Doubly Spread Signals for GNSS

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Abstract

In this paper we propose a new way of designing a GNSS signal using doubly spread signals in which the spreading operation is performed by means of frequency-domain and time-domain signature sequences. Doubly spread signals is a general formulation of *Code-Division-Multiple-Access* that includes both single-carrier DS-CDMA signals and multicarrier MC-CDMA signals. In a doubly spread signal, each user is allocated spreading matrix, which is a key parameter design, as it affects several aspects of the signal performance. In this work we propose a simple optimization strategy for the spreading matrix for either a GNSS pilot signal or a GNSS data channel signal, with the aim of minimising the *Multiple-Access-Interference* (MAI) and the *Peak-to-Average-Power-Ratio* (PAPR), while at the same time preserving a certain freedom for the spectral shaping of the signal.

INTRODUCTION

In recent years, multicarrier (MC) transmission schemes have been introduced into *Code-Division-Multiple-Access* (CDMA) systems to get such advantages as bandwidth efficiency, frequency diversity, lower speed parallel type of signal processing, and interference rejection capability [1]-[3]. A large variety of proposed techniques exist and they can be categorized into two types [4]: one performs frequency-domain spreading, known as *Multicarrier-Code-Division Multiple Access* (MC-CDMA) and the other performs time-domain spreading, known as *Multicarrier-Direct-Sequence-CDMA* (MC-DS-CDMA). In the former type the serial data stream is spread by a spreading code and each chip is transmitted by a different subcarrier; the latter type of techniques consists in the parallel transmission of narrowband direct sequence waveforms in the frequency domain.

MC transmission schemes are highly flexible and these two above mentioned types of techniques can also be applied simultaneously. The resulting signal is a multicarrier signal that conveys on each subcarrier a DS-CDMA signal multiplied by a complex factor, determined by the frequency spreading code. Since the signal is spread in two stages by spreading codes in both domains, such signal can be called *doubly spread* signals. Such a signal is a general form of *Spread-Spectrum/CDMA* (SS/CDMA) signal, as it encompasses both the classical single carrier DS-CDMA (if frequency spreading code contains one element), and a MC-CDMA transmission scheme (if the time spreading code contains one elements). This signal can be further generalised by using variable pulse shapes, frequency separations and chip rates, making use of a relatively new mathematical theory that goes under the name of Gabor Analysis [5].

In this work we define a general form of SS/CDMA, in order to adapt it entirely to GNSS applications, rather than using some signals developed for communication and later converted to ranging signal. Further, we show how the performance of such a doubly spread signal can be easily controlled by a proper design of the spreading codes. Finally, we propose a reasonable solution for the problem of designing the spreading matrix with the following objectives: 1) minimise the *Multiple-Access-Interference* (MAI) among satellites in the signal acquisition phase; 2) minimise the *Peak-to-Average-Power-Ratio* (PAPR) of the signal, which is essential for the power efficiency of a GNSS signal; 3) allow spectral shaping of the signal in order to fit the requirements of a given GNSS application or of a given spectral mask.

*The work of G. Seco was supported in part by the Spanish Ministry of Science and Innovation project TEC 2008-06305, by the Catalan Government under the grant 2009 SGR 298.

SIGNAL MODEL

A doubly spread MPSK-SS/CDMA signal can be modelled as

$$s(t) = \sqrt{\frac{P}{M}} d_k \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} c^u_{n,m} g(t - nT_c) e^{j2\pi m \Delta_f t}, \quad (k-1)T \leq t \leq kT \tag{1}$$

where d_k is the information symbol in the k-th time interval; in this paper we assume that data symbols belong to a Q-ary MPSK constellation: $d_k \in \left\{\frac{2\pi(q-1)}{Q}\right\}$, $q = 1, \ldots, Q$. P is the signal power, $g(t)$ is the unitary energy prototype filter ($\int_{-\infty}^{\infty} |g(t)|^2 \mathrm{d}t = 1$), T_c is the chip time, T the symbol time, with $T = NT_c$; the term Δ_f is the frequency separation among subcarriers.

The signal $s(t)$ in (1) is obviously a linear combination of the basis functions:

$$g_{n,m}(t) = g(t - nT_c) e^{j2\pi m \Delta_f t} \tag{2}$$

weighted by the complex coefficients $c^u_{n,m}$. The set of basis functions, generated by time and frequency translations of the same prototype pulse, or window function, g(t), can be seen as Weyl-Heisenberg system [5], if M and N are sufficiently large. The description of a Weyl-Heisenberg system is completed by another dual set of functions $\{\gamma_{n,m}(t)\}$, generated by time and frequency translations of a dual prototype filter $\gamma(t)$ in an analogous way. At the receiver the data is extracted performing the correlation with the dual signal:

$$\sigma(t) = \sqrt{\frac{P}{M}} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} c_{n,m} \gamma(t - nT_c) e^{j2\pi m \Delta_f t}, \quad (k-1)T \leq t \leq kT \tag{3}$$

in order to recover data

$$\hat{d}_k = < s(t), \sigma(t) > = \int_{-\infty}^{\infty} g(t) \sigma^*(t) \mathrm{d}t. \tag{4}$$

The function sets $\{g_{n,m}(t)\}$, $\{\gamma_{n,m}(t)\}$, known also analysis and synthesis function sets respectively, need not be the same, and the functions of each set need not be mutually orthogonal [6]. Moreover, the density of the time-frequency grid, given by the product $T_c\Delta_f$ can be chosen accordingly to different necessities, provided that the prototype filter is chosen consequently [7]. These degrees of freedom allow the analysis and synthesis function sets to be designed in many different ways. The matrix:

$$\mathbf{C}^u = \begin{pmatrix} c^u_{0,0} & c^u_{0,1} & \cdots & \cdots & c^u_{1,M-1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c^u_{n,0} & \cdots & c^u_{n,1} & \cdots & c^u_{n,M-1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c^u_{N-1,0} & c^u_{N-1,1} & \cdots & \cdots & c^u_{N-1,M-1} \end{pmatrix} \in \mathbb{C}^{N \times M} \tag{5}$$

is called *spreading matrix* and the superscript u indicated that this matrix identifies the u-th user (eg. satellite). The object of this paper is to present a reasonable methodology to design this matrix in a way to optimise the signal (1) for GNSS applications.

We remark that the signal in (1) can be well regarded as the most general expression of a SS/CDMA signal, and thus it provides a general framework for the design of GNSS signals based on the SS/CDMA technique. If $M = 1$, the reader will recognise in (1) the classical DS-CDMA signal used in GPS and Galileo; if $N = 1$, the signal (1) becomes a MC-CDMA signal, where the spreading is performed only by means of a frequency-domain code. If $M, N \neq 1$, we perform spectral spreading using codes in both domains, and thus such a signal can be called *doubly-spread*. In this work we define a certain pulse shape and the density of the time-frequency grid, by choosing , $g(t) = \mathrm{rect}\left(\frac{t}{T}\right)$ and $T_c\Delta_f = 1$. This is equivalent to a *Orthogonal-Frequency-Multiplexing* (OFDM) scheme.

SPREADING MATRIX OPTIMIZATION

The spreading matrix (5) strongly affects the acquisition and tracking performance, data transmission capabilities, spectral occupancy , and MAI of the signal (1). The absolute values of the elements of the n-th row of the spreading matrix determine the power spectrum of the signal during the n-th chip time, in other words these absolute values describe the power distribution among the subcarriers. The tracking accuracy directly depends on the shape of the signal spectrum. Each satellite is given a certain spreading matrix (5) that uniquely identifies the satellite; as a consequence of that, the acquisition performance and the MAI are dependent on the spreading matrix. In this section we formalise the properties that the spreading matrix must have. These can be summed up in the following points:

1. Guarantee high acquisition robustness.

2. Maximise power efficiency in terms of a low PAPR of the signal $s(t)$.

3. Provide sufficient degrees of freedom for spectral shaping of the signal $s(t)$.

Acquisition robustness shall include a low level of MAI and a low level of the side-peaks of the autocorrelation function of the signal. As a metric to quantify MAI we define the maximum of the cross-correlation functions, among signal coming from different satellite. This metric is relevant for the MAI in the acquisition phase, where MAI reduction algorithms cannot be applied [9].

Optimising properties 1 and 2 alone is a relatively difficult task. Designing spreading matrices with a minimised mutual MAI implies being able to design the matrices for the two elementary cases $M = 1$ and $N = 1$. The former case amounts to designing spreading codes for asynchronous single carrier DS-CDMA signals, and the properties these codes must have are clear and literature abounds. Nevertheless, the design of spreading codes for asynchronous MC-CDMA systems ($N = 1$) is highly problematic and the literature in this field is not as rich. The main problem is that the way spreading codes affect the MAI of a asynchronous MC-CDMA system is particularly complicated [8], optimal codes have not been found yet, neither have theoretical bounds to be used as benchmark. The minimization of the PAPR of a multitone signal is a highly non-linear problem that has been confronted by a large portion of literature in the last 40 years (e.g. [20], [21]). Moreover, when this problem is transposed from a neatly mathematical point of view into a communication system design context, it becomes connected with the spectral shape of the signal.

The solution that we propose for the spreading matrix optimization stems from the consideration that the PAPR of the signal (1) is determined only by the rows of the spreading matrix $\mathbf{C}^u$. For this reason, we can simplify the problem by decomposing it in two sub-problems: using the rows of $\mathbf{C}^u$ (i.e. frequency-domain spreading code) for the PAPR minimization and spectral shaping and on the other hand the columns of $\mathbf{C}^u$ (i.e. time-domain spreading codes) for satellite separation and thus MAI minimisation. Such an approach will be of course sub-optimal, but it has the advantage of being considerably simpler, as results developed for other similar problems can be applied. Thus we rewrite the elements of $\mathbf{C}^u$ as

$$c^u_{n,m} = c^u_T[n] \cdot c_F[m], \tag{6}$$

with $\{c^u_T[n]\}$, $n = 0, \ldots, N-1$ the time-domain spreading code and $\{c_F[n]\}$, $n = 0, \ldots, N-1$ the frequency domain spreading code. The user separation is performed by means of the time-domain spreading codes and each satellite is given a unique time-domain spreading code. Frequency-domain codes are chosen the same for all satellites and are constant over time. Both spreading codes are in general complex. Frequency-domain spreading codes must not be constant envelope, unless a flat spectrum is desired. Time-domain spreading codes must be constant envelope, if the PAPR is minimised by the frequency domain codes. With condition (6), the signal (1) becomes

$$s(t) = \sqrt{\frac{P}{M}} d_k \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} c^u_T[n] c_F[m] \cdot \big(g(t) \otimes \delta(t - nT_c)\big) e^{j2\pi m \Delta_f t}. \tag{7}$$

Two ways to interpret and also to implement the signal (1) can be derived from (7): either as a DS-CDMA signal pulse-shaped by a multitone signal $g_{MT}(t)$

$$s(t) = \sqrt{\frac{P}{M}} d_k \sum_{n=0}^{N-1} c^u_T[n] \delta(t - nT_c) \otimes \overbrace{\sum_{m=0}^{M-1} c_F[m] g(t) e^{j2\pi m \Delta_f t}}^{g_{MT}(t)}, \tag{8}$$

or as a MC-DS-CDMA signal in which all subcarriers are modulated by the same a DS-CDMA signal $s_{DS}(t)$ with different complex gains

$$s(t) = \sqrt{\frac{P}{M}} d_k \underbrace{\sum_{n=0}^{N-1} c^u_T[n] g(t - nT_c)}_{s_{DS}(t)} \cdot \sum_{m=0}^{M-1} c_F[m] e^{j2\pi m \Delta_f t}. \tag{9}$$

Time-Domain Spreading Codes

In our signal the time-domain spreading codes have the task of separating users. DS-CDMA spreading codes should have good correlation properties with the minimal length of the sequence. This feature is useful both in designing a data signal or a pilot signal. In the former case, spreading codes with better correlation properties allow a certain user separation level within shorter times, and higher data rate transmissions become possible. In the latter case, it is possible to keep the code length constant and improved correlation properties yield a lower MAI, and in particular, acquisition robustness. By using complex, rather than real spreading codes, we can substantially improve the correlation properties of the time-domain spreading codes.

Relatively recent results [15] [16] have shown that complex sequences exhibit improved auto- and cross-correlations, with a gain up to 3 dB in the maximum periodic correlation with respect to binary real Gold sequences. Moreover, complex signature sequences provide better anti-jamming performance than real signature sequences [10]. For this reason the application of complex sequences in DS-CDMA has attracted a lot of research interests [17] [18] and modern wireless transmission systems already use polyphase DS-CDMA techniques (e.g. IS-95 CDMA mobile cellular system). GNSS is a very good field for application of polyphase DS-CDMA due to the small number of users, and thus, of required codes.

The aperiodic correlation between two complex sequences $\mathbf{u} = [u_0, \ldots, u_{N-1}]^{\mathrm{T}} \in \mathbb{C}^{1\times N-1}$ and $\mathbf{v} = [v_0, \ldots, v_{N-1}]^{\mathrm{T}} \in \mathbb{C}^{1\times N-1}$ of length N are defined by:

$$C^{aper}_{\mathbf{u},\mathbf{v}}(\tau) = \begin{cases} \sum_{n=0}^{N-1-\tau} u_n v^*_{n+\tau}, & 0 \le \tau \le N-1 \\ \\ \sum_{n=0}^{N-1+\tau} u_{n-\tau} v^*_{n}, & N-1 \le \tau < 0 \\ \\ 0, & |\tau| \ge N \end{cases} \tag{10}$$

The correlation between the received data-modulated sequence and the locally generated sequences (spreading sequence and data sequence) depends on the aperiodic correlation function of the spreading sequence and on the data symbols in this way:

$$C_{\mathbf{u},\mathbf{v}}(\tau) = \lambda_{-1} C^{aper}_{\mathbf{u},\mathbf{v}}(\tau) + \lambda_0 C^{aper}_{\mathbf{u},\mathbf{v}}(N-\tau) \tag{11}$$

with λ_{-1} and λ_0 the scalar products between the incoming data symbol and the reference symbol, at the time instant $t = -T$ and $t = 0$ respectively:

$$\lambda_{-1} = d_{-1}\hat{d}^*_{-1} = e^{j\delta_{-1}} \tag{12}$$
$$\lambda_{0} = d_{0}\hat{d}^*_{0} = e^{j\delta_{0}} \tag{13}$$

The terms δ_{-1} and δ_0 indicate the phase offsets between the received and local data symbols. These quantities take values in the alphabet of the data modulation, i.e.: $\left\{\frac{2\pi(q-1)}{Q}\right\}$, $q = 1, \ldots, Q$. Bearing in mind that the sign of (11) is not relevant for the code acquisition, for the special case of BPSK modulation (Q=2), the reader can recognise in (11) the expression for the even and odd correlation functions [19]. In general, for higher data modulation alphabets, there exist more than only two correlation functions. Our objective is to maximise acquisition robustness; this can be proven to be equivalent to minimising the maximum absolute value of (11) [19], thus minimising the following metrics:

$$f_1 = \max_{\lambda_{-1},\lambda_0} \left\{ \max_{\tau} \left| C_{\mathbf{u},\mathbf{v}}(\tau) \right| \right\}, \forall \mathbf{u} \neq \mathbf{v} \tag{14}$$
$$f_2 = \max_{\lambda_{-1},\lambda_0} \left\{ \max_{\tau} \left| C_{\mathbf{u},\mathbf{v}}(\tau) \right| \right\}, \mathbf{u} = \mathbf{v}, \tau \neq 0$$

In general this is a very difficult problem to solve, and in the course of this work we content ourself with giving a solution that is optimal in the case of a pilot signal, while still being a practical solution for a data channel with binary symbols (BPSK), which even outperforms Gold sequences, as shown in the following. This solution is provided by the Zadoff-Chu codes [12], [13], which are characterised by an ideal even auto-correlation function. A Zadoff-Chu code of length N is defined by:

$$a_r[k] = \begin{cases} \mathrm{e}^{j\pi\frac{rk^2}{N}} & N \text{ even}, \\ \mathrm{e}^{j\pi\frac{rk(k+1)}{N}} & N \text{ odd}, \end{cases} \tag{15}$$

where r is an integer relatively prime to N, and it can be called the *root* of the code. We synthetically denote Zadoff-Chu code by (N, r), since a Zadoff-Chu is uniquely determined by its length and its root. In Fig. 1 the correlation properties of Zadoff-Chu codes are depicted for variable code lengths and they are compared with the corresponding properties of Gold codes of 1023 chips. The cross-correlations of Zadoff-Chu codes have been computed taking two codes with roots r_1, r_2 such that $r_1 - r_2$ is relatively prime with N. This ensures that the Sarawate bound is achieved for the even correlations. Zadoff-Chu codes perform better than Gold codes in all correlation metrics, except for the odd periodic correlation, where they present a slightly increased peak in comparison with Gold codes. If one is willing to accept this reasonable cost, Zadoff-Chu codes can achieve the same correlation performance of Gold codes with less than 40% of their length. Thus, more than double the data rate with respect to a signal spread by Gold codes can be achieved assuming the same time-spreading factor and frequency-spreading factor. In the case of a pilot signal, the odd correlations vanish and the Zadoff-Chu codes achieve the Sarawate bounds [14] marking a 3 dB improvement with respect to Gold codes.

While Zadoff-Chu codes constitute an example of a complex spreading code design optimal for a pilot channel, near optimal designs for data channels can be achieved following the works of [16] and [15].

Frequency-Domain Spreading Codes

If in a given chip interval, all the subcarriers convey the same chip, then designing a frequency code with low PAPR amounts to the problem of minimizing the peak factor of a multitone signal by adjusting the subcarrier phases. This problem is well known in the literature (e.g. [20], [21], [22]) and can be so summarized: given the signal

$$x(t) = \sum_{m=0}^{M-1} c_F[m] e^{j2\pi m \Delta_f t}, \tag{16}$$

set the complex coefficients $c_F[m], m = 0, \ldots, M-1$ in order to minimise

$$\mathrm{PAPR} = \frac{\max_t \left\{|x(t)|^2\right\}}{\Delta_f \int_{\frac{1}{\Delta_f}} |x(t)|^2 \mathrm{d}t}. \tag{17}$$

The signal (16) represents the signal (1) in a chip-time interval, with the assumptions (6) holding. The square envelope of the frequency domain code determines the power

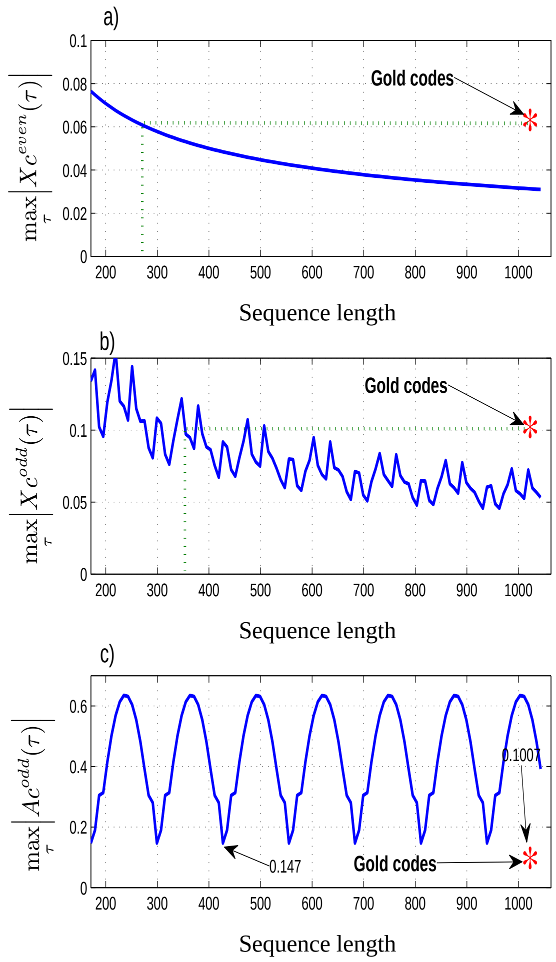

Fig. 1 Correlation properties of Zadoff-Chu Codes: a) even cross-correlation, b) odd cross-correlation , c)odd autocorrelation. The even correlation of Zadoff-Chu codes is ideal and as such it is not plotted. The performane of Gold codes are plotted only for length 1023.

distribution among the subcarriers, which determines the spectrum of the signal (1). A large part of the literature (e.g. [20], [21], [22]) proposed several solutions to this problem with the a constraint that all subcarriers have the same amplitude, and thus the signal has a flat power spectrum. This constraint is not only unnecessary for the problem at hand, but also counter-productive because we want both to minimise the PAPR of a multitone signal and to shape the spectrum of the signal (1). Moreover, none of these solutions proposed by the above mentioned literature can claim to be optimal. On the contrary, our approach consists in using both degrees of freedom of the frequency-domain spreading code, that is amplitudes and phases, to achieve a theoretical optimum for the peak minimization problem. Then spectral shaping and PAPR minimization become interdependent and consequently have to be solved jointly.

The frequency-domain codes that we propose are the Huffman codes [23][1]. It can be proved that these codes are the only codes to achieve the Friese's lower bound [22] for PAPR minimization of a multitone signal. This bound sets the minimum PAPR of a multitone signal between 0 dB and 3 dB, depending on the spectral shape. The use of Huffman codes as frequency-domain spreading codes guarantees that for a given spectral shape of the signal (1) the arrangement of subcarrier phases provides an optimum solution for the PAPR minimization problem. Since Huffman spreading codes determine also the spectrum of the doubly spread signal, it is necessary to spend some words on how spectral shaping is possible.

A Huffman code of length M is fully described by two parameters: the *roots pattern vector* $\mathbf{b} = [b_0, \dots, b_{M-2}]^{\mathrm{T}}$, with $b_m \in \{0, 1\}$ and a scalar γ. The m-th element b_m of $\mathbf{b}$ indicates the location of the root of the polynomial associated to the Huffman code that lies on the angle $2\pi \frac{m-1}{M-1}$; if $b_m = 1$ the root lies on the outer circle, if $b_m = 0$ on the inner circle , as defined in the original publication by Huffman [23]. The parameter γ is determined by the *Central-to-Side-lobe Ratio* (CSR) of the autocorrelation function of the Huffman code:

$$\mathrm{CSR} = \frac{\|\mathbf{c}_F\|^2}{|c_F[0]|\,|c_F[M-1]|} \quad , \tag{18}$$

$$\gamma = 1 + \frac{2}{\mathrm{CSR}} \quad , \tag{19}$$

where $\|\mathbf{c}_F\|^2 = \sum_{m=0}^{M-1} |c_F[m]|^2$. It can be proven that the parameter γ of a Huffman code $\mathbf{c}_F$ is equal to the PAPR of the multitone signal whose carriers have amplitudes and phases adjusted according to $\mathbf{c}_F$ [24]. For this reason, it is convenient to express γ in dB. If we use a Huffman code as frequency-spreading code for the signal (1), we can generate a doubly spread signal as defined in (1) with a controlled PAPR equal to γ for an arbitrary number of subcarriers, provided that the constraint (6) is fulfilled.

We conclude this section by observing that the use of Huffman codes as frequency-domain spreading signature sequences is optimal for the PAPR minimisation, but on the other hand not all spectral shapes are possible.

CASE STUDY

In this section we consider an example of signal design with a null-to-null bandwidth of $B = 20.46$ MHz. We remind that our doubly spread signal does not have a symmetric baseband spectrum, and so the concept of one-sided baseband bandwidth becomes a bit problematic. Therefore,

[1] These codes are multilevel complex codes that were developed for radar pulse compression and their use as frequency-domain spreading codes was first proposed by [24].

we adopt the unambiguous concept of null-to-null bandwidth. We choose $N = 555$ and $M = 17$. This amounts to an overall spreading factor of $10 \log_{10}(N\,M) = 39.74$ dB. The bandwidth is divided in 17 sub-bands, each with a main lobe of 1.11 MHz, which corresponds to a chip time of $T_c = 0.87\,\mu$ s. A period of our doubly spread signal consists of 555 chips and so it is equal to $T \approx 0.5$ ms. The time-domain spreading code is a Zadoff-Chu code (555,1), which guarantees correlation properties that are better (binary data signal, i.e. $d_k = \pm 1$) or much better (pilot signal, $d_k = 1, \forall; k$ than current GPS signals. The frequency-domain spreading code is a Huffman code with roots pattern vector $\mathbf{b} = [000010010110111]$. We consider Huffmann codes with three different values of γ: 0.3 dB, 0.8 dB and 1.4 dB. As a comparison we have chosen a DS-CDMA signal with a rectangular pulse and the same null-to-null bandwidth , and a ten times higher chip rate. In Fig.2 the normalised power spectral densities of the three doubly spread signals along with the rectangular pulse DS-CDMA signal are depicted and the signals are strictly bandlimited to B. The meaning of γ is intuitive to interpret: the higher γ, the higher is the signal power pushed to the edges of the band. This is a degree of freedom very easy to use in the trade-off between synchronization accuracy and synchronization robustness. Indeed, the higher the power at the edges of the band, the lower the tracking jitter becomes, whereas the higher the signal power around the center of the band, the lower the secondary peaks of the autocorrelation function of ranging signals become. This concept is further illustrated in the following figures: in Fig. 3 the absolute value of the autocorrelation function is depicted, within a chip time interval; in Fig.4 the *Cramer-Rao Lower Bound* (CRLB) for time-delay estimation error is depicted for an equivalent noise bandwidth $B_n = 1$ Hz. In Fig.5 we represent the multipath envelope for the different signal designs with a *Signal-to-Multipath Ratio* SMR= 3 dB.

CONCLUSIONS

In this work a general mathematical framework to design SS/CDMA signals for GNSS was introduced. The proposed framework offers the considerable advantage of determining key performance of the signal only by selecting the spreading matrix. This indeed is very useful achieving high flexibility in the signal design especially if such a signal is to be modified over time with the minimal impact on space-, ground-, and user-segment. Further, it introduces the concept of a scalable spectrum, which is typical of multicarrier modulations, into the GNSS signal design: the spectrum is divided in several sub-bands, that can be flexibly and dynamically allocated to different GNSS services. In this work we focused on devising a spreading code allocation scheme that minimises MAI and PAPR.

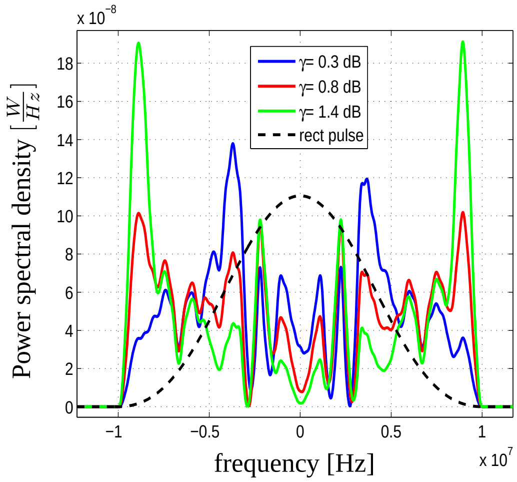

Fig. 2 Spectra of doubly spread signals with M=17, N=555 with different values of γ.

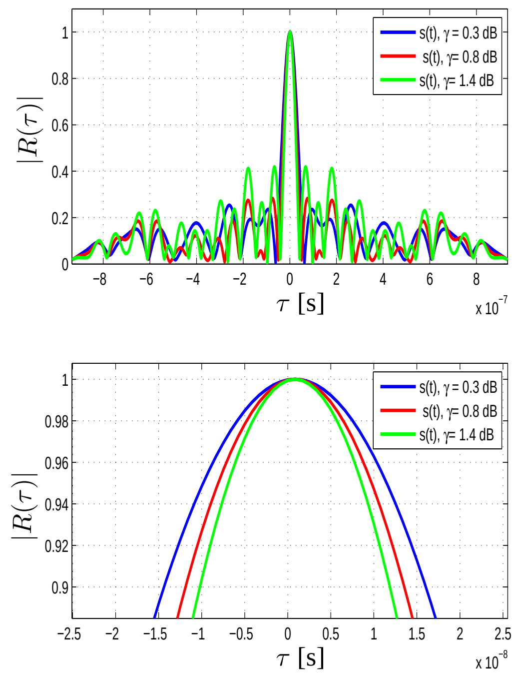

Fig. 3 Normalised autocorrelation function of the considered doubly spread signals, for lags laying within a chip interval. A chip interval is 0.9 μs.

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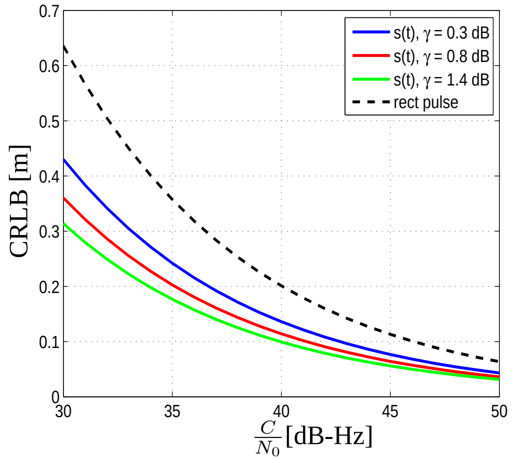

Fig. 4 CLRB for time-delay estimation.

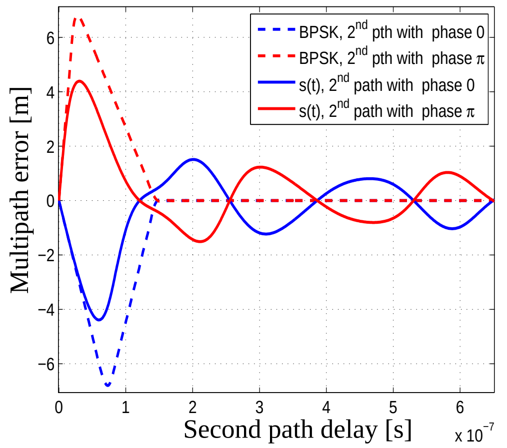

Fig. 5 Multipath error envelope of a doubly spread signal with $\gamma = 0.3$ dB.

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