Charging station selection optimization for plug-in electric vehicles: an oligopolistic game-theoretic framework

J. Joaquín Escudero-Garzás, *Member, IEEE,* and Gonzalo Seco-Granados, *Senior Member, IEEE*

***Abstract*—In this paper, we describe a framework for the selection of the best charging station when plug-in electric vehicles (PEV) need to recharge their batteries, while at the same time the power utilities, which own the charging stations (CS), optimize their revenue. We use two-way communication to transmit positioning information, which is a key factor so that the plug-in electric vehicles evaluate its required energy. Within this framework, we also describe a procedure that is implemented in a distributed manner and is based on a non-cooperative oligopoly game that makes use of differentiated products theory and conjectural variations to provide a Nash equilibrium in prices and quantities.**

***Index Terms*—Battery recharge, conjectural variations, game theory, oligopoly, plug-in electric vehicle.**

I. Introduction

MANY agree that electric power systems are experiencing a profound change driven by the need for environmental compromise and energy conservation. To this end, plug-in electric vehicles (PEVs) are considered key actors in the new electric power framework due to their potential to reduce CO_2 emissions and transportation costs. On the other hand, electric power grid itself must be utterly transformed into a "smart grid", where computing and communication technologies and services are integrated with the electric power infrastructure [1].

Progressive deployment of PEVs is predicted for the next years (Fig.1). For instance, the U.S. Department of Energy (DOE) gives estimates for the amount of plug-in vehicles that will be more than 1.2 million by the end of 2015 [2]. Yet, this deployment can lead to undesired situations if it is made without control: a number of PEVs plugged into, e.g., a parking garage could overload the grid, which is often working close to its operational limit. Moreover, the unbalanced load conditions may result in degradation of power quality and damage utility equipments and customer appliances [3], [4]. These and some more factors, such as the need of unexpected recharging, make think that, for the PEVs, a recharging infrastructure is required with the same functionalities as the currently deployed gas station networks.

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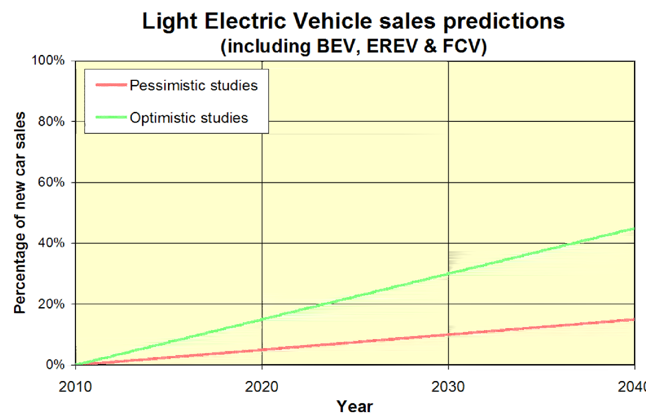

Figure 1. Expected sales of electric vehicles, in percentage of total vehicles sales [5].

One of the major challenges that emerges with the arrival of PEVs is the existence of three levels of charge specifications, namely Level 1 (residential and commercial buildings), Level 2 (specific charging facilities) and Level 3 (fast charging). They provide quite different charge times because of the different VAC levels: for a 10 kWh battery, we require about 5,5 hours for Level 1, 1–2 hours for Level 2 and less than 1/2 hour for Level 3. Although Level 2 additional infrastructure may make sense in residential garages and commercial parking lots, this Level 2 infrastructure have the potential problem of finding a free plug in our garage. Additionally, the global system cost may not be advantageous compared to other types of infrastructures [6] and we still need DR mechanisms. In second place, residential and commercial charging facilities are not adequate due to their limited availability, mainly owing to locational and timetable restrictions.

Although demand response (DR) and demand-side management (DSM) schemes may provide satisfactory solutions for the existing grid, they do not cover challenges related to PEVs such as the three levels of charging or where to do the recharge if we are running out of batteries. DR mechanisms can encourage reduction of prices and enhance robustness since they include interactions among participants such as wholesale markets, retail utilities and customers. Even customers may adjust their demand in response to external signals such as grid failures or price changes. Along with DR, DSM enables the users to employ the energy more efficiently. These DR mechanisms can be used to optimize the charge of electric

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vehicles seeking to schedule the charge when the electricity is more economic [7]–[9]. In [7], PHEV (plug-in hybrid electric vehicles) agents communicate their desired level of charging to a PHEV manager. The PHEV manager optimally distribute the available energy after communicating the total required energy to an energy hub. In this scheme different factors such as price and amount of energy are considered for the optimal distribution of energy. In [8], the authors propose a centrally controlled charging scheme based on centralized PEV aggregators that can act on the power market to minimize the cost of charging the PEVs. This scheme considers some practical constraints in the low-voltage distribution grid and the outcome is an optimal scheduling of the PEVs charging. A centralized approach is also followed in [9], where a charge plan is elaborated by a charge controller to find the state of charge curve that maximizes the vehicle's owner benefit.

It is then clear that to make a broad adoption of PEVs possible a dedicated infrastructure besides of residential and industrial connections must be available to facilitate Level 2 and Level 3 charging when it is necessary. From this public infrastructure viewpoint, two strategies can be adopted: battery charging (plugging it into the grid) and battery swapping (exchanging it for a charged battery) [10]. Battery swapping is quicker but it causes a considerable increase in the global number of manufactured batteries and, moreover, the associated costs in storage and transportation may raise batteries price, making its use inadvisable. On the other hand, battery charging provides a similar service to that provided by gas stations, where a PEV is plugged directly to a charging station. Their main drawback of current battery technologies is the charge time, although it is expected that coming improvements in PEV batteries technologies as long as the apparition of Level 3 fast chargers reduce the charge time in the next future to as little as fifteen minutes [11].

In this paper we study the problem of selecting the best charging station (owned by utility providers)[1] for a given PEV, according to criteria of price and distance. Since DR and DSM mechanisms enable utility providers to adapt power distribution and prices in real time according to users demand, we propose a game-theoretic approach that exploits such mechanisms and wireless communications to optimize the charging cost for the PEV. We thus take advantage of a two-way communications system that exchanges information among the PEVs and the charging stations. The two-way communication associated to the battery charging process allow power generators to adjust their production to the demand in real time. The joint use of DR schemes and traffic patterns or average consumption data have been recently applied to the problem of charging batteries for PEVs [12], [13]. However, these data may become outdated and are not useful when the demand must be adapted in real time, as it is expected to happen in the next years with the continuous growing in the number of PEVs.

Although some studies have addressed the problem of

[1]In this work, we assume that charging stations are owned by utility providers, or simply utilities, and that utility providers, also known in the related literature as retail utilities, acts also as electricity generator companies. Then, throughout this paper we use these terms indistinctly.

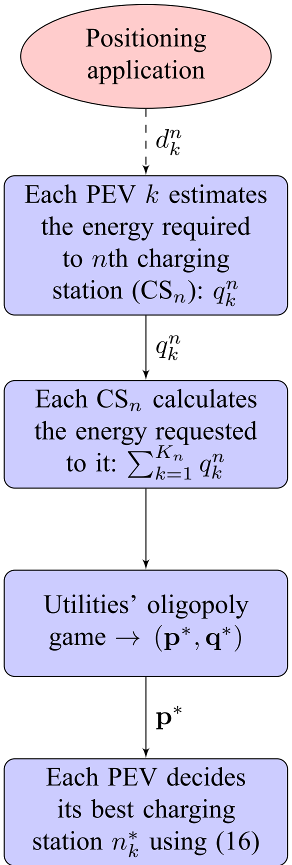

Figure 2. Oligopolistic game-theoretic framework for charging station selection: main steps

optimizing the charge of PEVs, as the above mentioned, all of them consider that a central entity controls the process. However, the charging stations are likely to use renewable energy sources such as photovoltaic and wind power stations and therefore the existence of such central coordinator hub is not easy to implement. This is why we propose a distributed approach based on oligopoly models that uses the PEV's positioning to provide the best charging station (in economic terms) to each PEV. In this direction, a very few works have addressed the optimal choice of charging station. To the best of our knowledge, the only related work is that of Pan et al. [14], which tackles somehow this problem, although the studied scenario is different: they study the problem of how to optimally site the charging stations (swapping stations in this case) from a statistical perspective.

We make the distinction between fully-battery powered electric vehicles (PEV) and plug-in hybrid electric vehicles (PHEV). Most studies consider PHEV. Although the possibility in these vehicles to use gasoline provides a backup energy when they run out of batteries, this has two major drawbacks. On the one hand, the PHEV must be provided with two engines, one fuel-powered and one battery-powered. For this reason, the cost, and obviously the price, will be higher. On the other hand, different works show the impact of PHEV penetration on power generation [15], but this impact is more notorious for the case of PEV and has to be considered for

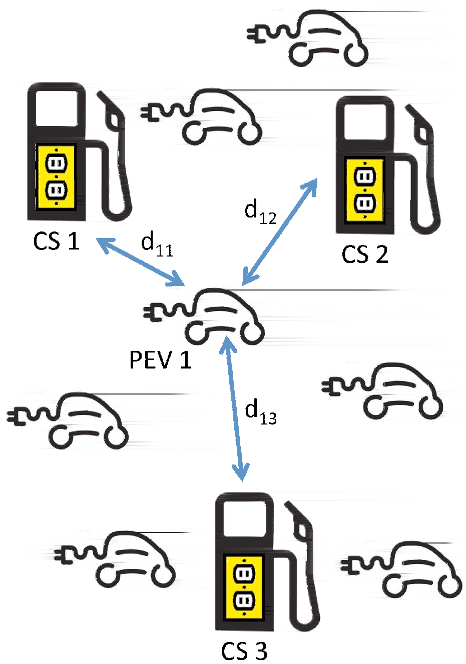

Figure 3. System model: distance from a plug-in electric vehicle (PEV1) to the available charging stations

the future. In this paper we focus on PEV because our believe is that in the long term most electric vehicles will be fully battery-powered.

The above exposed ideas are collected in the framework presented in this paper, which is schematically shown in Fig.2. Basically, the framework consist of the following steps. First, the PEVs obtain CSs' positioning information and request energy form different CS. Once the PEVs' requests are received, each CS establishes its energy price by means of a non-cooperative oligopoly game, and communicates the price to the PEVs. Finally, each PEV determines the best CS according to the price and the distance to the CSs.

The details of the different steps of the framework are given in the forthcoming sections. In Section II, we introduce the energy model for the PEVs. In Section III, the oligopoly game to determine the prices at which the CSs charge the energy is described. Section IV explains how the PEVs select the best charging station. Finally, some conclusions are extracted in Section VI.

II. PEV ENERGY MODEL

For our analysis, we consider a given area where N charging stations have been deployed, and K users are present and can reach some of the CSs. For the sake of simplicity, it is assumed that each charging station corresponds to a different utility provider (see Fig. 3). Each user k expresses her willingness to charge her battery by communicating the desired level of charging q_k^{ch} (kWh), which may range from a full recharge of the batteries up to an estimated quantity that guarantees, for instance, the return home. The amount of energy consumed to arrive to each CS is also considered, and is calculated as $\bar{q}_k^c d_{kn}$, being $\bar{q}_k^c$ (kWh/km) the estimated consumed energy per unit of length[2] and d_{kn} the distance the user should travel from her current position to the nth CS. Therefore, positioning information about its location and CS's one must be available to the PEV (for example, by means of GPS or UMTS communications). In this work, this information is taken for granted so that each PEV k determines d_{kn} for all desired charging stations. Therefore, the total estimated energy required by PEV k from CS$_n$ is

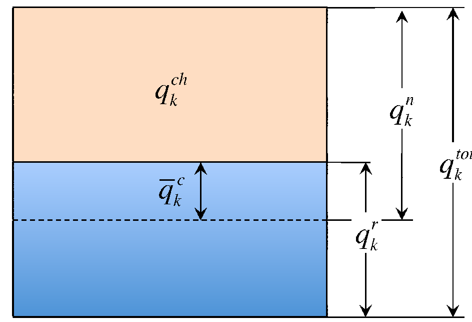

Figure 4. Plug-in electric vehicle energy model.

$$q_k^n = \bar{q}_k^c d_{kn} + q_k^{ch} \tag{1}$$

and q_k^n is communicated to CS$_n$ in order to calculate the estimated total energy requested by the users.

The battery of the kth PEV is characterized by its total energy capacity q_k^{tot}. We assume that the required energy q_k^{ch} is the energy necessary to fill up the batteries, so we have

$$q_k^{tot} = q_k^r + q_k^{ch}, \tag{2}$$

where q_k^r is the available energy in the batteries at the moment user k realizes the request (Fig.4). However, without loss of generality, we consider that all the batteries have the same total capacity, so $q_k^{tot} = q_T, forall\ k = 1, \ldots, K$.

III. THE OLIGOPOLY FRAMEWORK

The ultimate objective of both PEVs and CSs is to optimize their benefit, i.e., in the case of PEVs is to pay as less as possible for the energy, and in the case of CSs, to charge to the users a price and an energy amount as high as possible.

In our scenario there are some PEVs' owners who desire to charge the PEV batteries. The N charging stations, i.e. the utilities, receive the requests from the users during an adequate period of time and try to maximize their benefit, but, at the same time, the users want to pay the lowest possible price for the energy. This can be achieved by the following two-step scheme. First, the CSs establish their energy prices to maximize their benefits, and second, the PEVs choose the adequate CS to minimize the cost.

Given that utilities compete with each other for the users, a natural framework for this situation is game theory. In this case, the number of competing players (the utilities) is

[2]It is assumed, without loss of generality, that $\bar{q}_k^c$ is constant for all k. Nevertheless, different driving energy consumption patterns depending on variables such as driving style or climate conditions (heating, a/c) can be constructed from on-board collected data and taken into account in our proposed model.

sufficiently low to consider that each utility can influence the energy price at which PEVs will buy the energy, and that a single PEV (the consumer) is not so powerful to influence the energy price, that is, the PEVs' owners are price takers. The other possibility is to consider that the PEVs are not price takers and can discuss the price with the CSs [16]. In this case, the figure of an aggregator is necessary to gather a significative aggregated energy demand to encourage a price negotiation with the CSs. Nevertheless, this focus requires a previous agreement among the sufficient number of PEV and a non-real time bargaining mechanism among the aggregator and the CSs.

The scenario described in the former paragraphs can be modeled as an oligopoly game [17] that has been largely used to model energy markets (see, for instance, [18]), in which the users' energy demand is the sum of the demanded energy. In this case, we are interested in prices rather than in quantities, so we formulate our game as a Bertrand's oligopoly game for which the price equilibrium vector is represented as $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_N^*)$, where p_n^* denotes the price that CS$_n$ will charge at equilibrium. Also, the demanded energy is kept constant when the utilities determine their energy prices, since it is assumed that the PEVs maintain their demanded energy q_k^n irrespectively the prices they are offered. Afterwards, each CS communicates its own price to the PEV, who decides which CS minimizes her charging cost.

A. Charging station model

Here we will assume that the energy generator is directly operating the CS. In a more general model, the CS owner, for instance the utility company, incur in different costs and revenue functions. Nevertheless, our model may be straightforwardly modified to suit to this more general scenario.

A charging station is modeled as follows. As each CS pertains to an utility company, we assume that the market demand model for generation companies can be translated to the CSs. Then, each charging station is characterized by its quantity q, its unitary price p and its cost function C. The relationship between quantity and price is represented by a linear demand function; thus we have

$$q = A - Bp, \tag{3}$$

where A and B are the positive coefficients of the linear demand function, specific to each CS. The total energy required to the CS by the PEVs is represented by A, that is $A = \sum_{k=1}^{K_n} q_k^n$, being $K_n \leqslant K$ the number of users that send a request to charge their batteries in a given CS. The total profit π is given by

$$\pi = pq - C, \tag{4}$$

and we assume a quadratic utility's cost function C [19]

$$C = aq^2 + bq + c, \tag{5}$$

being a, b and c the coefficients of the utility's cost function with $a > 0, b \geqslant 0$ and $c \geqslant 0$, specific to each CS.

B. Differentiated products

Contrary to what happens when we have a Cournot's oligopoly game, the only possible equilibrium for the Bertrand's oligopoly game is $\mathbf{p}^* = \mathbf{0}$ if the players are sellers of homogeneous products [17]. Nevertheless, if goods are heterogeneous, we talk about *differentiated products* and in this case goods are substitutive [20]. In the context of this paper, goods are substitutive in the sense that generators produce different quantities of an unique product (an energy unit with certain price and cost) but different to the energy unit produced by the other generators, and the customers (PEVs) only buy to one producer, that is, the PEV charges its battery in one and only one CS.

In our case, consumers (PEV's owners) have preferences about the products (the energy) offered by the generators. For a given PEV's owner, the energy offered by CS_1 at price p_1 that is very close to him has a higher subjective value than the energy offered by a distant CS_2 at p_2. However, the customer's final choice is determined by his minimum cost and this is conditioned to prices p_1 and p_2, so if CS_2 gives a price p_2 sufficiently low, it may worth for the customer to travel to CS_2 instead of to the closer station CS_1.

The way how energy is differentiated is of paramount interest. It is commonly assumed that the cost function C_n is the same for all producers. However, for the smart grid, where there is a variety of producers ranging from renewable producers to nuclear plants, this hypothesis is not suitable. Even more, the energy price is different from one CS$_n$ to any another CS_m at equilibrium ($n \neq m$), which is a second source of energy differentiation.

We can reformulate our CS model of Section III.B to include differentiated products by considering *cross-elasticity*, defined as the change in CS$_n$'s demand caused by a deviation in the price of CS$_m$:

$$\varepsilon_{nm} = \frac{\partial q_n / q_n}{\partial p_m / p_m}, \tag{6}$$

where $\varepsilon_{nm} = 0$ for homogeneous products and $\varepsilon_{nm} \neq 0$ for differentiated products [21]. Suppose that utilities choose quantity (or output) levels. The CS$_n$'s profit maximizing q_n, given by (3), depends on the quantities chosen by its rivals, denoted by $\mathbf{q}_{-n} = (q_1, q_2, \ldots, q_{n-1}, q_{n+1}, \ldots, q_N)$. Let f_n be the function that reveals the interaction, $f_n = f_n(q_1, q_2, \ldots, q_{n-1}, q_{n+1}, \ldots, q_N) = f_n(\mathbf{q}_{-n})$. We can rewrite (3), for each CS$_n$, such that the interaction is included as

$$q_n = A_n - B_n p_n - f_n(\mathbf{q}_{-n}), \tag{7}$$

which implicitly depends on the rivals' prices via $f_n(\mathbf{q}_{-n})$. The interaction function f_n can then be expressed by

$$f_n(\mathbf{q}_{-n}) = \sum_{m=1, m\neq n}^{m=N} \varepsilon_{nm} q_m. \tag{8}$$

C. Interaction among charging stations

The interaction among competing CSs can be specified in different forms. The utilities *reaction function* specifies this relationship, giving its profit maximizing output as a function of $\mathbf{q}_{-n}$. The slope of this reaction function is the rate at which the maximizing output changes with a change in a rival's output, and it can be represented by the cross-elasticities ε_{nm} of (8). However, the rival's reaction function is not known to the other utilities, but CS_n may have conjectures about it. In particular, the firm may make conjectures about the slopes of the rivals' reaction functions. These conjectured slopes are called *conjectural variations* (CV) [22]. Finally, a *supply function* relates the quantity that a producer will sell to the market price when its faces uncertain demand [23].

In our model, we adopt the conjectural variations approach to express the interaction among CSs. Since $q(n) = q(q_n, \mathbf{q}_{-n}) = q(p_n, \mathbf{p}_{-n})$, we have:

$$\frac{\mathrm{d}q_n}{\mathrm{d}p_n} = \frac{\partial q_n}{\partial p_n} + \frac{\partial q_n}{\partial p_{-n}} = \frac{\partial q_n}{\partial p_n} + \sum_{m=1,m\neq n}^{m=N} \varepsilon_{nm} \frac{\partial p_m}{\partial p_n}. \tag{9}$$

In our framework, the conjectural variation associated to CS_n is denoted by δ_n, and it is defined as the rate at which CS_n believes that the price of the other CSs changes if CSn's own changes once the cross-elasticities are known, that is

$$\delta_n \triangleq \frac{\partial p_{-n}}{\partial p_n} \triangleq \sum_{m=1,m\neq n}^{m=N} \varepsilon_{nm} \frac{\partial p_m}{p_n}, \tag{10}$$

where p_{-n} denotes the price of other CSs than n. We note that the conjectural variation is defined as a derivative of p_n since prices are the problem variables, as stated in (7), while conjectural variations are mostly expressed as $\partial q_{-n}/\partial q_n$ [21].

D. Utilities' oligopoly game based on differentiated products

Roughly speaking, most models of generators' competition are based on the general approach of defining a market equilibrium as a set of prices and quantities that simultaneously maximize the participants' benefits, while clearing the market, i.e supply equals demand (see [24], [18] and references therein). When oligopoly games are applied to power markets, the usual approach is the standard static Cournot oligopoly model. In this game, generators make their quantity decisions simultaneously, and each generator behaves independently to maximize its own profit. Generators communicate their power forecast demand and the ISO (Independent System Operator) puts a market clearing price. This spot market perspective is beneficial for generators, but imposes the same market price for all users.

Moreover, scenarios such as those we describe in this paper are expected to happen frequently in the new smart grid, so a different approach is necessary. Let us think about a wind or photovoltaic power station. It generates power and does not need any other utility company for selling its power to PEV when its location is adequate. This also gives us an idea about the heterogeneity of the types of generation companies, from traditional hydroelectric or nuclear generation to biomass, solar and wind power generation. Therefore a distributed oligopoly game may better model the described scenario.

We present a game-theoretic model, named as the *differentiated product oligopoly game* (DPOG), that tries to maximize the generators' profit: instead of agreeing on a market price, we propose a negotiation of the prices among the generators based on differentiated products. In other words, each generator gives a different price for the energy that is sold at its CS. Later, users may also benefit form the DPOG by selecting the more suitable CS. The N-players DPOG $\{\mathcal{N}, \{p_n\}, \{\pi_n\}\}$ determines the prices at which the CSs sell their energy to the PEVs, where the set $\mathcal{N} = \{1, \ldots, N\}$ represents the N charging stations; p_n is the unitary price that the nth utility charges (in €/kWh) and embodies the strategy of CS_n; and π_n is the total profit obtained by the nth utility and can be calculated by (4)

$$\pi_n = p_n q_n - C_n, \tag{11}$$

being q_n the energy produced by CS_n, and C_n the associated costs given by (5). In the DPOG, each CS wants to maximize its own revenue, what can be formally expressed as

$$\max_{p_n} \pi_n = p_n q_n - C_n, \text{for all } n \in \mathcal{N}. \tag{12}$$

The resulting game is noncooperative: although the players (CSs) do not reach an explicit agreement in prices, if a Nash equilibrium exists, this equilibrium is in itself an implicit agreement among the CSs. The only information available to player CS_n is the strategies of the other charging stations, that is, the price vector $\mathbf{p}_{-n} = (p_1, p_2, \ldots, p_{n-1}, p_{n+1}, \ldots, p_N)$. So no bargaining is previously agreed to the oligopoly game.

The objective with the DPOG is to reach an equilibrium, at which no CS has anything to gain by changing his own strategy (his price in this case) unilaterally [20]. This equilibrium, also known as Nash equilibrium, is found by simultaneously solving the following set of equations:

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= 0 \\ &\vdots \\ \frac{\partial \pi_N}{\partial p_N} &= 0 \end{aligned} \tag{13}$$

The introduction of CV endows with a dynamic behaviour to our non-dynamic utilities' oligopoly game. Indeed, this game is a one-shoot game, whose outcome is the Nash equilibrium $(\mathbf{p}^*, \mathbf{q}^*)$, and this equilibrium is a by-product of interactions among the CSs until they reach the point at which no charging station has incentive to modify its price.

With respect to the information flow, we may also consider that CSs know the PEVs' position and how much energy is left. The fact that CSs know the PEVs' location can induce to think that they can take advantage of this information and establish high prices to those PEVs that are in need of charging, that is to say, they are very close to a CS and the remaining battery is quite low. However, the game is an imperfect information game, given that the costs and policy prices of each utility are not known by the other utilities, so it is our believe that they cannot freely raise the prices at the risk of that PEVs go to some other CS.

E. Interpretation of the utilities' oligopoly game outcome

The Nash equilibrium outcome of the utilities' game can be written as $(\mathbf{p}^*, \mathbf{q}^*)$, where $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_N^*)$, and $\mathbf{q}^* =$

$(q_1^*, q_2^*, \ldots, q_N^*)$ are calculated once the prices p_n^* are obtained at equilibrium through the corresponding demand functions. Therefore it may be possible that q_n^* corresponding to a certain CS$_n$ does not be equal to the sum of the quantities initially demanded q_n. If $q_0 = \sum_{k=1}^{K_0} q_k^n$ represents the energy initially demanded to CS$_n$ by K_0 PEVs, there exist two possibilities:

- $q_n^* \geqslant q_0$. In this case the PEVs' demand is guaranteed since the produced energy quantity is larger than the total energy required by the PEVs.
- $q_n^* \leqslant q_0$. At a first glance, CS$_n$ does not provide sufficient energy to satisfy PEVs' demand. However, PEVs ask several CSs for energy, i.e they make a manyfold petition to the surrounding CSs. As a result, the total energy produced is several times the sum of the total energy required, and with very high probability the CSs will attend all the petitions once the PEVs have made their choice of their best charging station.

Of course, some extra energy may be produced by some CSs, but we cannot forget that not only the PEVs communicating with the CSs will recharge their batteries, but also some additional capacity allow to serve possibly unexpected vehicles.

IV. Optimizing the PEVs cost

The K PEVs seek for minimize the total price paid to the utilities. Then, once the charging stations have announced their prices to their corresponding PEVs, the users have to independently solve their optimization problem, which consists in selecting the CS n to minimize her cost, namely n_k^*, and it is formulated as

$$\min_n c_k^n = q_k^n p_n = (\bar{q}_k^c d_{kn} + q_k^{ch}) p_n, \quad (14)$$
$$\text{for all } n \in \mathcal{N}, \text{ for all } k = 1, \ldots, K.$$

In (14), the first term between brackets reflects the energy demand, for user k, to get to CS$_n$, and the second term expresses unit price if charges the batteries at CS$_n$.

However, PEVs must take into consideration whether they have energy enough to reach the selected CS, that is, the remaining battery q_r^k must be larger than the consumed battery to get to the CS$_n$, which implies that the condition

$$d_{kn} < d_{th} = \frac{q_k^r}{\bar{q}_k^c} \quad (15)$$

must hold, where d_{th} expresses de maximum distance the PEV can cover before running out of batteries. Thus they must incorporate this restriction to the optimization problem and the resulting charging station selection problem, for each user k, is formulated as

$$\min_n \; c_k^n, \text{ for all } n = 1, \ldots, N, \quad (16)$$
$$\text{s.t. } d_{kn} < d_{th} = \frac{q_k^r}{\bar{q}_k^c}.$$

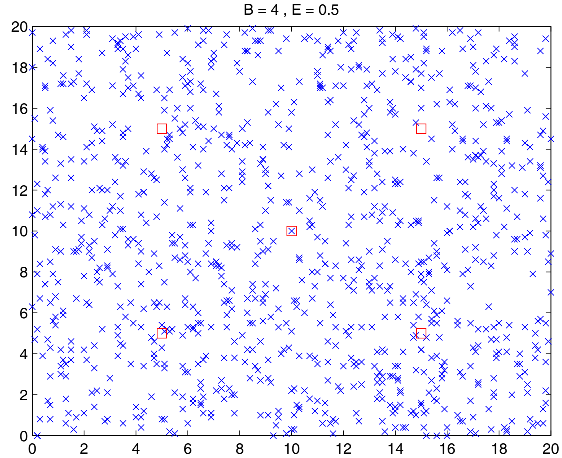

Figure 5. Distribution of charging stations (CS) and electric vehicles (EV).

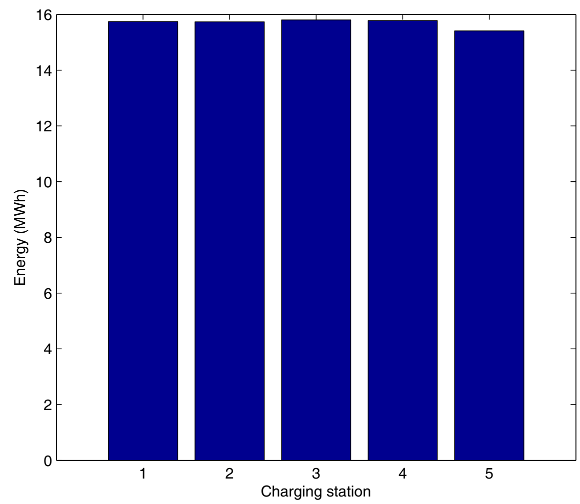

Figure 6. Total energy requested to each charging station.

V. Case study and results

For the evaluation of the proposed framework the following case study is considered. A given area of 20km×20km is homogeneously served by 5 CSs, and 1350 EVs are assumed to be randomly distributed over this area (Fig. 5) [25]. The battery capacity is randomly chosen between 24kWh (e.g. Nissan Leaf model) and 35kWh (e.g. Mini E model), and all the EVs has a consumption of $1/6$ kWh/km, an usual value []. The linear demand function of CS$_n$, following (7), is $q_n = A_n - B_n q_n - \sum_{m \neq n}^{m=N} \varepsilon_{nm} q_m$, where A_n represents the total energy requested to CS$_n$ (see Fig.6). Product differentiation is considered to be due only to the different cost functions, which have been taken from [19] and the corresponding coefficients are reported in Table I, taking then cross-elasticities the same value (in this case $\varepsilon_{nm} = 0.5$) for all CSs [22]. We compare the OPDG prices with the prices calculated by using standard Bertrand equilibrium for different values of the conjectural variation $\delta_n = \delta$, for all CS$_n$. Fig. 7 illustrates this comparison

Charging station	a_n[\$/MW^{2}h]	b_n[\$/MWh]	c_n
1	0.04	2	0
2	0.0166	3.25	0
3	0.035	1.75	0
4	0.125	1	0
5	0.05	3	0

Table I
GENERATING COST COEFFICIENTS

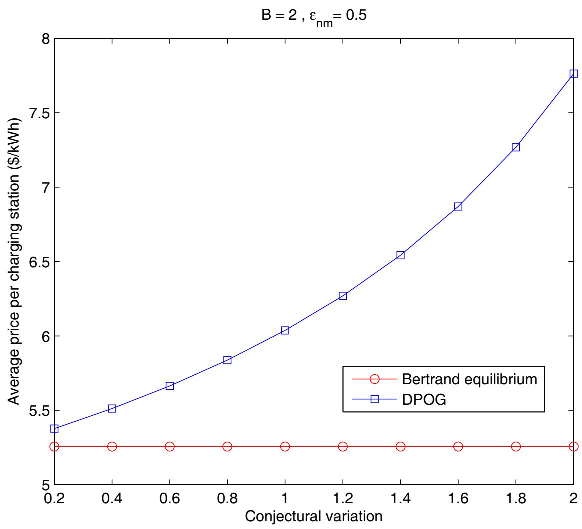

Figure 7. Average price per charging station for Bertrand equilibrium and DPOG approaches, $B_n = 2$ and cross-elasticities $\varepsilon_{nm} = 0.5$ for all n.

in terms of the average price (in \$/kWh) per CS, calculated as $\frac{1}{N}\sum_{n=1}^{N=5} p_n$, where we observe that the price for DPOG is higher than the price for the Bertrand equilibrium. We note that the Bertrand equilibrium's price is constant with δ, since Bertrand equilibrum corresponds to the case of $\delta = 0$. This difference is more appreciable when the benefit per charging station is compared for both cases, as it is shown in Fig. 8.

We also analyze the total cost charged to the users. The payment made by the users is lower if they are allowed to choose the CS as it is described in Section IV, compared to the strategy of charging in the nearest CS, as it is shown in Fig. 9, where k_1 represents the percentage of users that obtain a better (lower) payment if they do not choose to charge at the nearest CS. It is observed that at least about 40% of the EVs obtain a lower price ($\delta = 0.2$) and, with respect to the charged cost, a maximum average saving of 11.5% can be achieved, for $\delta = 0.2$.

The effect of the conjectural variation can be observed in the above figures: price is increasing with δ, that is, a higher expectation in the quantity q_{-n} due to an increase in p_n leads to higher prices for all CSs, and also to lower benefits (higher costs) for the EVs.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we introduce a framework for plug-in electric vehicles in which the problem of charging station selection is formulated and analyzed, and where positioning information and wireless two-way communications play a key role. This

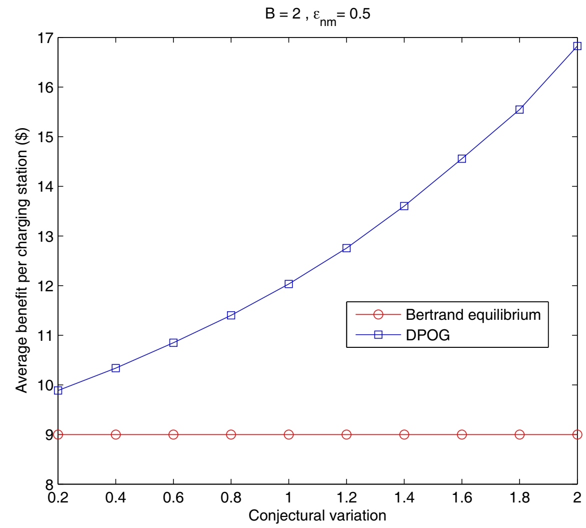

Figure 8. Average benefit per charging station for Bertrand equilibrium and DPOG approaches, $B_n = 2$ and cross-elasticities $\varepsilon_{nm} = 0.5$ for all n.

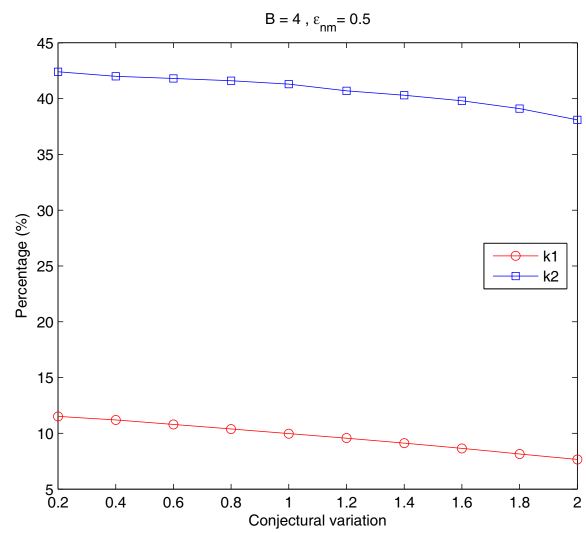

Figure 9. Percentage of number of EV that obtain a better price with DOPG with respect to the closest CS, represented by k_1, and the average percentage of cost saving per user of DOPG with respect to the closest CS, represented by k_2, $B_n = 4$ and cross-elasticities $\varepsilon_{nm} = 0.5$ for all n.

framework is based on a non-cooperative oligopoly game that makes use of differentiation products theory to provide a Nash equilibrium in prices and quantities. It is also considered that conjectural variations model the expected reaction that the rest of charging stations have with respect to variations in the price of a given charging station. We have shown that the proposed DPOG outperforms the Bertrand oligopoly scheme for the participating charging stations, and electric vehicles benefits from the DPOG when they select the charging station according to the proposed criteria.

The next steps are the analytical study of how cross-elasticities impact the performance of the proposed DPOG, and conjectural variations consistency analysis. The adoption of EV driving patterns and different EV area distributions, such as urban EV distribution, are also considered for future case studies.

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