Signal-level Integrity and Metrics Based on the Application of Quickest Detection Theory to Multipath Detection

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ABSTRACT

Multipath is one of the major impairments that can threat the integrity of mass-market GNSS receivers (i.e. those mainly used in terrestrial environments). In this context, the purpose of this work is to adopt a quickest detection framework for multipath detection in single-antenna GNSS receivers. This is done with the aim of providing signal-level integrity in GNSS applications. Three different approaches, all of them using the correlator output samples, are proposed in order to cope with a wide range of multipath and NLOS conditions. The results obtained in real field tests confirm the suitability of the proposed post-correlation metrics and the quickest detection framework to improve the navigation performance and to perform real-time quality monitoring.

The novelty of this work is the proposal of sequential tests for multipath detection based on quickest detection theory, which provides an optimum level of signal integrity in terms of the trade-off between delay in detecting integrity threats and time between false alarms. This is in contrast to classical detection techniques, where the goal is to maximize the detection probability subject to some probability of false alarm, but where "time" is not explicitly considered.

INTRODUCTION

With the widespread use of Global Navigation Satellite Systems (GNSS) in liability- and safety-critical applications [1], one of the major challenges to be solved is the provision of integrity to different types of users beyond the civil aviation community, where this feature is already well established. Position integrity is typically provided in civil aviation by Receiver Autonomous Integrity Monitoring (RAIM) algorithms and Satellite Based Augmentation Systems (SBAS) [2]. However, in general such methods are not sufficient to provide integrity in road and urban environments, due to the predominance of local degradation effects, such as multipath, fading, non-line-of-sight (NLOS) propagation and interference signals [3]. This is the reason why integrity analyses on the received signal (i.e. signal integrity) should be considered instead, thus truly contributing to the subsequent provision of PVT integrity. Note that this has not been traditionally the case in civil aviation applications, where it is assumed that local effects have a controlled influence on the received signal.

In this paper, we will concentrate on multipath and NLOS as the major impairments that can threat the integrity of GNSS signals in urban environments. Multipath mitigation has attracted the attention of many researchers during the past years, leading to a plethora of contributions in the existing literature such as the narrow and strobe correlator [4], the early/late slope or the multipath estimating delay lock loop (MEDLL) [5]. Nevertheless, detection of these threats has often remained in a secondary place, when indeed it is even more important than mitigation, especially for NLOS. The reason is that before using mitigation techniques, we can benefit from knowing whether multipath is present or not. Moreover, in many cases, eliminating multipath is not as relevant as knowing if the signal is heavily affected by multipath. Many users can be satisfied with that, thus not requiring complicated multipath mitigation techniques.

In the past years, different contributions for multipath detection in GNSS have appeared. Most of them propose the use of external information like map-matching for identifying local threats (e.g. signal blocking obstacles) [6], external sensors for obtaining redundant information [7], or fisheye cameras to obtain a sky plot and determine the geometrical distribution of satellites in view [8]. Nonetheless, the use of external aid needs prior information about the user environment or external hardware, which is not always available in mass-market GNSS receivers. In addition, up to now, detection of degrading effects has been addressed adopting a classic detection framework, which is often not well suited to fulfill the requirements of liability- or safety-critical applications. In these applications, the key objective of signal-level integrity method should be to detect the occurrence of a degrading effect as quick as possible, within the current batch of samples being processed.

In this work, we take a leap forward in the field of multipath detection, and we propose the adoption of a family of CUSUM-based on-line change detectors. The CUSUM algorithm is probably the most popular change detector [9]. It was derived as a solution to "statistical change detection" problems, where the incoming measurements exhibit a sudden change in either their statistical parameters (e.g. mean, variance) or even in the type of its probability density function. This approach fits very well into the kind of threats and abnormal events (in particular, multipah) that a GNSS receiver may experience in real life, where sudden changes in the properties of the received signal are often common. In order to cope with any kind of multipath situation, we propose a set of analyses based on values directly or indirectly available at the tracking loop outputs (i.e. postcorrelation metrics).

While the quickest detection framework has been extensively applied into a wide range of fields, it has barely been used in the GNSS domain. Based on this observation, we already addressed this problem for the case of multi-antenna GNSS receivers (see [10], [11]), and for single-antenna receivers in [12]. Moreover, in [13] we provided a framework to introduce and stimulate the use of quickest detection in GNSS. In the present paper, though, we focus only on multipath detection for singleantenna GNSS receivers. Therefore, our contribution in this work is twofold: (i) to provide a complete set of quickest multipath detection techniques able to cope with different situations; (ii) to provide real signal results of the proposed techniques in order to show the capability of our techniques to work in real conditions.

Theoretical results on quickest detectors are complemented in this work with very extensive experimental tests using real signals, which were gathered in the framework of the "Integrity Receivers" project (iGNSSrx) funded by the European Commission, being UAB part of the consortium that carried out the project. The performance of the proposed detection algorithms in real working conditions is presented. This serves as a reference for tuning the parameters of algorithms to be used later on in real receivers. The results confirm the suitability of the proposed algorithms for real-time integrity monitoring, thus improving the navigation quality.

Next, we introduce the signal model used for developing the detection algorithms. In the following section, we present the quickest multipath detectors and their configuration. Finally, we present numerical results obtained with real life signals, assessing the performance of the proposed methodology.

SIGNAL MODEL

Let us consider a sequence of independent observations $\mathbf{x} = [x(0), x(1), ..., x(v), ..., x(K-1)]^T$, where v is the time instant at which an integrity threat appears (e.g. multipath). Consequently, it is assumed that before v (i.e. at hypothesis \mathcal{H}_0) the observation x(n) follows a given statistical distribution f_0 , whereas after the change (i.e. at hypothesis \mathcal{H}_1) it follows a different one, f_1 :

$$\begin{aligned} &\mathcal{H}_0: \ x(n) \sim f_0\big(x(n)\big), \quad n < v \\ &\mathcal{H}_1: \ x(n) \sim f_1\big(x(n)\big), \quad n \ge v. \end{aligned}$$

Based on these premises, sequential change detection aims at finding the strategy that minimizes the detection delay, while keeping the mean time between false alarms larger than a conveniently set value. For this purpose, the CUSUM algorithm was proposed, which is based on the logarithm of the likelihood ratio, defined by

$$LLR(n) \doteq \ln \frac{f_1(x)}{f_0(x)}$$
(2)

and referred to as the log-likelihood ratio (LLR). For the sake of clarity we have omitted the time index n from the random variables x, keeping in mind that each variable correspond to a given time instant (i.e. x(n)).

From [13], we know that if the LLR is completely known, the CUSUM is defined as the next decision rule:

$$g(n) \doteq \left(g(n-1) + \text{LLR}(n)\right)^{+} \ge h \tag{3}$$

for some threshold h, where $(x)^+ = \max(0, x)$. By doing so, it is known that the CUSUM algorithm minimizes the detection delay (i.e. $\overline{\tau}$) subject to a false alarm rate constraint (i.e. $\overline{T} \ge N_{fa}$). Specifically, the optimality of the CUSUM is achieved with the following results:

$$T \ge e^{h},$$

$$\overline{\tau} \le \frac{h}{K[f_1, f_0]},$$
(4)

with $K[f_1, f_0] \doteq E_1[LLR(n)]$ the Kullback-Leibler divergence, and $E_1[\cdot]$ the expectation under f_1 .

For the particular case of multipath detection in GNSS, the detection must be carried out at the acquisition and/or tracking stage (i.e. after despreading), where measurements such as the estimated C/N_0 , code discriminator output (i.e. DLL) and the shape of the correlation curve fluctuates with the presence of NLOS

and multipath [1]. Consequently, we will be able to detect NLOS and multipath based on the fluctuations of these measurements. Let us first define the signal model for the multipath detection problem. We assume that we operate at the output of a bank of *L* correlators with a given post detection integration time (PDI). With this scheme, the following hypotheses can be written, depending on the presence (\mathcal{H}_1) or absence (\mathcal{H}_0) of multipath:

$$\mathcal{H}_0: \mathbf{y}(k) = \boldsymbol{\alpha}_0(k) + \mathbf{w}(k),$$

$$\mathcal{H}_1: \mathbf{y}(k) = \boldsymbol{\alpha}_0(k) + \boldsymbol{\alpha}_1(k) + \mathbf{w}(k),$$
 (5)

where $\mathbf{y}(k)$ is the $(L \times 1)$ vector with the *L* correlator outputs at the *k*-th PDI period. $\boldsymbol{\alpha}_j(k)$ is the vector with the *L* complex amplitude, with $j = \{0,1\}$ corresponding to the LOS and multipath post-correlation outputs components, respectively. Finally, $\boldsymbol{w}(k)$ is the noise vector whose components are the post-correlation complex Gaussian noise.

Without loss of generality only one multipath ray is assumed, representing either a single dominant reflector or a virtual reflector, the latter being the mathematical equivalent of a combination of physical reflectors. This model is even more complicated than the showed one, since the complex amplitudes $\alpha_j(k)$, besides being correlated, might be modeled as random variables because they depend on the multipath parameters. This is so because the multipath parameters are unknown in practice and then they might also be modeled as random parameters. For this reason the statistical characterization of the multipath metrics may be difficult to obtain, and we proceed as follows:

- Despite the complexity of the signal model, the multipath metrics can be approximated as Gaussian variables in both absence and presence of multipath.
- We obtain the statistical characterization experimentally, by evaluating the distribution of real data. This is done in order to confirm the Gaussian distribution.
- Hence we can use the CUSUM for a change on the mean, variance or both of a Gaussian variable.

QUICKEST MULTIPATH DETECTION

In this section, we propose some sequential or "automatic" approaches in order to detect the presence of multipath. These automatic approaches make use of the tracking measures (i.e. multipath metrics) and are intended to provide quickest detection, with the aim of detecting as soon as possible the presence of any kind of multipath. We show the experimental statistical characterization that is required for quickest detection on different multipath detection metrics (i.e. C/N₀ estimate, DLL output and the Slope Asymmetry Metric, SAM). In particular, we will focus on the C/N₀-based threat detection and we will provide a summary of the main results for the DLL- and SAM-based techniques, which are further analyzed in [12] and [13], respectively. The results that we will show for presenting the statistical characterization of C/N₀ values are obtained by analyzing data captured with a real GNSS receiver under the framework of the iGNSSrx project. These data were gathered by a moving vehicle in a dense urban environment, in London's (UK) downtown. The vehicle was under benign conditions (i.e. \mathcal{H}_0) the first 200s, and then it changed to harsh conditions (i.e. \mathcal{H}_1) until the end of the data record. We discriminate between benign and harsh conditions with the aid of a truth reference for calculating the position error. For the data under benign conditions we obtain a 2D mean positioning error of 2m, whereas for the data under harsh conditions we obtain a mean positioning error of 50m.

Analysis of the C/N₀

Under benign conditions the C/N_0 should be relatively constant in a given range, normally 42–47dB-Hz. On the other hand, under harsh conditions (i.e. with multipath) the C/N_0 increases or decreases depending whether the multipath is constructive or destructive, respectively, and shows higher variance. Hence, this is equivalent to having a change on the mean of the C/N_0 . This is shown in Figure 1, which presents the C/N_0 of one of the satellites in view for the case of being in benign conditions the first 200 s and in harsh conditions the rest of the gathered data.



Figure 1 - C/N₀ time-evolution of one of the satellites in view.

We see how the C/N_0 presents a change on the mean after second 200. Before the change, the mean value is about 44dB-Hz with variations up to 3dB, whereas after the change, the mean is around 33dB-Hz and the C/N_0 values take more variations than before the change. Thereby, we can contribute to solving the problem of multipath detection as a C/N_0 mean change detection. To do so, we have to statistically characterize the C/N_0 . Since the C/N_0 estimates are based on the average of the prompt correlator (i.e. aligned with the estimated time-delay of the received signal) [14], the C/N_0 estimates can be approximated, using the central limit theorem, by a Gaussian random variable. Therefore, we can write the following hypotheses:

$$\begin{aligned} \mathcal{H}_{0} \colon x_{\mathrm{c}}(m) &\sim \mathcal{N}\left(\mu_{0}^{(\mathrm{c})}, \sigma_{0}^{2^{(\mathrm{c})}}\right), \qquad m < v \\ \mathcal{H}_{1} \colon x_{\mathrm{c}}(m) &\sim \mathcal{N}\left(\mu_{1}^{(\mathrm{c})}, \sigma_{1}^{2^{(\mathrm{c})}}\right), \qquad m \geq v. \end{aligned}$$
 (6)

This is shown in Figure 2, which presents the statistical characterization of the C/N_0 estimates (linear units) corresponding to the values of Figure 1. In the left plot,



we see how the histogram in benign conditions is centered about 44dB-Hz (i.e. 2.5e4), and we can see how the main component of the histogram fits pretty well to a Gaussian distribution with mean and variance equal to 43.7dB-Hz and 1.12e7, respectively. Hence, we can conclude that the C/N_0 values in linear magnitude can be modeled as a Gaussian random variable, in absence of multipath. On the other hand, in the right plot, we see how the histogram in harsh conditions reflects the variation of the mean value after change. This may be due to the variation of multipath conditions, producing a non-stationary histogram. However, we see how the main component of the histogram is quite Gaussian shaped.

In gross terms, we could say that the C/N₀ distribution in both hypotheses \mathcal{H}_0 and \mathcal{H}_1 can roughly be approximated by a Gaussian distribution with different mean and variance. However, the variance after change will depend on the multipath parameters and then it will be unknown. In addition, the change on the mean is large enough to neglect the fact that there is a slight change in variance, too. Hence, we can use the CUSUM algorithm for a change in the mean of a Gaussian distribution in order to detect the presence of multipath. To do so we propose the following configuration of the Gaussian distribution:

- $\mu_0^{(c)}$: Under ideal conditions the mean of the C/N₀ should be around 45dB-Hz, however depending on the number of bits of the ADC and environment conditions it may be slightly lower. Then we might fix the mean under \mathcal{H}_0 to 43-44dB-Hz, since it is the expected value under benign conditions. Nevertheless, it is possible that for any reason the mean value may be lower, even in the case of absence of multipath (e.g. receiver losses, shadowing, ...). It is for this reason that it may be convenient to use the estimate of the mean C/N₀ value. Therefore we can sequentially estimate the mean C/N₀ value and use it as the mean before the change for the CUSUM algorithm. In this way, the mean before the change is set more precisely, but we have to make sure that the estimation corresponds to the hypothesis \mathcal{H}_0 and the change has not occurred yet. This is done, by fixing a limit for the mean before the change (e.g. 35dB-Hz), and if the estimate of the mean C/N₀ value is below this limit, the estimate is discarded and we use the previous estimate that is above the limit.
- $\sigma_0^{2^{(c)}}$: As we said, the variance of the C/N₀ is unknown a priori. We know that it depends on the noise power and some expressions may be found in the literature showing the relationship between the C/N₀ variance and noise

power. However, these expressions may also depend on other parameters that are not under our control. It is for that reason that is difficult to have a perfect knowledge on the actual variance of the C/N_0 . Hence, we propose to fix the variance under benign conditions according to the maximum allowable variation due to peaks of attenuation. In this way, fixing this maximum variation we can define the variance of the C/N_0 under benign conditions as:

$$\sigma_0^{2^{(c)}} \doteq \left(\frac{\Delta (C/N_0^{lin})_{\max}}{3}\right)^2.$$
(7)

This is so because we know that for a Gaussian distribution the 99.86% of the values are comprised in the interval $\mu \pm 3\sigma$. Equation (7) uses the maximum variation in linear magnitude, which is obtained form dBs as follows: $\Delta (C/N_0^{lin})_{max} = \mu_0^{(c)} \cdot \left(10^{\frac{\Delta (C/N_0^{dB})_{max}}{10}} - 1\right)$, with the

 C/N_0 mean in linear units. Therefore, for instance in our case we see that the C/N_0 varies around 2-3dB due to attenuation or some negligible multipath, and then using (7) for the worst case (i.e. $\Delta(C/N_0^{dB})_{max} = 3$ dB) we obtain a variance equal to $\sigma_0^{2^{(c)}} = 5.77$ e7.

• $\mu_1^{(c)}$: The mean after change for the CUSUM algorithm must be fixed as follows:

$$\mu_1^{(c)} \doteq \mu_0^{(c)} / 10^{\Delta \mu} , \qquad (8)$$

with $\Delta \mu$ the minimum fixed mean change in dBs and

$$\mu_0^{(c)} \doteq \overline{(C/N_0)} | \mathcal{H}_0 \tag{9}$$

the mean before the change for the CUSUM algorithm, which is fixed as the average of the obtained C/N_0 , making sure that the values correspond to the hypothesis \mathcal{H}_0 .

With this configuration we are able to use the CUSUM algorithm for a mean change in a Gaussian distribution. This is so because if we configure the mean before the change as the estimated C/N0 mean value under ideal conditions, whenever the mean C/N₀ value changes due to multipath, it will be detected. We note that the fixed variance before change (i.e. $\sigma_0^{2^{(c)}}$ 5.77e7) is not equal to the variance of the fitted Gaussian distribution (see left plot in Figure 2). This is so because the variance of the fitted distribution corresponds to the variance of the presented C/N_0 results, where the variations with respect the mean value are always below 3dB. However, in order to have into account those spikes, we have to increase the configured variance before the change in the CUSUM algorithm. Thus, we prevent false detection due to the increment of the attenuation, which can be confused by the presence of multipath.

On the other hand, in this case the change for the C/N_0 under harsh conditions is about 10dB, but maybe it is too large. This suggests that we are under extreme harsh conditions (i.e. either NLOS or high amount of reflections), and then for LOS or more moderate multipath, the change might be smaller. Therefore a more

general mean change (i.e. $\Delta \mu$) might be $\Delta \mu = 5-7$ dB, which is a change large enough to be due to multipath but not to mere fast fading. Moreover, this mean change is small enough to allow the detection of multipath also in less harsh conditions (i.e. LOS conditions), where the mean change is lower than that reflected in Figure 1.

We know that the presence of multipath can incur in either an increase or decrease of the C/N_0 mean, and then we should apply a two-sided mean change CUSUM in order to detect both possible changes. That is:

$$LLR_{c}^{\pm}(k) = \frac{\mu_{1}^{\pm^{(c)}} - \mu_{0}^{(c)}}{\sigma_{0}^{2^{(c)}}} \left(x_{c}(k) - \frac{\mu_{1}^{\pm^{(c)}} + \mu_{0}^{(c)}}{2} \right), \quad (10)$$

with $\mu_0^{(c)}$, $\sigma_0^{2^{(c)}}$ defined above, $\mu_1^{\pm^{(c)}} = \mu_0^{(c)} \pm \Delta \mu$, and $x_c(k)$ the C/N₀ estimate at the *k*-th PDI. Thereby, we can use the decision rule in (3) with the two LLR in (10) (i.e. + and -), leading to the performance in (4) with

$$\mathbf{K}[f_1, f_0] = \frac{\left(\mu_1^{+(c)} - \mu_0^{(c)}\right)^2 + \left(\mu_1^{-(c)} - \mu_0^{(c)}\right)^2}{2\sigma_0^{2(c)}}.$$

Figure 3 presents the CUSUM behavior for the data displayed in the left plot of Figure 1. We present two configurations: (i) fixed minimum change of $\Delta \mu = \pm 5 \text{dB}$ and fixed maximum variation under \mathcal{H}_0 of $\Delta(C/$ N_0^{lin} _{max} = ±3dB (i.e. test 1), and (ii) $\Delta \mu = \pm 6$ dB and $\Delta (C/N_0^{lin})_{max} = \pm 4$ dB (i.e. test 2). The left plot shows the results for the test 1. With this configuration, we see how before the change the CUSUM is close to 0 except for two peaks that appear around 50 and 130 seconds. We see that the spikes exceed the threshold, which is fixed in order to obtain a time between false alarms of 1 hour (i.e. h = 12), and then the CUSUM decides for the presence of multipath. For these spikes, multipath is not present, and then the corresponding decisions when exceeding the threshold become false alarms. A possible solution to these false alarms is to increase a bit the maximum allowable variations under \mathcal{H}_0 as well as the minimum change due to multipath. This is done by the configuration in test 2, whose results are shown in the right plot of Figure 3. Doing so, we see how the CUSUM metric is 0 under \mathcal{H}_0 and it starts drifting upwards just after 200s. This increase of the CUSUM metric is big enough to exceed the detection threshold just at the moment when multipath appears. Therefore, using this new configuration we reduce the number of false alarms due to peaks of attenuation.



Figure 3 - Multipath detection for test 1 (left) and test 2 (right).

Notwithstanding, multipath that produces either smaller changes on the C/N_0 mean than 6dB or smaller variations than 4dB will not be detectable.

Analysis of the code discriminator output

We know that under benign conditions (i.e. \mathcal{H}_0), the DLL is close to zero, with all variations due to the noise and to the small corrections needed to track the code dynamics (i.e. user movement). However, when a single multipath ray is present (i.e. \mathcal{H}_1) we see how the DLL output exhibits a spike in order to compensate for the shift in the code position due to multipath. Afterwards, the DLL reverts to zero. Nevertheless, since multipath conditions vary in practice, the DLL output will exhibit different spikes along the period when multipath is present, which is translated into an increase of the variance of the DLL.

This problem was addressed in [12], which formulates the DLL-based as a variance Gaussian change as follows:

$$\begin{aligned} &\mathcal{H}_{0} \colon x_{\mathrm{d}}(m) \sim \mathcal{N}\left(\mu_{0}^{(\mathrm{d})}, \sigma_{0}^{2^{(\mathrm{d})}}\right), \qquad m < v \\ &\mathcal{H}_{1} \colon x_{\mathrm{d}}(m) \sim \mathcal{N}\left(\mu_{0}^{(\mathrm{d})}, \sigma_{1}^{2^{(\mathrm{d})}}\right), \qquad m \ge v \end{aligned}$$
 (11)

with the following configuration for the Gaussian distribution parameters:

 $\blacksquare \quad \mu_0^{(d)}: \text{ We can fix it to } 0:$

$$\mu_0^{(d)} = 0. \tag{12}$$

• $\sigma_0^{2^{(d)}}$: As for the C/N₀ case, this value is unknown a priori. Thus, as for the C/N₀ we fix the variance under benign conditions according to the maximum allowable variations on the DLL values under \mathcal{H}_0 , as follows:

$$\sigma_0^{2^{(d)}} \doteq \left(\frac{\Delta(DLL_0)_{\max}}{3}\right)^2.$$
(13)

For example, in [12] we fixed the maximum allowable variations to $\Delta (DLL_0)_{\text{max}} = 0.04$ chips, leading to a variance before the change equal to $\sigma_0^{2^{(d)}} = 1.78\text{e-}4$.

• $\sigma_1^{2^{(d)}}$: Similarly, we fix the variance under harsh conditions as the minimum detectable variability DLL due to multipath:

$$\sigma_1^{2^{(d)}} \doteq \left(\frac{\Delta(DLL_1)_{\min}}{3}\right)^2.$$
(14)

For instance, in [12] we fixed $\Delta (DLL_1)_{\min} = \pm 0.07$ chips, which results in $\sigma_1^{2^{(d)}} = 5.5e-4$.

Hence, with this configuration we are able to use the CUSUM algorithm for a Gaussian variance change, which has the following LLR: LLR_d(k) =

$$= \ln\left(\frac{\sigma_0^{(d)}}{\sigma_1^{(d)}}\right) + \frac{\left(x_d(k) - \mu_0^{(d)}\right)^2 \left(\sigma_1^{2(d)} - \sigma_0^{2(d)}\right)}{2\sigma_0^{2(d)}\sigma_1^{2(d)}},$$
 (15)

with the variances and mean defined above, and $x_d(k)$ the DLL output at the *k*-th PDI snapshot. Thereby, we can use the decision rule in (3), leading to the performance in

(4), with
$$K[f_1, f_0] = \ln\left(\frac{\sigma_0^{(d)}}{\sigma_1^{(d)}}\right) + \frac{\left(\sigma_1^{2(d)} - \sigma_0^{2(d)}\right)}{2\sigma_0^{2(d)}}$$
. For a

detailed analysis of the DLL-based detection see [12].

Analysis of the correlation curve

We know that under benign conditions the correlation curve (i.e. correlation between local replica and received signal) is symmetrical, but it loses the symmetry under harsh conditions. This can be measured by the slope asymmetry metric (SAM), which under ideal conditions should be close to cero (indicating symmetry), whereas when multipath is present it should depart from zero (indicating asymmetry). The main idea is to compare the left and right slopes of the correlation curve, so that when it is symmetrical both slopes are equal, but sign reversed, and thus their sum is zero. On the other hand, when the curve is not symmetrical, the slopes are not identical and then the difference departs from zero. The metric can be defined as:

$$x_s(k) \doteq \hat{a}_l(k) + \hat{a}_r(k), \tag{16}$$

where \hat{a}_l and \hat{a}_r are the estimated slopes of the left and right sides of the correlation peak, respectively.

The SAM was analyzed in [13], which formulates the problem as follows:

$$\begin{aligned} &\mathcal{H}_{0}: \ x_{s}(m) \sim \mathcal{N}\left(\mu_{0}^{(s)}, \sigma_{0}^{2^{(s)}}\right), \qquad m < v \\ &\mathcal{H}_{1}: \ x_{s}(m) \sim \mathcal{N}\left(\mu_{1}^{(s)}, \sigma_{1}^{2^{(s)}}\right), \qquad m \geq v \end{aligned}$$

with the following configuration for the Gaussian distribution parameters:

• $\mu_0^{(s)}$: It should be equal to 0, but in practice it is slightly larger:

$$\mu_0^{(s)} = \kappa \sim 0. \tag{18}$$

• $\sigma_0^{2^{(s)}}$: Fixed according to the maximum allowable variations on the SAM under \mathcal{H}_0 , as follows:

$$\sigma_0^{2^{(s)}} \doteq \left(\frac{\Delta(SAM_0)_{\max}}{3}\right)^2.$$
(19)

• $\mu_1^{(s)}$: Unknown, but it might be fixed as follows:

$$\mu_1^{(s)} \doteq \mu_0^{(s)} \pm \delta$$
, (20)

with δ a proper value selected experimentally.

• $\sigma_1^{2^{(s)}}$: Fixed as the minimum detectable variability of the SAM due to multipath:

$$\sigma_1^{2^{(d)}} \doteq \left(\frac{\Delta(SAM_1)_{\min}}{3}\right)^2.$$
 (21)

Hence, with this configuration we can use the CUSUM algorithm for a Gaussian change in both mean and variance. The LLR in this case is as follows:

$$LLR_{s}(k) = \ln\left(\frac{\sigma_{0}^{(s)}}{\sigma_{1}^{(s)}}\right) + \frac{\left(x_{s}(k) - \mu_{0}^{(s)}\right)^{2}}{2\sigma_{0}^{2(s)}} - \frac{\left(x_{s}(k) - \mu_{1}^{(s)}\right)^{2}}{2\sigma_{1}^{2(s)}},$$
(22)

with the variances and mean defined above, and $x_s(k)$ the SAM at the *k*-th PDI snapshot defined in (16). Thereby, we can use the decision rule in (3), leading to the performance in (4), with

 $K[f_1, f_0] = \ln\left(\frac{\sigma_0^{(s)}}{\sigma_1^{(s)}}\right) + \frac{\left(\mu_1^{(s)} - \mu_0^{(s)}\right)^2}{2\sigma_0^{2(s)}} - \frac{1}{2}.$ However, as we indicated in [13], we know that the same start

indicated in [13], we know that the mean change of the SAM is predominant in LOS, whereas the variance change is predominant in NLOS. Therefore, we can use two different CUSUM algorithms, one for detecting the change in variance (i.e. NLOS) and another for detecting the mean change (i.e. LOS). The expression for the two LLR would be like (15) and (10), respectively, but with the SAM parameters. For further analysis see [13].

REAL DATA ANALYSIS

This section describes the integration of the previous signal integrity algorithm (i.e. DSP anomaly detector) with the GNSS navigation and PVT integrity algorithm. The approach that will be herein proposed works offline and is executed sequentially:

- 1) The raw GNSS data is processed with the DSP anomaly detector, to generate an output file with validity flags for the processed epochs.
- 2) The PVT navigation and integrity algorithm is executed (PVT+I), using the file in step 1 as an input in order to exclude those measurements declared as faulty from the computation of the navigation solution.

In order to test the performance of this methodology, a test including one urban scenario is presented here. In particular, we have selected urban data captured by a moving vehicle in a dense urban environment, in London's (UK) downtown (see trajectory in Figure 4). Since the trajectory is in a urban area, the scenario covers both static (due to traffic) and dynamic cases with a maximum velocity of 50km/h. Finally, it is important to say that we use here an offline approach in order to analyze the integration between the DSP and PVT algorithms. However, in a practical implementation, the output flags from the DSP detector will feed the navigation algorithm online (i.e. real-time).



Figure 4 – Truth trajectory of the analyzed scenario with data captured in London's (UK) downtown.

The analysis performed in this chapter has illustrative purposes, and therefore it will be simplified as follows:

1) A brief description of the fault detection and exclusion (FDE) implemented by the DSP anomaly

detector will be performed, in order to output the fault flags file.

- 2) The impact of these flags in the rejection of GPS measurements will be evaluated, paying special attention to the missed detection and false alarms probabilities of pseudo-range measurements.
- 3) We will observe the impact of the measurement rejection in the navigation performance, in the particular case of using a dual GPS+GLONASS constellation.

Generation of the signal integrity flags

This section describes the generation of the fault measurement flags at signal level that will be used by the PVT+I algorithm in order to discard the satellites with any disturbing effect from the positioning calculation. We generate a flag for every satellite in view for every second, which is the time configured in the used PVT+I algorithm. The methodology for the fault measurement flags generation is the following:

- 1) Generation of the detection metrics given by processing the input data with the DSP anomaly detector (i.e. multipath metrics). The behavior of these metrics indicates the presence or absence of any disturbing effect in the received signal.
- 2) Generation of signal integrity flags from the CUSUM values of the generated metrics in such a way that if the CUSUM is above the detection threshold we activate the flag indicating that the measurement of the corresponding satellite at this time is a fault.
- 3) Since we have three different metrics, in the previous step we generate three signal integrity flags. Hence, we generate a unique flag with the combination of the three generated flags.
- 4) The flag in the previous step is generated for every satellite but at a snapshot time given by the DSP anomaly detector. However, we have to generate one flag every second, which is the configuration of the used PVT+I algorithm. Then, we generate the fault measurement flag every second by combining (with a certain logic that will be described later on) consecutive flags generated in the previous step.

These flags are used by the PVT+I algorithm with the aim of deciding which satellites are used for the navigation solution. We configure the DSP anomaly detector for generating metrics with a snapshot time of 20ms and then we have a flag every 20ms (at step 3) that must be converted to a flag every second (step 4). Consequently we have to average 50 flags of 20ms in such a way that if there are more flags indicating the presence of a threat than a pre-established threshold (e.g. more than 10 flags of 20ms), we indicate a fault in the 1s flag.

The detection algorithms implemented in the DSP anomaly detector and described in the previous section are formulated for detecting a change from nominal conditions (i.e. no threat) to harsh conditions (i.e. integrity threat). But, in real life conditions the integrity threats can appear and disappear at any time, and we need to detect this situation. Otherwise, when the integrity threat disappears, the CUSUM algorithm would take a too long time to switch back again to the initial benign decision. This is shown in the left plot of Figure 5, where we see (upper plot) a change of the metric at sample 100 and it presents an opposite change at sample 200. Using the CUSUM configuration for detecting only the presence of threat (left plot), we see in the lower plot how the CUSUM raises the threshold just when the change appears (i.e. sample 100) and it continues increasing until sample 200, when it starts decreasing. However, because of the large value that has been accumulated by the CUSUM up to sample 200, it takes a very long time to decrease it and to cross the threshold again for declaring the restoration of the initial working conditions.



Figure 5 - CUSUM change detection in the case of changing from good to bad conditions and vice versa. Single (left) and double CUSUM (right).

In order to circumvent this limitation and to allow the promptly detection of the disappearance of threats, we have to use another CUSUM aimed at detecting this inverse change. Based on the definition of the loglikelihood ratio (LLR) in (2) it is easy to show that this CUSUM has the same expression but using the negative value of the log-likelihood ratio. Thereby, when we are in nominal conditions we use the CUSUM for detecting the appearance of threats. When this CUSUM declares the presence of a threat, we start using the other CUSUM (negative LLR). This 'negative' CUSUM will remain close to zero as long as the threat remains present, and it will start increasing just when the threat disappears. Then, when the 'negative' CUSUM raises the threshold, we can restart the 'positive' CUSUM and be able to detect either the appearance or disappearance of integrity threats (see right plot of Figure 5).

Next we show the results of processing the data collected with the modification above. The results show the multipath detection metrics as well as the CUSUM values generated by the anomaly detector for different GPS satellites in view. Among all the satellites in view we can do the following classification:

SVN in view all the time (15, 17 and 24): These satellites are in view during the whole observation time, as it is shown for SVN 15 in Figure 6. The results show how it is in nominal conditions most of the time. This is seen looking at the metrics, which in general present a nominal behaviour but they present harsh behaviour in some moments (e.g. SVN15 is affected by harsh propagation conditions between minute 70 and 100), or at the CUSUM





behaviour, which in general is below the threshold (black line) but it is above in some moments.

- SVN in view most of the time (12, 18 and 26): These satellites are in view most of the time but they are out of view in some moments. This is shown for SVN 12 in Figure 7, which shows how it is in harsh conditions most of the time except for some moments (e.g. min. 110-115).
- SVN out of view most of the time (5, 6, 8, 9, 14, 22, 25, 28): They are only in view a small portion of time. This is shown in Figure 8, which shows how SVN 28 is in benign conditions most of the time it is in view.



Figure 8 - Results for SVN 28. C/N₀ (left), DLL (middle) and SAM (right).

With the results shown above we generate the fault measurement flags from the CUSUM values. We do so in such a way that if the CUSUM is above the threshold, we activate the flag indicating that the measurement at the corresponding time is a fault, and then it should be rejected. Doing so we can see the availability of each GPS satellite in view (i.e. time they do not provide faulty measurements) given by every metric. Figure 9 shows how SVN 15, 18, 24, 26, 28 are available (i.e. do not present faulty measurements) more than the 50% of the time they are in view, while, the rest of SV present different availabilities but all of them are below the 50% of the time. Moreover, we see how the results for the DLL and the SAM are quite similar, indicating that they declare a faulty measurement roughly at the same time. In contrast, the C/N_0 exhibits more restrictive results in the sense that it detects more faulty measurements than the other two, and then the use of the satellites for the positioning calculation is smaller. This is because the



Figure 9 - Percentage of time that the different satellites are decided to be used for the different metrics. C/N_0 (left), DLL (middle) and SAM (right).

 C/N_0 as well as presenting variations due to multipath it also varies due to other effects such as shadowing or peaks of attenuation that do not present variations in the other metrics (i.e. DLL and SAM).

This is better seen in Table 1, which shows the availability in percentage for the most available satellites. For instance, we see for the SVN17 how the DLL and SAM exhibit a percentage of 49% and 47%, respectively, whereas the C/N_0 becomes 29%. On the other hand, we can also see how there are some exceptions where the DLL and SAM do not coincide so much. For instance, we see for the SVN 12 that the values for the C/N_0 and DLL are similar (i.e. 27% and 30%, respectively), but the SAM presents a higher value of 41%.

 Table 1 - Numerical percentage of time that the most available satellites are used.

| SVN | 12 | 15 | 17 | 24 | 28 |
|-----|-----|-----|-------------|-----|-----|
| CN0 | 27% | 68% | 29% | 84% | 68% |
| DLL | 30% | 70% | 49 % | 88% | 75% |
| SAM | 41% | 71% | 47% | 89% | 75% |

These differences between metrics indicate a disagreement between integrity detection techniques, in the sense that one is declaring benign conditions while some other is declaring harsh ones. In order to circumvent this situation, we present two alternatives for generating the flags to be fed into the PVT+I algorithm. First of all, we have to generate one flag from the three flags given by every metric:

Restrictive flag: This option is the default one for generating the fault measurement flags in integrity algorithms. It involves declaring a faulty measurement whenever any of the metrics declares some threat. The expression for the restrictive flag at snapshot *i* is as follows:

$$T_r(i) \doteq T_c(i) | T_d(i) | T_s(i)$$

with T_c , T_d and T_s the flag for the C/N₀, DLL and SAM, respectively, and | the logical OR operator. This flag is named restrictive since in this way we obtain the maximum number of faulty measurements.

Permissive flag: This option is proposed in order to compensate the restrictiveness of the C/N₀ metric and then avoid detections due to other effects different from multipath like shadowing or attenuation. The idea is to declare a faulty measurement only when at least two metrics declare a fault. Thereby, we also provide robustness against false alarms because a false alarm will be declared only when two metrics produce a false alarm at the same

time, which is an unlikely situation. The expression for the permissive flag at snapshot *i* is as follows:

$$T_p(i) \doteq (T_c(i) + T_d(i) + T_s(i)) > 1.$$

In this case the flag is named permissive since it produces less fault measurements.

Next, once we have generated the flag from the three metrics, we have to generate one flag every second from the previous flags, which are generated every 20ms. This is done averaging 50 flags of 20ms in the following ways:

Restrictive averaging: This option declares a faulty flag if at least 5 flags of 20ms are declared faulty:

$$F_r(m) \doteq \operatorname{sum}^{(m)}(T_r) \ge 5,$$

with *m* the new snapshot index (i.e. 1s snapshot), and the operator $sum^{(m)}$ indicating the summation of the 50 averaged flags corresponding to the snapshot *m*.

Permissive averaging: This option declares a faulty flag if at least 20 flags of 20ms are declared faulty:

$$F_p(m) \doteq \operatorname{sum}^{(m)}(T_p) \ge 20.$$

Figure 10 shows the availability of every GPS satellite in view for the two options of generating the fault measurement flags at signal level. The two plots show how for the restrictive flag the availability of the GPS satellites is smaller than for the permissive flag. Indeed, we see that for the restrictive case, only three satellites are available more than the 50% of the time, whereas for the permissive one, up to five of them are close to 50% of time. This fact only reflects the amount of time that the different satellites are used for the position calculation. In the sequel, we will use both flags configurations into the integrity PVT+I algorithm at positioning level and see which one produce better results.



Figure 10 - Percentage of time that the different satellites are decided to be used for the restrictive (left) and permissive (right) flags.

Navigation analysis

The first step to evaluate the performance of the DSP processing is to check whether the measurements flagged as faulty at this stage correspond actually to potential outliers and that healthy measurements are consequently flagged as valid. The first topic is covered with the concept of missed detection probability, while the second topic uses the false alarm probability to measure the effectiveness of the FDE algorithm. The threshold value used to determine whether a pseudo-range error can be considered to be healthy or an outlier is 20m (in absolute value). Last but not least, we will evaluate the impact of the DSP anomaly detector FDE into the navigation performance, using the GPS+GLONASS constellation.

The Table 2 summarizes the values of the probabilities of false alarm and missed detection in each configuration (i.e. permissive and restrictive). These probabilities are obtained by using the standard configuration for the CUSUM algorithms explained in the previous section for the different multipath metrics. In the light of the obtained results, we can conclude that the DSP anomaly detector FDE is capable of detecting most of the outliers (around 90% for both FDE configurations), which should imply an improvement of the accuracy in the navigation solution. On the other hand, we see that the False Alarm probability is quite high, and measurements whose error is actually low are discarded (around 39% and 50% for the permissive and restrictive configurations, respectively). Fortunately, the fact that we use the dual GPS+GLONASS constellation, which increases the measurements availability, makes that we could expect an improvement in the positioning algorithms in terms of the accuracy performance.

Table 2 - False Alarm and Missed Detection probabilities for the standard configuration.

| Flag Configuration | PFA | PMD |
|--------------------|------|------|
| Permissive | 0.39 | 0.09 |
| Restrictive | 0.50 | 0.07 |

This is validated in Figure 11, which represents the horizontal positioning error (HPE) performance of the PVT+I algorithm with each set of flags (permissive and restrictive), compared to the nominal solution, where no fault flags file is used. However, it does not mean that faulty measurements are strictly used in the nominal navigation solution, since an additional FDE is implemented at this level by the PVT+I algorithm. It is worth mentioning that the three configurations use the same data previously to the satellite exclusion (i.e. they have the same amount of satellites in view in each epoch, then each configuration excludes different satellites, which produce different positioning performances). We see how the permissive configuration improves the performance of the navigation solution. Notwithstanding, for the restrictive configuration there is not improvement with respect to the nominal one, and then the use of the signal level flags produces poor navigation results. This is due to the high false alarm rate in the restrictive configuration, which produces a large amount of discarded measurements. Therefore the number of satellites for computing the navigation solution is low, and then the positioning performance is also low.

For the permissive configuration, the false alarm rate is also high but it is low enough for maintaining sufficient measurements in order to improve the accuracy. Anyhow, additional work should be performed in order to improve the false alarm rate of the signal level FDE. The problem with the false alarms may be due to:

1) **Relation signal and positioning outlier**: We consider an outlier those measurements that present a pseudo-range error greater than 20m. However, the configuration of the CUSUM parameters might not



Figure 11 - HPE cumulative distribution for different configurations (standard configuration).

correspond to an error of 20m. Specifically, we see that with this configuration we obtain fault measurements with errors greater than 5-10m.

2) **Snapshot time averaging**: This average is not convenient at all because it may be possible that the metrics at the signal level produce a fault flag, but in average it is not a fault metric.

Therefore, a first approximation for improving the performance in terms of false alarm of the FDE at signal level, is to modify the configuration of the CUSUM parameters in a more permissive way (i.e. declaring fault measurements those errors close to 20m). In addition, the multipath metrics should be generated with a snapshot time equal to the snapshot used in the navigation algorithm (i.e. 1s). Doing so, we obtain two different configurations: (i) Maintaining the previous configuration (i.e. LowMD), but using a snapshot time of 1s (then the CUSUM parameters vary). (ii) Modifying the CUSUM parameters in order to obtain lower false alarm rate (i.e. LowFAR). These new configurations for the CUSUM algorithms are the following:

| | C/N_0 (dB) | | DLL | | SAM | |
|--------|-------------------|----|-------------------|-------------------|-------------------|-------------------|
| | $\Delta_{0,\max}$ | Δμ | $\Delta_{0,\max}$ | $\Delta_{1,\max}$ | $\Delta_{0,\max}$ | $\Delta_{1,\max}$ |
| LowMD | 5 | 7 | 0.003 | 0.005 | 0.03 | 0.06 |
| LowFAR | 10 | 11 | 0.01 | 0.02 | 0.1 | 0.15 |

The results in terms of probabilities of these configurations are shown in Table 3. We see how the LowMD results are similar to the obtained with the previous configuration (see Table 2), whereas for the LowFAR we obtain better results in terms of false alarms but at the expense of increasing the missed detections. The results in terms of HPE are presented in Figure 12, which shows how the performance is improved by using the fault flags file for both configurations. In fact, we can notice that it is the LowMD configuration the one that provides the best performance. Therefore, we can conclude that is better to detect as many threats as possible (i.e. low missed detections) instead of having a few false alarms at the expense of increasing the missed

 Table 3 - False Alarm and Missed Detection

 probabilities for the modified configuration.

| Flag Configuration | PFA | PMD |
|--------------------|------|------|
| Permissive_LowMD | 0.37 | 0.09 |
| Permissive_LowFAR | 0.17 | 0.28 |



Figure 12 - HPE cumulative distribution (modified configurations).

detections. This is in line with the expected results, considering the dual GPS+GLONASS constellation, which increases the measurement availability, and then the false alarm rate impact on the accuracy is not as critical as not detect the actual faults. Moreover, we present in Figure 12 the curve corresponding to the perfect flags for measurement errors of 20m (i.e. activate the flags when the actual error in pseudo-range measurements is greater or equal to 20m). We see how the LowFAR curve fits the perfect curve until almost the 70th percentile, and then it is always below the perfect curve and approaching the nominal one. This is because the 70% of the time the LowFAR configuration is linked with a measurement error.

On the other hand, we see how the LowMD configuration outperforms the perfect curve. This is due to the fact that this configuration is linked with a measurement error lower than 20m, and then it improves the accuracy, with respect to the perfect flag, since we are discarding measurements with lower errors. This is reflected in Figure 13, which shows how the LowMD configuration is close to the perfect10 (i.e. measurement error threshold of 10m) curve and it is always below the perfect5 (i.e. 5m error). Therefore, we can conclude that the LowMD configuration is linked with a measurement error threshold between 5m and 10m (closer to 10m than 5m), as we expected when configured the algorithms. Therefore, the real probabilities will vary with respect to those presented in Table 3.



Figure 13 – HPE cumulative distribution comparison of different perfect flags.

 Table 4 – Analysis of the real-time processing capability.

| Multipath Metric | SAM | C/N0 | DLL |
|--|----------|---------|-------|
| Processing time for a data set with 10s observation time | 0.1652 s | 77.5 ms | 65 ms |

Real-time processing capability

Finally, we show some results on the time that the proposed algorithms take to process 10 seconds of signal. The results are obtained using a MacBook Pro computer with an Intel core i7 processor @ 2.2GHz, using 1 core. Results are presented in Table 4, which shows the time for processing the data of only one satellite. Therefore, the final results will depend on the number of satellites in view, however even in the hypothetic case we have all the GPS constellation in view (i.e. 32 satellites), taking into account the times in the table, real-time processing would be possible. This is so because here we only present the time needed to implement the CUSUM algorithm to the obtained metrics. These metrics are supposed to be already available within the GNSS receiver, and based on real-time calculations. For the SAM metric we need a multi-correlator receiver, then if the multi-correlation is real-time we can assume that the SAM metric calculation will also be so, as Table 4 shows. We see that the times for the C/N_0 and DLL metrics is quite lower than for the SAM, the reason for this is that for the SAM case we need to apply a bit synchronization in order to properly average the correlation curves for calculating the SAM metric. Moreover, there is the process itself of calculating the SAM, whereas for the C/N_0 and DLL, we only have to apply the CUSUM algorithm. Finally, we see how the application of the CUSUM, which is reflected with the C/N₀ and DLL metrics, takes around 70ms. This short time makes clear the applicability of the CUSUM algorithm into real-time integrity monitoring applications. Note that the C/N_0 CUSUM takes a slightly greater time than the DLL one. This is because for the C/N_0 we have to sequentially estimate the C/N₀ mean under ideal conditions, whereas for the DLL this is not needed.

CONCLUSIONS AND FUTURE WORK

Based on the results presented so far, we can summarize the main features of the proposed techniques in terms of their suitability to certain scenarios, the type of sequential algorithm they implement, and their main configuration parameters. A summary of the proposed multipath detection techniques, based on the big amount of data analyzed in the iGNSSrx project (which is impossible to present in a sole paper), is provided in Table 5. We can classify the proposed metric in two groups, namely Low and High multipath delay, depending on the relevance of the analysis for the two delays cases. As can be seen in the table, the classification between low and high delay corresponds to C/N₀ and DLL/SAM, respectively. These two groups of techniques are complementary, and in practice, they will have to be used jointly to ensure that we truly cover any possible multipath case. Moreover, the table below indicates the different variations of the

| Metric | C/N _o | DLL | SAM | |
|---------------------------|--|--|---|--|
| CUSUM type | Mean-change | Variance-change | Mean&Variance change | |
| Target multipath delay | Low | Hi | ligh | |
| Comment | Ideal conditions estimation | Spikes presence instead of magnitude change. | It can take nominal values even with MP (oscillations) | |
| Limits | $\begin{aligned} \Delta \mu &= \pm 11 dB \\ \Delta (C/N_0^{dB})_{\text{max}} &= \pm 4 dB \\ \mathrm{K}[f_1, f_0] &= \\ \frac{\left(\mu_1^{+(c)} - \mu_0^{(c)}\right)^2 + \left(\mu_1^{-(c)} - \mu_0^{(c)}\right)^2}{2\sigma_0^{2(c)}} \end{aligned}$ | $\Delta(DLL_0)_{\text{max}} = \pm 0.01 \text{chips} \Delta(DLL_1)_{\text{min}} = \pm 0.02 \text{chips} K[f_1, f_0] = \ln\left(\frac{\sigma_0^{(d)}}{\sigma_1^{(d)}}\right) + \frac{\sigma_1^{2^{(d)}}}{2\sigma_0^{2^{(d)}}} - \frac{1}{2}$ | $\begin{aligned} \mu_1^{\pm} &= 0.3/-0.1\\ \Delta(SAM_0)_{\max} &= \pm 0.3\\ \Delta(SAM_1)_{\min} &= \pm 0.5\\ \mathrm{K}[f_1, f_0] &= \ln\left(\frac{\sigma_0^{(s)}}{\sigma_1^{(s)}}\right) + \frac{\left(\mu_1^{(s)} - \mu_0^{(s)}\right)^2}{2\sigma_0^{2(s)}} - \frac{1}{2} \end{aligned}$ | |

 Table 5 - Properties of the proposed multipath detection algorithms.

CUSUM algorithm that the proposed multipath detection techniques are implementing (i.e. Mean-Change, Variance-Change and Mean&Variance-Change CUSUM for the C/N_0 , DLL and SAM, respectively).

The parameters that have been set before and after change also indicated (i.e. mean change are and maximum/minimum variations under ideal and harsh conditions, respectively). Those values have been chosen according to the experimental analyses of the proposed techniques with real signal captures, taking into account a snapshot time of 1s. This is done in order to represent as accurately as possible the conditions before and after change (i.e. ideal and harsh conditions) that may be found in a representative scenario. Those values determine the performance in terms of detection delay, which depends on the false alarm rate. For the latter, and since we are using the CUSUM algorithm, we have the logarithmic relationship with the threshold (i.e. $h = \ln(\overline{T})$), being $\overline{T} = t_{fa}/T_{sp}$ the metric samples between false alarms, with t_{fa} the time between false alarms. On the other hand, for the detection delay we have the known expression in (4), with $K[f_1, f_0]$ the Kullback-Leibler divergence given in the table for every metric.

Regarding the Kullback-Leibler divergence for every metric, we can say that in general the quickest detection is obtained with the SAM metric, since it uses a Mean&Variance-Change CUSUM, and the Kullback information is greater than for the two other cases. For the other two cases (i.e. C/N₀ and DLL) the delay will depend on the change magnitude. Nevertheless, the changes on the mean of the C/N₀ metric are usually larger than the changes on the variance of the DLL, and then the Kullback information for the former case will usually be greater and then give a quicker detection than by using the DLL metric. This will be true for the cases of low delay, where the C/N_0 is relevant. Nonetheless, if the multipath has a large delay, the effects on the C/N_0 will be lower than for the DLL case, and then the DLL will produce the quickest detection (with respect the C/N_0).

Hence, making use of different analyses simultaneously we maximize the insight that can be obtained during a fault event. In essence, some multipath analyses may fail in some circumstances, but it is unlikely that all of them will fail at the same time. Finally, it is also worth noting that the values presented in Table 5 might be used as defaults parameters for the CUSUM configuration in the DSP anomaly detector implementation, giving a good performance in terms of accuracy (i.e. provides better navigation results than the nominal configuration) and integrity (i.e. rises warnings related with the signal integrity that are not raised with the nominal configuration). However, further work is needed in order to obtain better results:

- Regarding the tuning of the CUSUM parameters, in order to obtain a relation between faults at signal level and pseudo-range error (e.g. configuration of parameters for declaring a fault an error greater than 20m), and then improve the results in terms of false alarms.
- The combination of flags generated by each metric should be further investigated.
- Improvements on the availability of the satellites may be beneficial for large HPE. Two possible alternatives are:
 - To fix the minimum number of satellites to use in the navigation algorithm.
 - To weight the satellite measurements instead of discarding them.

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