data) is approximately a 14% reduction for the distributions designed with the lower bound C'_{lb} as a cost function.

V. CONCLUSIONS

In this paper, we have considered the problem of power and subcarrier allocation for capacity optimization with a constraint on the time-delay estimation accuracy of the signal. We have discussed the optimal solutions to a relaxed version of the problem and showed how these solutions may be used to find close-to-optimal power distributions. Moreover, our decision to take into account channel estimation accuracy in the computation of capacity has been validated. As our results show, by using pilot and data subcarrier assignments obtained from the solutions to a relaxed version of the problem, we can obtain a suboptimal pilot and data power distribution that may be expected to perform close to the optimal one.

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