

Unified Framework for Flexible Multi-Carrier Communication Systems

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Abstract—The design of efficient architectures for Multi-Carrier (MC) communication systems can be a challenging task when it involves the adoption of Filter-Bank Multi-Carrier (FBMC) modulation techniques. The main design issues found in FBMC systems are generated by the incorporation of band-limited shaping pulses. Yet, these pulses have many advantages in terms of performance, such as providing either improved spectral confinement or no frequency overlap between adjacent subcarriers. They also benefit from robustness in front of narrowband interferences, and reduced out-of-band power emission, among others. However, the advantages of FBMC schemes are often obscured when it comes to the implementation point of view. This is particularly true for the flexible FBMC systems, where on top of incorporating band-limited shaping pulses, no restrictions are imposed on the signal parameters (*i.e.* symbol rate, carrier spacing or sampling frequency). In this context, the present paper will provide a unified framework to describe flexible FBMC signals when both signal design and implementation criteria are combined.

I. INTRODUCTION

Currently there is a growing interest in Filter Bank Multi-Carrier (FBMC) systems, particularly for applications involving digital subscriber lines [1], wireless communications [2] or cognitive radio, just to mention a few. The main advantage of FBMC with respect to traditional Orthogonal Frequency Division Multiplexing (OFDM) resides in the replacement of the rectangular shaping pulse by a band-limited (non-rectangular) one. This property provides a more robust system performance in front of carrier frequency mismatches and narrowband interferences. Moreover, FBMC signals can also be designed so as to preserve the subcarrier orthogonality without requiring the insertion of a cyclic prefix, in contrast to what occurs in OFDM. This advantage, together with the reduction of out-of-band power emissions, may lead to FBMC signals with a higher spectral efficiency than conventional OFDM. It is for this reason that FBMC is currently being considered for future software defined radio (SDR) and spectrally agile communication systems.

Different variations of FBMC schemes can be found in the literature, such as Filtered Multi-Tone (FMT), Cosine Modulated Multi-Tone (CMT), Discrete Wavelet Multi-Tone (DWMT) or Offset Quadrature Amplitude Modulated OFDM (OQAM/OFDM). However, each of these variations uses a specific and case-dependant signal model, thus making it difficult to perform a fair comparison among the possible alternatives. Attempts have been made to provide a general and unified formulation for the family of filterbank architectures. Most contributions are circumscribed to the field of digital signal processing, where analysis and synthesis filterbanks are widely adopted for speech coding or image compression [3]. The application of these results to the field of digital communications is not straightforward, since the conceptual approach is completely different (*i.e.* analysis and synthesis operations are exchanged) and new signal parameters, design constraints and performance metrics do appear. It is for this reason that current research efforts are

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being devoted to provide a suitable and generalized formulation for communications-oriented filterbank architectures [4]. The few existing proposals do encompass different multi-carrier modulations such as OFDM, DWMT and OFDM/OQAM, but they restrict some of the signal parameters, which cannot be freely adjusted. Such a limitation is a serious drawback for the case of flexible FBMC systems, where no constraints are imposed on the relationship between the different signal parameters. This is the case of Non Orthogonal Frequency Division Multiplexing (NOFDM), where neither the number of subcarriers, their spectral spacing, the shaping pulse (and its length), nor the symbol rate are specified [5]. Such a flexible scheme is an interesting approach that allows to freely optimize the transmitted signal so as to fulfill some predefined criteria in terms of out-of-band radiation, power/bandwidth efficiency, physical-layer security or synchronization performance.

In this context, the present contribution is intended to cover the gap between existing generalized FBMC formulations and the requirements of emerging flexible FBMC communication systems. A unified framework is proposed first, where any FBMC signal is mapped onto a quadruple of key parameters. Some multi-rate techniques and basic filter-bank theory are reviewed in order to support the derivation of flexible FBMC architectures as a function of the signal's key parameters. Then, implementation guidelines are provided through the extensive use of polyphase filters. Finally, equivalent efficient transmitter architectures are presented for different types of polyphase network layouts and signal parameter sets.

II. SIGNAL MODEL

In this section, we formulate a signal model that is flexible enough to encompass all existing MC signal formats by properly selecting the values of a few key parameters. Thus, if we have a scheme to generate this generic signal for any set of parameters, we will have a unique architectural framework that can be particularized to generate any MC signal. Let us consider the following continuous-time baseband equivalent model for a Flexible MC signal formed by N subcarriers with a frequency separation of $F_0 \doteq 1/T_0$

$$x(t) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} s'_n(l) g(t - lT) e^{j2\pi n \frac{t}{T_0}} e^{j\varphi_n(l)}, \quad (1)$$

where $s'_n(l)$ are the symbols to be transmitted (in general, $s'_n(l) \in \mathbb{C}$), $g(t)$ is the shaping waveform, $R = 1/T$ is the signaling rate (*i.e.*, T is the MC symbol period), and $\varphi_n(l)$ is a possibly additional phase term used in some cases to ensure that the symbols are separable at the receiver. For instance, in OQAM-OFDM a 90° rotation is alternatively applied in the frequency and time dimensions (which are represented by indexes n and l , respectively) to force that the symbols adjacent to a real one are imaginary, and vice versa. In order to simplify the notation, we can gather the symbols and the additional phases into an equivalent symbol term $s_n(l) = s'_n(l) e^{j\varphi_n(l)}$. The model in (1) can also be used to represent offset modulations; it is only necessary to interpret T as half of the actual symbol period.

The analog signal propagating through the channel is evidently independent of any sampling frequency. However, we are interested in transmit digital architectures, so we formulate the discrete-time version of (1) sampled at a rate $F_s \doteq 1/T_s$:

$$x[m] \doteq x(mT_s) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} s_n[l] g[m - lN_{ss}] e^{j2\pi n \frac{m}{P}}, \quad (2)$$

where the fundamental subcarrier discrete-time period (*i.e.*, expressed in samples) is $P = F_s/F_0$, and $N_{ss} = F_s T = F_s/R$ the number of samples per MC symbol. The discrete-time shaping pulse $g[k] \doteq g(kT_s)$ (so called *prototype filter*) has a length of L_g samples, and parameters P and N_{ss} are considered to be integer values. Note that this does not entail any loss of generality because (2) is a model at transmission, where there is a perfect control of the MC symbol rate and subcarrier frequency relative to the sampling rate regardless of any absolute bias in the transmitter frequency reference.

One of the main characteristics of MC modulation systems resides on the fact that a total of N source symbols $s_n[l]$ are involved in the generation of a single MC symbol. Normally, each $s_n[l]$ will be associated to an individual subcarrier frequency so the signaling rate of each subcarrier will be at least N times lower than the original input signaling rate. Such a decrease in the signaling rate leads to longer symbol periods, thus mitigating undesired channel effects like fast fading due to multi-path propagation and increasing the robustness of the communication system.

The format of an FBMC signal is uniquely defined by four parameters: N, N_{ss}, P, L_g or combinations thereof. In particular, we choose the quadruple:

$$\{N, D, Q, L'_g\} \doteq \{N, N_{ss}/N, N_{ss}/P, L_g/P\}. \quad (3)$$

The flexibility of the proposed model comes from the fact that any MC signal can be represented by specific values of these parameters. The value of $Q = F_0 T$ represents the subcarrier spacing normalized to the symbol rate. The minimum spacing that makes subcarrier orthogonality possible corresponds to the case of $Q = 1$. Since Q can take non-integer values, the model is also valid for the representation of Non-Orthogonal or Generalized MC signals. The parameter D can be regarded as an oversampling factor; and $D = 1$ corresponds to the so-called *critical sampling* condition.

Some illustrative examples are the paradigmatic case of OFDM with N subcarriers. It is sampled at N samples per symbol and characterized by $\{N, D, Q, L'_g\} = \{N, 1 + \beta, 1 + \beta, 1 + \beta\}$. A fraction β of the symbol time is devoted to the cyclic prefix. If a null guard interval is used instead of the cyclic prefix, then the representation is $\{N, 1 + \beta, 1 + \beta, 1\}$. Let us consider a FBMC signal with square-root raised-cosine shaping pulses of roll-off factor α (whose effective length is limited to L_g) that overlap each other in frequency at the half-amplitude point. If the signal is critically sampled, the representation is $\{N, 1, 1, L_g/N\}$. In case the pulses do not overlap, the signal format corresponds to the FMT case, whose characteristic quadruple is $\{N, 1 + \alpha, 1 + \alpha, L_g/N\}$. This formulation has the advantage of making apparent that, for instance, FMT is very similar to OFDM with guard interval from an structural point of view, but FMT simply uses a longer pulse.

III. MULTIRATE PRELIMINARIES FOR FBMC MODULATIONS

Prior to the derivation of the efficient architectures proposed in this work it is necessary to review some concepts from the field of multi-rate digital signal processing and filter banks [6]. In fact, the processes of FBMC signal transmission and reception usually require rate conversion operations. Fig. 1 shows a typical FBMC transmission architecture that generates the transmit digital signal in (2). As it can be noticed, the convolution between each sub-band signal

and the prototype filter $g[n]$ is carried out at the highest sampling rate. Additionally, each convolution involves all the prototype filter coefficients and it is replicated for each branch. Therefore, it becomes apparent that there is room for improvement in the computational efficiency and memory resources usage of this architecture. Then, it is interesting to gain more insight in rate conversion operations and their interaction with digital filters. First of all, we are going to define the discrete-time signal $y[m]$ as the output of the convolution between the input signal $u[m]$ (upsampled by a factor B) and a digital filter $g[n]$ (*interpolation filter*):

$$y[m] = \sum_{l=-\infty}^{\infty} u[l] g[m - lB] = \mathcal{I}_B\{u[m]\} * g[m], \quad (4)$$

where the notation $\mathcal{I}_B\{\cdot\}$ denotes an upsampling operator by a factor B before the filtering operation. Analogously, we will be interested in the expression of a digital filter followed by a downsampling operation by B (*decimation filter*):

$$y[m] = \sum_{l=-\infty}^{\infty} u[l] g[mB - l] = \mathcal{D}_B\{u[m]\} * g[m], \quad (5)$$

being $\mathcal{D}_B\{\cdot\}$ the notation for digital signal downsampling by B . Likewise, it is also interesting to introduce the complementary cases

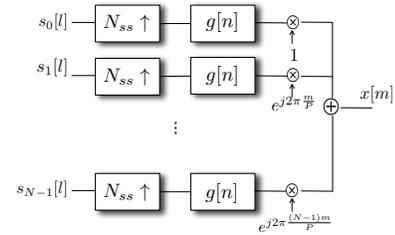


Fig. 1. General architecture for a flexible FBMC transmitter.

to the interpolation and decimation filters which correspond to the time-domain expression of a filter followed by an upsampling operation (6) and a filter preceded by a downsampling operation (8):

$$\tilde{y}[m] = \mathcal{I}_B\{y[m]\} = \mathcal{I}_B\left\{\sum_{l=-\infty}^{\infty} u[l] g[m - l]\right\}, \quad (6)$$

where:

$$\tilde{y}[m] \doteq \begin{cases} y\left[\frac{m}{B}\right] & \text{if } m = kB; \forall k \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

and

$$y[m] = \sum_{l=-\infty}^{\infty} u[lB] g[m - l] = \mathcal{D}_B\{u[m]\} * g[m]. \quad (8)$$

It should be noticed that these properties allow a direct matching between the conventional FBMC transmitter architecture (Fig.1) and the reference signal model in (2). For instance, a quick inspection of (2) reveals that the signal $x[m]$ is generated by adding up N different signals $s_n[l]$, being each one modulated by a different exponential term (subcarrier) and convolved with the interpolation filter $g[n]$ by a factor of N_{ss} (which is precisely what is shown in Fig. 1).

Furthermore, in this section we introduce the concept of *polyphase* structures which constitute a useful tool to provide efficient FBMC transmitters with reduced computational cost. Such structures allow to take advantage of the cyclic behavior of the MC signal complex exponentials. The prototype filter coefficients are grouped into subsets of samples (called *sub-filters*) according to the phase of the exponential term they are associated to in the convolution operation.

According to the polyphase theory [6], both the interpolation filter (4) and the decimation filter (5) can be decomposed into B polyphase

sub-filters. The coefficients of the i -th sub-filter ($i \in (0, 1, \dots, B - 1)$) are defined by the expression: $g_i[n] = g[nB + i]$ where B is considered herein as the order of the polyphase network. The obtained sub-filter $g_i[n]$ can be interpreted as a downsampled version (by a factor B) of the prototype filter with a *sampling offset* (or delay) of i samples. Hence, as a consequence of this re-arrangement, the rate of each polyphase component is B times lower than the serial signal $x[m]$. This is advantageous from an implementation point of view since it reduces the rate of operation of each individual branch.

IV. EFFICIENT MULTI-CARRIER TRANSMITTER ARCHITECTURES

Similarly to the MC signal formulation, efficient architectures have also been subject to unification efforts to provide a valid common framework under different design criteria [1], [3], [4], [7]. However, current hardware architectures do not exploit all the available degrees of freedom that flexible FBMC communication systems demand. Previous contributions have attempted to structure and homogenize the process of polyphase architecture definition. Efficient polyphase architectures have been proposed in [1], [7], [8], but they are restricted to FMT modulations. Other works like [9] do consider generic MC signal parameters but as in [1], the resulting architectures entail a time variation of the filter coefficients that requires a complex control and operation of memory buffers. Besides, these solutions do not solve the problem of relying on a time-variant architecture

In the present contribution, a truly unified framework has been derived in the form of a generic architecture for the implementation of flexible FBMC transmitters with unrestricted signal design parameters. This general architecture is schematically depicted in Fig. 2. Apart from the quadruple of key signal parameters described in Section 2, flexible FBMC architectures are mainly determined by two parameters: the order of the polyphase network B and the normalized frequency spacing factor Q . Typically, $B = \{\text{l.c.m.}(N, N_{ss}), L_g\}$ values have been adopted in the existing literature (where l.c.m stands for *least common multiple*). However, a more general approach suggests a wider range of possibilities. In particular, we consider the set of values $B = \{P, N_{ss}, \text{l.c.m.}(P, N_{ss})\}$ as the more representative cases for our study. The interpolation modules within Fig. 2 can easily accommodate integer values of Q . For rational values of Q , the role of these interpolation modules would lead to a time-variant input-output response of the polyphase network. Addressing such an extra complexity (usually avoided in practice) is one of the main contributions of this paper. In the first part of this

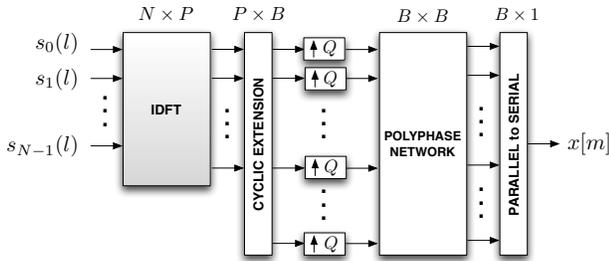


Fig. 2. General polyphase architecture for a flexible FBMC transmitter

section we introduce the architectures obtained for integer values of Q which are simpler in terms of implementation. Secondly, we focus on rational values of Q and study the implementation issues that arise with standard polyphase networks. Next, we provide a detailed derivation of alternative time-invariant polyphase architectures for different values of B .

A. Type A: Polyphase architectures for integer values of Q

The main advantages of the polyphase structures become apparent in this case since it is possible to obtain a polyphase network of

order P that minimizes the amount of required hardware. According to a given quadruple of values we can particularize the reference transmission signal model (2) as follows:

$$x[m] = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} s_n[l] g[m - lQP] e^{j2\pi n \frac{m}{P}}. \quad (9)$$

The P -points Inverse Discrete Fourier Transform (IDFT) operation over the source symbols can be directly inferred from (9). Assuming that $P \geq N$, we can define $\mathbf{s}[l] \triangleq [s_0[l], s_1[l], \dots, s_{N-1}[l]]^T$ and $S_{\text{mod}(m,P)}[l] \triangleq \text{IDFT}_{m,P}(\mathbf{s}[l]) = \sum_{n=0}^{N-1} s_n[l] e^{j2\pi n \frac{m}{P}}$, which leads to the following compact form of the signal model:

$$x[m] = \sum_{l=-\infty}^{\infty} S_{\text{mod}(m,P)}[l] g[m - lQP]. \quad (10)$$

The IDFT operation for MC modulations was initially introduced in [10] and it is truly convenient from an implementation point of view since it enables the use of Fast Fourier Transform (FFT) modules. The use of such modules is considered one of the catalysts in the success of multicarrier techniques and its widespread deployment in modern wireless communication systems during the last years. Considering the cyclic nature of the IDFT exponentials we introduce the following modulo operation in the signal model: $m = \text{mod}(m, P) + \lfloor \frac{m}{P} \rfloor P$. Then, we can re-write (10) as:

$$x[m] = \sum_{l=-\infty}^{\infty} S_{\text{mod}(m,P)}[l] g \left[\text{mod}(m, P) + \left(\left\lfloor \frac{m}{P} \right\rfloor - lQ \right) P \right]. \quad (11)$$

Given that $\text{mod}(m, P) \in (0, 1, \dots, P-1)$ we can interpret the signal obtained in (11) as a total of P digital convolution operations. Hence, in terms of polyphase decomposition we regard the term $\text{mod}(m, P)$ as a branch index that identifies the specific sub-filter involved in the filtering process. Moreover, since each sub-filter will operate at a sampling rate which is P times lower, we ought to apply a sub-filter decimation by a factor of P over the prototype filter $g[n]$. From this point on we propose the following notation to reflect the mentioned manipulations:

$$x[m] = \sum_l S_{\text{mod}(m,P)}[l] g_{\text{mod}(m,P)} \left[\left\lfloor \frac{m}{P} \right\rfloor - lQ \right] \quad (12)$$

$$= \mathcal{I}_Q \{ \text{IDFT}_{m,P}(\mathbf{s}[k]) \} * g_{\text{mod}(m,P)}[k] \Big|_{k=\lfloor \frac{m}{P} \rfloor}. \quad (13)$$

Therefore, the transmit signal $x[m]$ is generated as the result of the convolutions of each IDFT output $S_{\text{mod}(m,P)}[k]$ (upsampled by Q) and a downsampled version of the prototype filter $g_{\text{mod}(m,P)}[k]$ followed by an up-sampling operation by P . The resulting transmission architecture once the polyphase decomposition has been applied is depicted in Fig. 3.

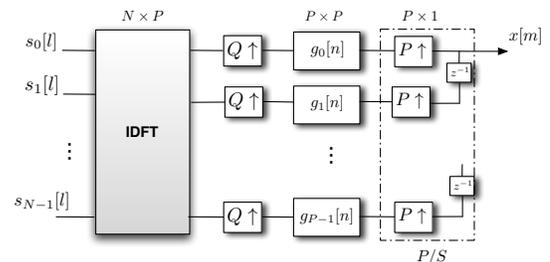


Fig. 3. General architecture for a flexible FBMC transmitter and an integer oversampling factor Q

B. Type B: Polyphase architectures for non-integer values of Q

It follows from (12) that in this case a rational Q up-sampling operation would be required prior to the sub-filter convolution, thus complicating the design of a time-invariant architecture. The main implementation obstacle is given by a rate imbalance between the symbol rate and the polyphase network output rate which is set by the order of the polyphase network. In particular, the output of a P -branch long polyphase transmitter generates blocks of P samples (one for each sub-filter output). However, the number of generated samples per symbol should be N_{ss} in order to meet the target output rate of the digital communication signal being transmitted. In other words, the symbol period in samples N_{ss} does not account for an integer number of periods of the fundamental subcarrier frequency, thus making it hard to exploit the cyclic nature of the FFT. Furthermore, since the duration of the symbol (in samples) is not a multiple of the order of the polyphase network, it becomes necessary to apply a different set of sub-filter samples to every symbol delivered by the IDFT block.

Despite these issues, we show in this work that if the polyphase network is conveniently designed it is certainly possible to obtain a time-invariant structure for any rational value of Q . It should be also noticed that the following architectures are essentially equivalent and they only differ in the layout of the polyphase network. The flexibility of the framework provided in this work is clearly highlighted by this fact since any of these schemes can be used indistinctly depending on the specific constraints of the application.

1) *Order of the polyphase network $B = P$* : let us express the index of the convolution l in (2) as $l = l_b P + l_r$ being $l_b \triangleq \lfloor \frac{l}{P} \rfloor$ and $l_r \triangleq \text{mod}(l, P)$. Then replacing it in (2) and according to (10):

$$x[m] = \sum_{l_r=0}^{P-1} \sum_{l_b=-\infty}^{\infty} S_{\text{mod}(m,P)}[l_b P + l_r] g[m - l_r N_{ss} - l_b P N_{ss}]. \quad (14)$$

Additionally, we can further decompose the term $m - l_r N_{ss} = \lfloor \frac{m - l_r N_{ss}}{P} \rfloor P + \text{mod}(m - l_r N_{ss}, P)$. Therefore, we can re-write $x[m]$ following a P -order sub-filter decimation:

$$\begin{aligned} x[m] &= \sum_{l_r=0}^{P-1} \sum_{l_b=-\infty}^{\infty} S_{\text{mod}(m,P)}[l_b P + l_r] \\ &\quad \cdot g_{\text{mod}(m-l_r N_{ss}, P)} \left[\left\lfloor \frac{m - l_r N_{ss}}{P} \right\rfloor - l_b N_{ss} \right] \quad (15) \\ &= \sum_{l_r=0}^{P-1} \mathcal{I}_{N_{ss}} \{ \mathcal{D}_P \{ S_{\text{mod}(m,P)}[k + l_r] \} \} \\ &\quad * g_{\text{mod}(m-l_r N_{ss}, P)}[k] \Big|_{k=\lfloor \frac{m-l_r N_{ss}}{P} \rfloor}. \quad (16) \end{aligned}$$

In this case there appears a delay of $l_r N_{ss}$ samples that affects each sub-filter convolution as well as the sub-filter indexes. Therefore, it is not possible to generate the transmit signal $x[m]$ with a single P -branches polyphase structure like the one shown in Fig. 3. However, it is certainly possible to consider separately the architecture defined by each value of l_r . This architecture can be interpreted as a hardware replica of a polyphase network of order P (being the index $l_r \in (0, 1, \dots, P-1)$ a replica index). The architecture defined by each l_r simply differs from the one in (12) in the introduction of a $l_r N_{ss}$ -samples delay that, according to the properties of the convolution, can be placed at the sub-filter outputs as shown in Fig. 4.

Moreover, it should be noticed in (15) that the branch index of the polyphase sub-filters and the IDFT output do not coincide. Then, it is necessary to introduce of a phase rotation over the input source symbols $s_n[l]$ in order to keep the phase continuity imposed by the transmitted signal model. According to the Fourier transform properties, we can define $\tilde{S}_{\text{mod}(m-l_r N_{ss}, P)}[l] \triangleq \text{IDFT}_{m-l_r N_{ss}, P}(\tilde{s}[l]) = \sum_{n=0}^{N-1} \tilde{s}_n[l] e^{j2\pi n \frac{m-l_r N_{ss}}{P}}$ where $\tilde{s}_n[l] = s_n[l] e^{j2\pi n \frac{l_r N_{ss}}{P}}$. Then,

$\tilde{S}_{\text{mod}(m-l_r N_{ss}, P)}[l]$ is defined as the IDFT of the source symbols array $\mathbf{s}[l]$ subject to a phase rotation that yields $\tilde{\mathbf{s}}[l]$. Finally, defining a replica-dependant row index $m(l_r) \triangleq m - l_r N_{ss}$ we are left with:

$$\begin{aligned} x[m] &= \sum_{l_r=0}^{P-1} \mathcal{I}_{N_{ss}} \{ \mathcal{D}_P \{ \text{IDFT}_{m(l_r), P}(\tilde{\mathbf{s}}[k + l_r]) \} \} \\ &\quad * g_{\text{mod}(m(l_r), P)}[k] \Big|_{k=\lfloor \frac{m(l_r)}{P} \rfloor}. \quad (17) \end{aligned}$$

It should be noticed that the phase rotation on the source symbols is constant for each replicated block. The resulting transmission architecture is depicted in Fig. 4.

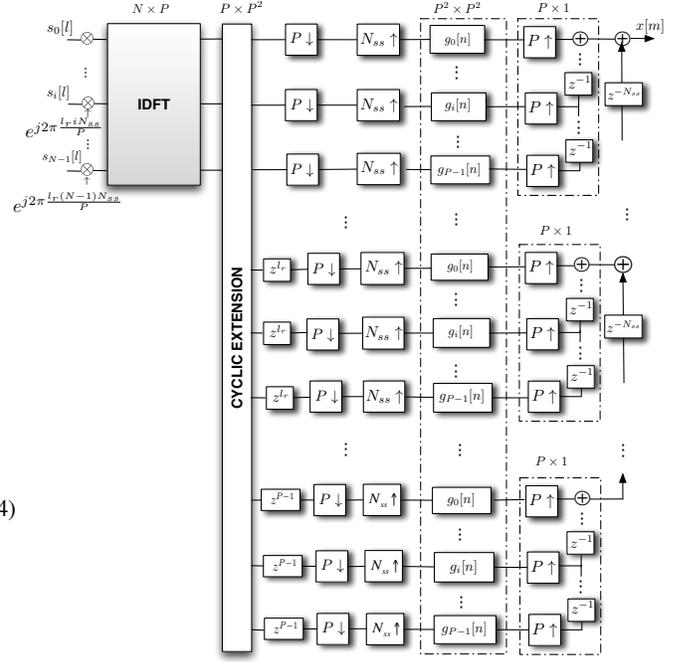


Fig. 4. General P^{th} -order polyphase architecture for a FBMC transmitter with a non-integer normalized frequency spacing factor Q

2) *Order of the polyphase network $B = N_{ss}$* : here it is convenient to apply the change of variable $l = \lfloor \frac{m}{N_{ss}} \rfloor - p$ in (10) to obtain:

$$\begin{aligned} x[m] &= \sum_{p=-\infty}^{\infty} S_{\text{mod}(m,P)} \left[\left\lfloor \frac{m}{N_{ss}} \right\rfloor - p \right] \\ &\quad \cdot g \left[m - \left\lfloor \frac{m}{N_{ss}} \right\rfloor N_{ss} + p N_{ss} \right]. \quad (18) \end{aligned}$$

Since, $m - \lfloor \frac{m}{N_{ss}} \rfloor N_{ss} = \text{mod}(m, N_{ss})$, we apply a N_{ss} -order sub-filter decimation to obtain the desired polyphase structure:

$$\begin{aligned} x[m] &= \sum_{p=-\infty}^{\infty} S_{\text{mod}(m,P)} \left[\left\lfloor \frac{m}{N_{ss}} \right\rfloor - p \right] g_{\text{mod}(m, N_{ss})}[p] \\ &= \text{IDFT}_{m,P}(\mathbf{s}[k]) * g_{\text{mod}(m, N_{ss})}[k] \Big|_{k=\lfloor \frac{m}{N_{ss}} \rfloor}. \quad (19) \end{aligned}$$

It is worth to notice that the order of the polyphase network N_{ss} is usually higher than the duration (in samples) of the subcarrier fundamental period. In this paper, we propose a solution based on a cyclic extension of the IDFT output (Fig. 5). The $N_{ss} - P$ extra samples generated can be seen as a cyclic prefix appended to the actual symbol. Indeed, the number of source symbols $s_n[l]$ required to generate an FBMC symbol is N , so if the target number of samples per symbols is higher (N_{ss}) there is a degree of freedom from a design point of view to fill the samples in the last part of the symbol.

Hence, the solution adopted in here is not unique and other solutions like zero-padding or pilot signaling are equally valid and do not have any meaningful effect on the presented architectures.

Moreover, due to the imbalance between the order of the polyphase network and the length of the IDFT, the phase continuity of the different subcarriers cannot be ensured. Since the order of the network $B = N_{ss}$ is not an integer number of subcarrier periods there appears a phase discontinuity at the end of each generated symbol forced by the characteristics of the implementation. Hence, once again it is convenient to resort to the phase rotation over the input source symbols to keep the signal phase continuous at every symbol transition (Fig. 5)

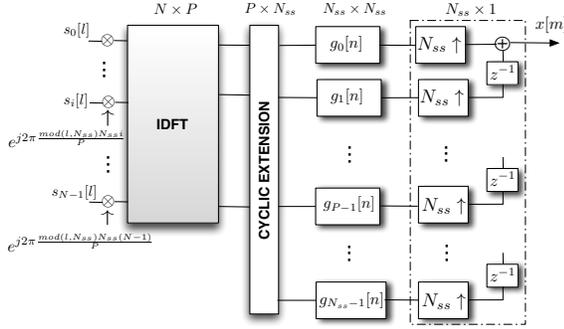


Fig. 5. General N_{ss}^{th} -order polyphase architecture for a FBMC transmitter with a non-integer normalized frequency spacing factor Q

3) *Order of the polyphase network* $B = \text{l.c.m.}(P, N_{ss})$: let us assume that $\text{l.c.m.}(P, N_{ss}) = P_o N_{ss} = N_{sso} P$ where both P_o and N_{sso} are integers. As in the previous architectures we conveniently decompose the convolution index $l = l_b P_o + l_r$ with $l_b = \lfloor \frac{l}{P_o} \rfloor$ and $l_r = \text{mod}(l, P_o)$. Thus, replacing in (10) we obtain:

$$x[m] = \sum_{l_r=0}^{P_o-1} \sum_{l_b=-\infty}^{\infty} S_{\text{mod}(m,P)}[l_b P_o + l_r] g[m - l_r N_{ss} - l_b P_o N_{ss}] \quad (20)$$

Similarly, we can consider $m - l_r N_{ss} = \lfloor \frac{m - l_r N_{ss}}{P_o N_{ss}} \rfloor P_o N_{ss} + \text{mod}(m - l_r N_{ss}, P_o N_{ss})$. Likewise, a phase rotation over the source symbols is also required to ensure carrier phase continuity. Then, according to the reasoning in (17) and applying a $P_o N_{ss}$ sub-filter decimation and replacing in (20) we obtain:

$$x[m] = \sum_{l_r=0}^{P_o-1} \sum_{l_b=0}^{P_o-1} S_{\text{mod}(m,P)}[l_b P_o + l_r] \cdot g_{\text{mod}(m - l_r N_{ss}, P_o N_{ss})} \left[\left\lfloor \frac{m - l_r N_{ss}}{P_o N_{ss}} \right\rfloor - l_b \right] \quad (21)$$

$$= \sum_{l_r=0}^{P_o-1} \mathcal{D}_{P_o} \{ \text{IDFT}_{m(l_r), P}(\tilde{\mathbf{s}}[k + l_r]) \} * g_{\text{mod}(m(l_r), P_o N_{ss})}[k] \Big|_{k = \lfloor \frac{m(l_r)}{P_o N_{ss}} \rfloor} \quad (22)$$

As in the case of a polyphase network of order P , the index l_r can be seen as a hardware replica index for blocks of order $P_o N_{ss}$. The signals $S_{\text{mod}(m,P)}[l_b P_o + l_r]$ correspond to downsampled versions by P_o of the IDFT outputs with a sampling offset of l_r samples that is constant for each replica. This implies that the signal delivered by the IDFT is going to be processed separately by different parts of the polyphase network. The resulting architecture is depicted in Fig. 6

V. CONCLUSION

We have presented a unified framework for the definition of FBMC signals based on a quadruple of key parameters that uniquely defines

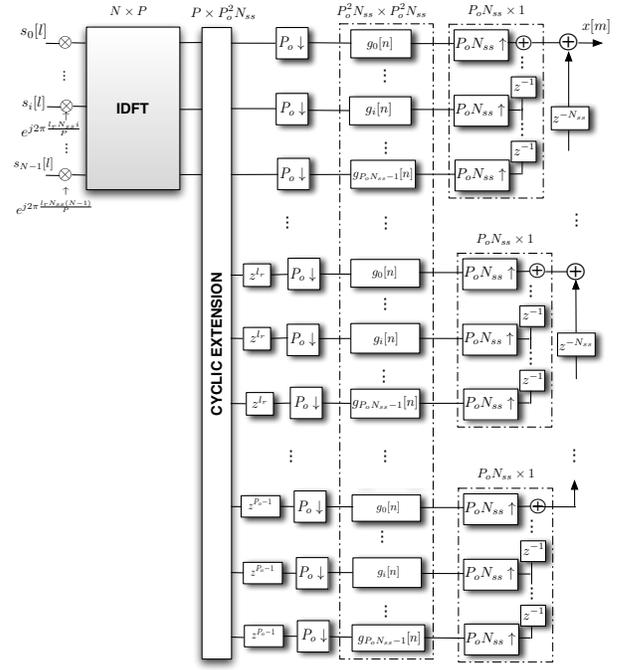


Fig. 6. General $\text{l.c.m.}(P, N_{ss})^{th}$ -order polyphase architecture for a FBMC transmitter with a non-integer normalized frequency spacing factor Q

any MC signal. Likewise, we show how to derive efficient FBMC transmitter architectures based on the polyphase decomposition of the discrete-time shaping pulse. Different polyphase architectures are provided for quadruple values that have been traditionally avoided in practical implementations because of their complexity or solved by means of time-variant filtering.

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