Mitigation of False Locks in the Acquisition of High-Order BOC Signals in HS-GNSS Receivers

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***Abstract*—This paper addresses the problem of acquiring weak high-order binary offset carrier (BOC) signals in the context of high-sensitivity global navigation satellite system (HS-GNSS) receivers. The use of high-order BOC modulations provides an improvement over conventional BPSK modulation in terms of positioning accuracy, at the cost of secondary peaks appearing in the correlation function. The acquisition of the main peak is a critical problem because the presence of secondary peaks often introduces a bias in the time-delay estimation. This problem is even more complicated to address when the signal is received with a low carrier-to-noise ratio. In this situation, we must resort to Post-Detection integration (PDI) techniques to acquire the received signal, and after that an estimator can be applied to identify the main correlation peak. The contributions of this work are twofold. Firstly, we analyze the performance of PDI techniques such as the Non-coherent PDI and the Differential PDI to acquire weak high-order BOC signals in static and dynamic channels. Secondly, after detecting the signal by using a PDI technique, we propose to apply the Maximum Likelihood (ML) and the Least Square (LS) estimators to estimate the time-delay of the main peak.**

I. INTRODUCTION

The acquisition of weak signals is a challenging problem nowadays because the attenuation of the received signal is large and it is not possible to detect the correlation peak correctly. Increasing the coherent integration time is the best way to obtain a gain in terms of signal detection. Nevertheless, the duration of coherent time is limited in practice. This fact is due to impairments arising from frequency offset or phase noise at the high-sensitivity global navigation satellite system (HS-GNSS) receiver. To circumvent these uncertainties, Post-Detection integration (PDI) techniques must be used. Although, these techniques are less efficien than coherent integration in terms of signal detection, they are much more robust against the frequency offset and the phase noise [1].

The situation is more complicated when the high-order binary offset carrier (BOC) signals are used. This modulation allows us to obtain better accuracy in terms of time-delay estimation than BPSK modulation. However, the drawback of BOC signals is that their autocorrelation functions are formed by several peaks. These peaks create ambiguities and in the presence of noise, makes it difficul to identify the main peak. The detection of a secondary peak or false lock is a dramatic problem for the high-order BOC signals because a bias is present in terms of positioning when a secondary peak is acquired. For this reason, the reduction of the probability of false lock is currently a challenging problem in HS-GNSS receivers [2].

Different works have tried to mitigate the probability of false lock at the tracking stage by applying various methods such as the Bump Jumping [3], the Double Estimator [4] and the Code-Subcarrier Smoothing [5]. Lately, also an open-loop multi-correlator approach, namely the Double Optimization Multi-correlator-based Estimator (DOME) [6][7] has been proposed for solving the unambiguous tracking estimation problem using as baseline the Maximum Likelihood (ML) estimator. Nonetheless, less attention has been paid to the problem of the main peak detection or of unambiguous estimation at acquisition stage. In this work, we focus on solving this problem at acquisition stage in HS-GNSS receivers where PDI techniques must be used to detect the received signal.

The aim of this paper is twofold. On the one hand, PDI techniques such as the Non-coherent PDI (NPDI) and the Differential PDI (DPDI) have been analyzed using high-order BOC signals in a AWGN static channel and a AWGN dynamic channel. It is important to analyzes a dynamic channel because the receivers often acquire the signal when they are moving. Then, the received signal suffers some variations in the Doppler frequency, which might cause a degradation in the performance of PDI techniques. On the other hand, after the signal detection by using the NPDI method or the DPDI method, we propose to apply estimators using as baseline the Least Square (LS) and the ML approaches, used for the unambiguous tracking estimation of high-order BOC signals in [2] and [6], respectively, to solve the unambiguous estimation problem at acquisition stage.

II. SIGNAL MODEL

The signal received by a HS-GNSS receiver can be represented in baseband as (see [8])

$$r(n) = \sum_{p=1}^{P} A_p s_p(n-\tau_p) b_p(n-\tau_p) e^{j(2\pi f_{d,p} n+\theta_p)} + \tilde{\omega}(n), \quad (1)$$

where A_p is the received amplitude of the pth satellite, $b_p(n)$ is the navigation data bit sequence, $s_p(n)$ is the pseudo-random code sequence modulated as a specifi high-order BOC signal,

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P is the number of GNSS satellites, $f_{d,p}$ is the Doppler frequency of the received signal, τ_p is the code delay, θ_p is the carrier phase, and $\tilde{\omega}(n)$ is the complex AWGN. In order to acquire a satellite, (which may be in view or may be not) for instance the lth satellite, a portion of the received signal with duration L_{ch} samples is correlated by $x(n)$ a replica of the transmitted signal as

$$x(n) = s_l(n - \tilde{\tau}_l)e^{j(2\pi \tilde{f}_{d,l} n)}, \tag{2}$$

where $\tilde{f}_{d,l}$ and $\tilde{\tau}_l$ are tentative value of the Doppler frequency $f_{d,l}$ and the code delay τ_l, respectively. The correlation is performed for all the possible values of $\tilde{f}_{d,l}$ and $\tilde{\tau}_l$. The resulting operation is define as

$$R_m(\Delta\tau, \Delta f) = L_{ch} A_l \text{sinc}(\Delta f T_{ch}) e^{j(2\pi m \Delta f + \theta)} v(\Delta\tau) + \omega_m, \tag{3}$$

where $R_m(\Delta\tau, \Delta f)$ is the output of the coherent correlation at time $m = 1, \ldots, N_{nc}$, N_{nc} is the number of non-coherent integrations, $\Delta f = f_{d,l} - \tilde{f}_{d,l}$ is the frequency offset between the replica of the signal transmitted and the received signal, $\Delta\tau = \tau_l - \tilde{\tau}_l$ is the delay offset, T_{ch} is the coherent integration time, $L_{ch} = f_s T_{ch}$ is the number of samples integrated coherently, f_s is the sampling frequency, $v(\Delta\tau)$ is the autocorrelation function of a high-order BOC signal, and ω_m is AWGN after the coherent correlation with zero-mean and variance σ^2. The $\text{sinc}(\Delta f T_{ch})$ term captures the degradation of the coherent integration owing to the frequency offset. Sometimes, because of the severe attenuation it is very difficul to detect the received signal and the integration time must be increased beyond the baseline value. Nonetheless, it is not possible to coherently integrate without bound due to the degradation caused by phase noise and residual frequency offsets. To circumvent these impairments a PDI technique, define as $Z_{\text{X}}(\Delta\tau, \Delta f)$, is performed by the use of a non-linear function of the outputs of the coherent correlation (see Section III). The goal of the metric $Z_{\text{X}}(\Delta\tau, \Delta f)$ is to discriminate between 2 hypotheses. Under H_0 the satellite is not in view (*i.e.* $A_l = 0$) and under H_1 the satellite is in view (*i.e.* $A_l \neq 0$).

The discrimination between the 2 hypotheses is performed comparing the maximum value of the metric $Z_{\text{X}}(\Delta\tau, \Delta f)$ with a threshold [1]. Then, the satellite is declared in view, if the maximum value of the metric $Z_{\text{X}}(\Delta\tau, \Delta f)$ exceeds the threshold. However, in the particular case of the high-order BOC signals, besides the problem of detection, there is the additional problem of false locks, produced by the multiple peaks present in the correlation. Due to this fact, the signal can sometimes be acquired in a main peak or in a secondary peak, unlike the BPSK modulation where the correlation shape of this signal only exhibits one peak. Fig. 1 shows the ideal correlation of a $\text{BOC}_{\cos}$(15,2.5) signal as a representative case of high-order BOC, which uses a cosine phased BOC subcarrier modulation of frequency 15.345 MHz and it has a chip rate of 2.5575 Mcps. In HS-GNSS receivers, the problem of false lock is more complicated because we acquire the signal by applying a PDI technique, which uses the absolute value operator (see Section III). This operator makes the negative peaks to become positive and therefore we have a larger number of secondary peaks. Fig. 2 shows the absolute value of the $\text{BOC}_{\cos}$(15,2.5) autocorrelation. Taking into account this Figure, we assume that a false lock is present when the estimated code delay is outside the region define between the two green lines in Fig. 2. From this assumption, the hypothesis H_1 can be broken down into two sub-hypotheses. Under H_{true} the acquisition of the signal by using the metric $Z_{\text{X}}(\Delta\tau, \Delta f)$ has been done in the main peak (*i.e.* the signal is acquired in a true lock) and under H_{false} the acquisition of the signal by using the metric $Z_{\text{X}}(\Delta\tau, \Delta f)$ has been done in a secondary peak (*i.e.* the signal is acquired in a false lock).

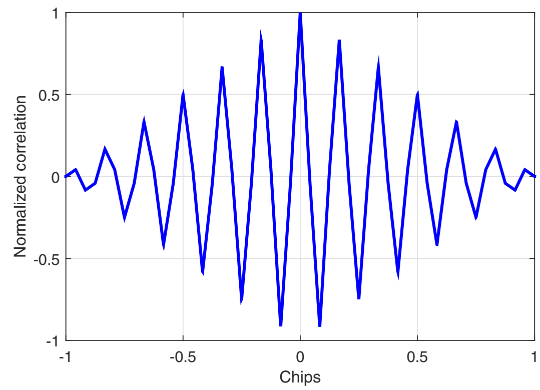

Fig. 1. Ideal autocorrelation of the $\text{BOC}_{\cos}$(15,2.5) signal.

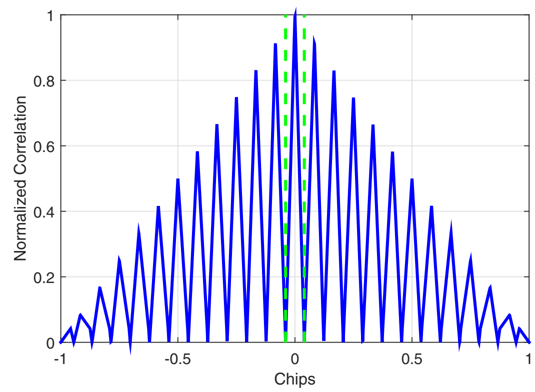

Fig. 2. Absolute value of a ideal autocorrelation $\text{BOC}_{\cos}$(15,2.5) signal.

III. PDI TECHNIQUES

The most common technique used to perform non-coherent integrations is the NPDI method [9].

$$Z_{\text{NPDI}}(\Delta\tau, \Delta f) = \sum_{m=1}^{N_{nc}} |R_m(\Delta\tau, \Delta f)|^2. \tag{4}$$

The NPDI method removes carrier impairments such as frequency offset by using the squared absolute value. The drawback of the NPDI technique is that it suffers from the squaring loss effect. That is, the mean of the noise is increased with respect to the coherent integration. Thereby, the total integration time must be larger to detect the signal with respect to the coherent integration under ideal conditions (assuming that the only disturbance is AWGN without frequency offset).

Alternatively, the DPDI method can be used [10], which is define as

$$Z_{\text{DPDI}}(\Delta\tau,\Delta f)=\left|\sum_{m=2}^{N_{nc}} R_m(\Delta\tau,\Delta f)R^*_{m-1}(\Delta\tau,\Delta f)\right|. \quad (5)$$

The idea behind this method is to eliminate the uncertainty of the frequency offset without increasing the mean of the noise. To do so, one output of coherent integration is multiplied by the next output of coherent integration conjugated. By doing so, the noise components of 2 consecutively outputs of the coherent integration are uncorrelated, while the signal components are highly correlated. Using this approach the mean of the noise is not increased, and therefore in a static channel with AWGN, the DPDI method provides a gain over the NPDI method in terms of signal detection.

IV. Fine acquisition of High-order BOC signals

The acquisition of the main peak for a high-order BOC signal is a challenging problem because the secondary peaks of its autocorrelation have practically the same energy as the main peak. The conventional approach used at acquisition stage, that is, to estimate the code delay as the position of the maximum of the correlation, might not be a good solution to solve this problem in certain conditions where the correlation function is impacted by the sampling frequency used or by distortions introduced by the receiver front-end.

To solve the problem of unambiguous estimation, we propose to use the ML estimator and the LS estimator. Although, these estimators have been already proposed [2] and [6], they have been applied from a point of view of the tracking stage. However, in this document, we use these estimators from a point of view of acquisition where PDI techniques must be used to detect the signal. After that, a fin acquisition is performed by applying the ML estimator or the LS estimator to solve the problem of unambiguous estimation. These estimators exploit the fact that the autocorrelation function of high-order BOC signals is known. Taking into account this information the correct peak can be acquired and the probability of false lock can be improved in certain conditions where the usage of the maximum of the correlation is not the optimum approach.

In this sense, the procedure is as follows: the NPDI method or the DPDI method is used to acquire a weak signal. Then, a coarse estimation of the code delay τ_l, in units of samples, and the Doppler frequency $f_{d,l}$ is obtained as $\hat{\tau}_l$ and $\hat{f}_{d,l}$, respectively. From these estimations, we defin the vector $\mathbf{g}$ containing a set of $K=2Q+1$ samples as follows:

$$\mathbf{g} \doteq [Z_{\text{X}}(\hat{\tau}_l-Q,\hat{f}_{d,l}),\dots,Z_{\text{X}}(\hat{\tau}_l+Q,\hat{f}_{d,l})]^T, \quad (6)$$

where $Z_{\text{X}}(\hat{\tau}_l-Q,\hat{f}_{d,l}),\dots,Z_{\text{X}}(\hat{\tau}_l+Q,\hat{f}_{d,l})$ is the output of the NPDI method or the DPDI method for the estimation of the Doppler frequency $\hat{f}_{d,l}$ and for the K samples of the correlation function contained around $\hat{\tau}_l$ in $\pm$ 1 chip; Q is the number of samples contained in one side of the correlation function during 1 chip, that is, the ratio between the sampling frequency and the chip rate of the BOC signal. The expected correlation with an unknown code delay τ_l can be define as follows:

$$\mathbf{v}(\tau_l) \doteq [v(\tau_l-Q),\dots,v(\tau_l+Q)]^T. \quad (7)$$

One way to estimate the code delay that is more robust against the false lock problem consists in findin the value of the delay that provides the best f t between the measured correlation and the theoretical one. This value of the delay is obtained as the argument that minimizes the following non-linear least square cost function:

$$J(\tau_l,\alpha)=||\mathbf{g}-\alpha\mathbf{v}(\tau_l)||^2, \quad (8)$$

where α is the unknown amplitude. This function depends on two unknown parameters τ_l, α and it can be solved by separation of variables. To do so, the unknown amplitude is replaced by $\hat{\alpha}=(\mathbf{v}(\tau_l)^T\mathbf{v}(\tau_l))^{-1}\mathbf{v}(\tau_l)^T\mathbf{g}$ [11]. After that, the problem can be solved by using an iterative algorithm because an analytical solution does not exist in a closed-form. Moreover, the noise in post-correlation is colored Gaussian noise and this can be used to write this problem considering the noise is colored. That is, we can apply the ML estimator taking into account the covariance matrix of the noise. Then, the cost function can be reformulated as follows [6]:

$$G(\tau_l,\alpha)=(\mathbf{g}-\alpha\mathbf{v}(\tau_l))^T\mathbf{C}^{-1}(\mathbf{g}-\alpha\mathbf{v}(\tau_l)), \quad (9)$$

where the $K\times K$ covariance matrix $\mathbf{C}$ is non-diagonal matrix owing to the colored noise generated at post-correlation level. The resulting matrix is known a priori because it depends on the autocorrelation of the BOC signal. This cost function can be minimized by the same procedure used to minimize the cost function in (8), but substituting the unknown amplitude by the following expression that contains the covariance matrix $\hat{\alpha}=(\mathbf{v}(\tau_l)^T\mathbf{C}^{-1}\mathbf{v}(\tau_l))^{-1}\mathbf{v}(\tau_l)^T\mathbf{C}^{-1}\mathbf{g}$.

It should be added that this estimator is not strictly the ML for the NPDI method and the DPDI method because the noise produced by these methods is not Gaussian. Nonetheless, the noise of the NPDI metric follows Chi-square distribution under hypothesis H_0, but when the number of N_{nc} is large the noise tends to be Gaussian by the central limit theorem. On the other hand, the pdf of the DPDI metric is not known in a closed-form. Therefore, the minimization of the cost function in (9) for the DPDI method might be far to be the optimal ML estimator. However, this estimator has a good performance for the DPDI method as we will see later on.

V. Detection threshold

This section deals with the definitio of the detection threshold for the methods NPDI and DPDI. The threshold can be determined from a individual probability of false alarm P_{fa}, as follows:

$$\begin{aligned} P_{fa} &= \int_{\gamma}^{\infty} f(Z_{\text{X}}(\Delta\tau,\Delta f);H_0) \\ &= 1-\int_{-\infty}^{\gamma} f(Z_{\text{X}}(\Delta\tau,\Delta f);H_0) \\ &= 1-\text{cdf}_{Z_{\text{X}}}(\gamma), \end{aligned} \quad (10)$$

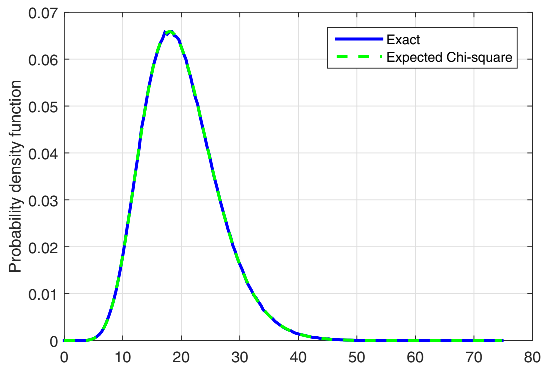

Fig. 3. Empirical pdf and expected Chi-square for $N_{nc} = 10$.

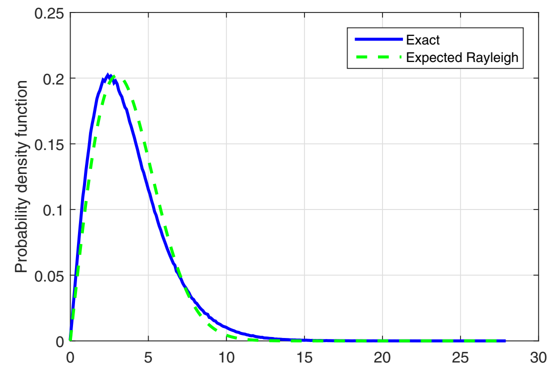

Fig. 4. Empirical pdf and expected Rayleigh for $N_{nc} = 10$.

where $f(Z_X(\Delta\tau,\Delta f);H_0)$ is the probability density function (pdf) under the hypothesis H_0 using the metric $Z_X(\Delta\tau,\Delta f)$, $\text{cdf}_{Z_X}(\gamma)$ is the cumulative density function under the hypothesis H_0 and γ is the detection threshold, which can be obtained by the following equation:

$$\gamma = \text{cdf}^{-1}_{Z_X}(1-P_{fa}). \quad (11)$$

The threshold is usually obtained by fixin a given value of global probability of false alarm (P_{FA}). The relationship between the P_{FA} and P_{fa} can be expressed as

$$P_{FA} = 1-(1-P_{fa})^N, \quad (12)$$

where N is the number of independent points of frequency and time in $Z_X(\Delta\tau,\Delta f)$. On the one hand, the cdf of the NPDI metric under H_0 follows a Chi-square distribution. Then, the threshold for the NPDI can be obtained, normalizing $\sigma = 1$ (noise in the output of the coherent correlation), as follows:

$$\gamma = \text{cdf}^{-1}_{Z_{\text{NPDI}}}\left(1-P_{fa}, 2N_{nc}\right), \quad (13)$$

where $2N_{nc}$ is the number of degrees of freedom. Fig. 3 shows that empirical pdf and the expected Chi-square are identical.

On the other hand, the pdf of the DPDI metric is not known in closed-form. For this reason, an approximation is performed to set a threshold for the DPDI method and thus being able to solve (11) for this metric. For large number of N_{nc}, the pdf of the DPDI metric under H_0 tends to be Rayleigh by the central limit theorem. Then, the pdf of the DPDI metric can be approximated as

$$p_{\text{DPDI}}(u) = \frac{u}{\sigma^2_{\text{DPDI}}} e^{-\frac{u^2}{2\sigma^2_{\text{DPDI}}}}, \quad (14)$$

where $\sigma^2_{\text{DPDI}} = (N_{nc}-1)\sigma^4$ and σ^2 is the variance of the output of the coherent correlation. From this approximation and normalizing $\sigma = 1$, the threshold for the DPDI is given by:

$$\gamma = \text{cdf}^{-1}_{Z_{\text{DPDI}}}\left(1-P_{fa}, \sqrt{N_{nc}-1}\right). \quad (15)$$

The accuracy of the approximation by using the central limit theorem is shown in Fig. 4 and Fig. 5 for $N_{nc} = 10$ and

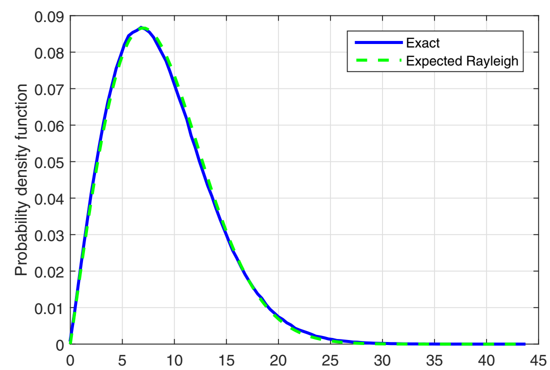

Fig. 5. Empirical pdf and expected Rayleigh for $N_{nc} = 50$.

$N_{nc} = 50$, respectively. Intuitively, the larger the N_{nc}, the better the approximation.

VI. Simulations

Simulations have been performed using a $\text{BOC}_{\cos}(15{,}2.5)$ signal as a representative case of a high-order BOC signal. The sampling frequency used is 50 MHz, which is the maximum value of a good USRP nowadays [12]. The frequency search space is within -500 Hz to 500 Hz because we assume that we have some assisted information about the Doppler frequency of the satellite [1]. The coherent time used is 10 ms and the steps of the search frequency are every 50Hz.

This section is divided into two parts. On the one hand, the firs subsection analyzes the performance of the methods NPDI and DPDI by using the Receiver Operating Characteristics (ROC) curves. This analysis have been done in a static and dynamic channel with complex AWGN. On the other hand, the second subsection deals with the comparative between different estimators in order to mitigate the probability of false lock.

A. Detection performance

Fig. 6 shows the comparative between the DPDI method and the NPDI method in a static channel with Gaussian noise.

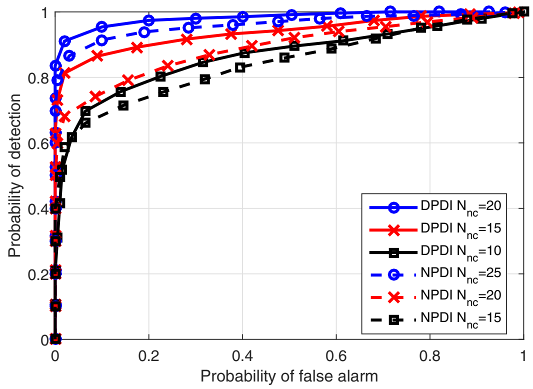

Fig. 6. Performance comparison of the methods NPDI and DPDI for $CN_0 =$ 27 dBHz in a static Gaussian channel.

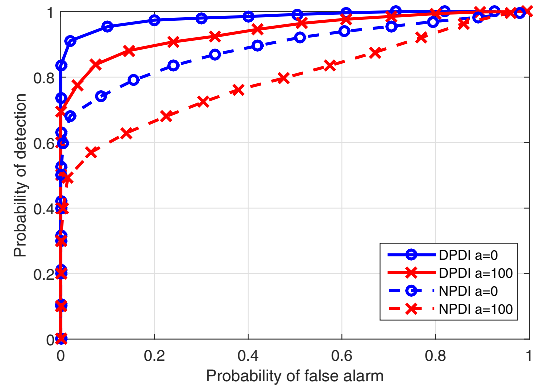

Fig. 7. Performance comparison of the methods NPDI and DPDI for $CN_0 =$ 27 dBHz and $N_{nc} = 20$ in a dynamic Gaussian channel.

In the case of the NPDI method, we have to perform a larger number of N_{nc} than using the DPDI method to obtain a good performance in the ROC curves. In other words, the DPDI method needs less integration time to obtain a good probability of detection. For this reason, the DPDI method provides a gain over the NPDI method in terms of signal detection.

The influenc of dynamic channels due, for instance, to acceleration of a car is usually ignored at acquisition stage because it should not affect to acquire signals using a short integration time. However, for a large integration time, the influenc of accelerations in a dynamic channel could affect the performance of the non-coherent techniques because the Doppler frequency changes with the acceleration. The Doppler frequency depends on the acceleration as follows [13]:

$$f_d(t) = f_{d_0} + \frac{f \cdot a \cdot t}{c}, \tag{16}$$

where f_{d_0} is the initial Doppler frequency, a is the acceleration, c is the light speed, f is the carrier frequency, and t is the time. Fig. 7 analyzes the effect of Doppler variations owing to the acceleration using $N_{nc} = 20$. We use a large value of acceleration to see some degradation in the ROC curves. This value is 100 m/s^2 and it is very large compared to the acceleration of an ordinary car, which is around 4 m/s^2. Then, the conclusion is that the Doppler variations due to the acceleration of a car in practice should not affect in the performance of the ROC curves for the methods NPDI and DPDI using a coherent time of 10 ms and using a total integration around 0.2 seconds. Comparing the two PDI techniques, the ROC performance degradation for these two methods in presence of accelerations is more or less the same for a value of acceleration. Therefore, the DPDI remains the best performing technique in presence of varying Doppler frequency due to the accelerations.

B. Probability of false lock

In this subsection, the probability of false lock at acquisition is assessed for three different methods: the conventional approach used at acquisition stage (to estimate the code delay as the position of the maximum of the correlation), the ML

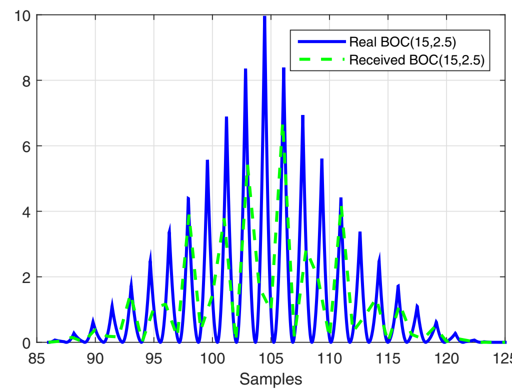

Fig. 8. Example of a false lock due to the sampling frequency.

estimator, and the LS estimator. It is clear that the performance of the three methods is affected by the noise level of the received signal because it distorts the correlation function. Moreover, the performance of the conventional approach used at acquisition stage is limited by the sampling frequency or the space between the samples of the correlation function. This happens because sometimes the maximum of the correlation is sampled in a secondary peak instead of a main peak. Fig. 8 is a illustrative plot, showing a false lock owing to the sampling frequency.

Fig. 9 shows the comparison between the conventional approach used at acquisition stage (referred to as “max” in the plot), the ML estimator, and the LS estimator in a AWGN static channel. We have a high probability of false lock owing to the sampling frequency by applying the conventional approach used at acquisition stage, though we have a good carrier-to-noise ratio (C/N_0). Nevertheless, this effect can be mitigated by using the ML and LS estimators as proposed in this work. For the particular case considered in the simulation, the best method from the ones assessed to reduce the probability of false lock is the ML because it takes into account the correlation of the noise in post-correlation. Moreover, the larger the number of N_{nc}, the better

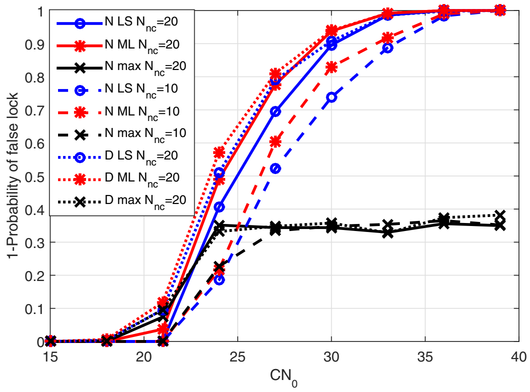

Fig. 9. Comparison between the conventional approach used at acquisition stage, ML, and LS for a $P_{FA} = 0.05$ ("N" means NPDI, "D" means DPDI).

performance the method. This is because we are increasing the total integration time and it allows to reduce the effect of the noise. Although we are not using the optimal ML for the DPDI, the performance of this estimator can still surpass the one of the NPDI. Comparing the estimators applied over the NPDI method and the DPDI method, we can see that these estimators have better performance by using the DPDI method in terms of probability of false lock.

VII. CONCLUSIONS

On the one hand, we have analyzed the performance of the methods NPDI and DPDI using $BOC_{cos}(15,2.5)$ signals in a static and dynamic channel with Gaussian noise. The DPDI method outperforms the NPDI method in both channels in terms of signal detection. Moreover, the DPDI method is as robust as the NPDI method against frequency changes owing to the channel dynamic using 10 ms of coherent integration. On the other hand, the probability of false lock obtained by applying the conventional approach used at acquisition stage is limited by the sampling frequency. We have proposed two estimators the ML and the LS to solve the problem of false lock. These estimators reduce the probability of false lock with respect to the conventional approach used at acquisition stage. In addition, the application of these estimators after using the DPDI method provides a gain over the application of these estimators after using the NPDI method in terms of probability of false lock.

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