A FINITE MOVING AVERAGE TEST FOR TRANSIENT CHANGE DETECTION IN GNSS SIGNAL STRENGTH MONITORING

Daniel Egea-Roca⋆*, Gonzalo Seco-Granados*⋆*, José A. López-Salcedo*⋆*, H. Vincent Poor*†

⋆ Department of Telecommunications and Systems Engineering,
Universitat Autònoma de Barcelona (UAB), Bellaterra, Spain
† Department of Electrical Engineering, Princeton University, Princeton, NJ, USA

ABSTRACT

Due to the increasing interest in Global Navigation Satellite Systems (GNSSs) for safety-critical applications, one of the major challenges to be solved is the provision of integrity to urban environments. In the past years, it has been noted that to do so, the integrity of the received signal must be analyzed with the aim of detecting any local effect disturbing the GNSS signal. Moreover, the detection of such disturbing effects must be done with a bounded delay. This is desirable because the presence of any local effect may cause large position errors. This work addresses the signal integrity problem as a transient change detection problem by proposing a stopping time based on a finite moving average. The statistical performance of this stopping time is investigated and compared, in the context of multipath detection relying on C/N_0 monitoring, to different methods available in the literature. Numerical results are presented in order to assess their performance.

***Index Terms*—** Transient Change Detection, Stopping Time, Finite Moving Average, GNSS, Signal Integrity

1. INTRODUCTION

There is an increasing interest in Global Navigation Satellite System (GNSS) -based safety-critical and liability-critical applications [1]. These applications, which are often associated with terrestrial environments, have very stringent requirements in terms of accuracy, continuity and integrity of the provided position solution. In addition to providing an accurate navigation solution, GNSS has to provide timely warnings to the user when the system should not be used; this capability is referred to as the *integrity* of the system.

Current integrity algorithms are conceived mainly for civil aviation [2], where local effects such as interference and multipath have a controlled influence on the GNSS signal. However, this is not the case in terrestrial environments where these local effects strongly affect the position errors and, thus, position integrity [3]. Hence, in terrestrial safety-critical applications, it is of paramount importance to promptly detect any possible anomaly or misleading behavior that could be endangering the received GNSS signal; we refer to this capability as the *signal integrity*. Moreover, for integrity purposes, an acceptable detection delay is limited by a given value m of samples. Late detections (i.e. delayed more than m) are considered as missed.

This kind of detection lies on the field of sequential change detection and it deals with obtaining the stopping time T at which the change (i.e. anomaly) is detected, including *quickest* change detection and *transient* change detection. The traditional *quickest* change detection problem deals with a change of infinite duration. The optimality criterion in this case is to minimize the detection delay subject to a level of false alarms. Comprehensives overviews of the *quickest* change detection can be found in [4] and [5]. In this work we focus on non-Bayesian approaches, *i.e.* when the time v at which the change appears is unknown but non-random. For this kind of approaches, the CUSUM algorithm was proposed in [6] and its optimal properties were analyzed in [7] and [8].

In contrast, the *transient* change detection problem deals with finite change duration and then a bounded detection delay is desired. Unfortunately, the traditional *quickest* detection criterion does not completely fit into this problem. In this case, we wish to minimize the probability of missed detection (i.e. late detections) subject to a level of false alarms. This kind of criterion was proposed in [9] and [10] for the *transient* change detection problem, and it suits perfectly the GNSS integrity problem [11]. The first optimal results, though, for this new criterion were provided very recently in [10] for the particular case of a transient duration of one sample.

For the more general case of finite transient duration greater than one sample, and to the best of the authors' knowledge, no optimal solution has been found for the *transient* change detection problem. We know however, that the use of a Finite Moving Average (FMA) test has been shown to be the optimal windowed CUSUM solution (i.e. using the last m samples) for the mean-Gaussian change case [9]. Based on these observations, we propose the use of an FMA test for approaching the general problem of *transient* change detection, in general, and signal integrity monitoring in GNSS, in particular.

Hence, the contribution of this paper is twofold. Firstly, we investigate the statistical performance of the FMA stopping time for any general case. This leads to the provision of tight bounds valid for any kind of change and not restricted to the mean-Gaussian change. Secondly, we show with numerical results that this stopping time outperforms other methods available in the literature for this new optimality criterion of probability minimization. This is done in the setting of signal level integrity in GNSS, when dealing with multipath detection. Specifically, for the sake of clarity we focus on C/N_0 monitoring because it is easy to analyze. The rest of the paper is organized as follows: Section 2 introduces the C/N_0 monitoring problem, Section 3 provides background on sequential change detection and Section 4 presents the statistical performance of the FMA stopping time. Finally, Section 5 presents our numerical results, while Section 6 concludes the paper.

2. SIGNAL MODEL

Let $\{x_n\}_{n\geq 1}$ be an independent and identically distributed (iid) random sequence. We consider a family $\{\mathbb{P}_v\}_{v\geq 1}$ of probability

This work was partly supported by the Spanish Grant TEC2014-53656-R and the U.S. National Science Foundation under Grant ECCS-1549881.

measures, such that, under $\mathbb{P}_v$, the observations $x_1, \ldots, x_{v-1}$ and $x_{v+m}, \ldots, x_\infty$ are iid with a fixed marginal probability density function (pdf) f_0, with v the unknown but non-random change time. On the other hand, $x_v, \ldots, x_{v+m-1}$ are iid with another marginal pdf $f_1 \neq f_0$. For the sake of simplicity, a mean-Gaussian change is considered for the further numerical analysis, that is

$$x_n \sim \begin{cases} \mathcal{N}\left(\mu_0, \sigma^2\right) & \text{if } 1 \leq n < v \text{ or } n \geq v+m \\ \mathcal{N}\left(\mu_1, \sigma^2\right) & \text{if } v \leq n < v+m \end{cases}, \tag{1}$$

where μ_i, with $i = \{0, 1\}$ is the mean under normal conditions and when the transient change is present, respectively, σ^2 is the variance and $\mathcal{N}(\mu, \sigma)$ the Gaussian pdf with mean μ and variance σ^2. For integrity purposes, the maximum permitted detection delay is a priori fixed. It is for this reason that we assume that the acceptable detection delay is equal to the transient duration m, so that if a detection is delayed more than the permitted m it is considered to be missed.

All parameters are assumed to be known, included the duration m. This is often the case of signal integrity in GNSS, such as the case of the C/N_0-based multipath detection presented in [12]. Let us define the log-likelihood ratio (LLR) for the observation x_n as $\mathrm{LLR}(n) \doteq \ln(f_1(x_n)/f_0(x_n))$. Thus, we can write the LLR as

$$\mathrm{LLR}(n) = \frac{\mu_1 - \mu_0}{\sigma^2}\left(x_n - \frac{\mu_1 + \mu_0}{2}\right), \tag{2}$$

and then from (1) it is easy to see that, when the change is present, the LLR is a Gaussian variable with mean and variance given by

$$\begin{aligned} \mu_{\mathrm{c}} &= \frac{(\mu_1 - \mu_0)^2}{2\sigma^2}, \\ \sigma_{\mathrm{c}}^2 &= \frac{(\mu_1 - \mu_0)^2}{\sigma^2}. \end{aligned} \tag{3}$$

Similarly, under normal conditions, the LLR is Gaussian too but with mean $-\mu_{\mathrm{c}}$ and variance σ_{c}^2, and thus we can write

$$\mathrm{LLR}(n) \sim \begin{cases} \mathcal{N}\left(-\mu_{\mathrm{c}}, \sigma_{\mathrm{c}}^2\right) & \text{if } 1 \leq n < v \text{ or } n \geq v+m \\ \mathcal{N}\left(\mu_{\mathrm{c}}, \sigma_{\mathrm{c}}^2\right) & \text{if } v \leq n < v+m \end{cases}. \tag{4}$$

3. FUNDAMENTALS OF SEQUENTIAL CHANGE DETECTION

In this section, we firstly recall the problem of *quickest* change detection. Then, we introduce the problem of *transient* change detection. Finally, the FMA stopping time proposed in this paper is presented.

3.1. Quickest change detection: CUSUM stopping time

A change detection procedure is completely defined by its *stopping time* T at which the change is detected. Denoting $\mathbb{E}_v$ as the expectation under the distribution $\mathbb{P}_v$, with $m = \infty$, the following criterion has been traditionally used for *quickest* change detection [7]:

$$\inf_{T \in C_\gamma} \left\{ \sup_{v \geq 1} \operatorname{essup} \mathbb{E}_v \left[(T - v + 1)^+ \, | x_1, \ldots, x_{v-1} \right] \right\} \tag{5}$$

among all stopping times satisfying the false alarm constraint $C_\gamma = \{T : \mathbb{E}_\infty(T) \geq \gamma\}$, where $(x)^+ = \max(0, x)$, essup denotes the essential supremum, and $\gamma > 0$ a finite constant. It is shown in [8] that the CUSUM stopping time

$$\boxed{T_{\mathrm{C}} \doteq \inf\left\{ n \geq 1 : \max_{1 \leq k \leq n} S_k^n \geq h \right\}; S_k^n \doteq \sum_{i=k}^{n} \mathrm{LLR}(i),} \tag{6}$$

with h the fixed threshold for guaranteeing a certain level of false alarms, is optimal in the sense of criterion (5).

However, as shown in [13], the requirement of having large values of $\mathbb{E}_\infty(T)$ does not guarantee small values of the probability of false alarm $\mathbb{P}_\infty(l \leq T < l + m_\alpha)$ within a fixed interval of length m_α, for all $l \geq 1$. As a result, [13] proposed to replace the traditional constraint in (5) by the following constraint on the worst-case probability of false alarm within any interval of length m_α, which is more convenient for safety-critical applications:

$$\mathbb{P}_{\mathrm{fa}}(T) \doteq \sup_{l \geq 1} \mathbb{P}_\infty \left(l \leq T < l + m_\alpha \right) \leq \alpha. \tag{7}$$

It was shown in [13] that the CUSUM stopping time T_{C}, with the threshold h chosen so that

$$m_\alpha e^{-h} = \alpha, \tag{8}$$

minimizes, when $\alpha \to 0$, the detection delay, satisfying (7).

3.2. Transient change detection: Shewhart stopping time

Unlike the *quickest* change detection problem, the change duration in the *transient* change detection problem is assumed to be finite. Thereby, we would like to have $v \leq T < v + m$. Stopping within the prescribed interval constitutes a desirable event while stopping at $T \geq v + m$ is considered a missed detection. In this sense, we use through this paper the criterion of optimality introduced in [9] and [10], which involves the minimization of the following probability:

$$\inf_{T \in C_\alpha} \left\{ \mathbb{P}_{\mathrm{md}}(T) = \sup_{v \geq 1} \mathbb{P}_v \left(T > v + m - 1 | T \geq v \right) \right\} \tag{9}$$

among all stopping times $T \in C_\alpha$ satisfying

$$C_\alpha = \left\{ T : \mathbb{P}_{\mathrm{fa}}(T) = \sup_{l \geq 1} \mathbb{P}_\infty \left(l \leq T < l + m_\alpha \right) \leq \alpha \right\}, \tag{10}$$

where $\mathbb{P}_{\mathrm{md}}$ and $\mathbb{P}_{\mathrm{fa}}$ stand for the worst-case probability of missed detection and false alarm within any interval with length m_α, respectively.

Very recently, the optimal solution for the non-Bayesian *transient* change detection for $m = 1$ was shown to be the Shewhart test [10]

$$\boxed{T_{\mathrm{S}} \doteq \inf\left\{ n \geq 1 : \mathrm{LLR}(n) \geq h \right\}.} \tag{11}$$

Indeed, this is the only available result on optimality for the criterion in (9)–(10). However, for the case of finite $m > 1$, which is of practical interest and it is the case of integrity monitoring, no optimal solution has been found yet.

3.3. Windowed solutions: WLC and FMA stopping times

Since the Shewhart test, which uses information of one sample, is optimal for the non-Bayesian *transient* detection with $m = 1$ and the CUSUM test, which uses information about all the past samples, is optimal for *quickest* detection (i.e. $m = \infty$), it is intuitive to think that the optimal solution for the *transient* change detection problem with $1 < m < \infty$ would be some test statistic between these two techniques, or equivalently, a test statistic using information about m samples (i.e. a windowed solution).

In this context, a window-limited CUSUM (WLC) test is proposed in [9] by using at each moment the m last observations only. The stopping time is given by

$$\boxed{T_{\mathrm{WL}} \doteq \inf\left\{ n \geq m : \max_{n-m+1 \leq k \leq n} S_k^n \geq h \right\},} \tag{12}$$

and it was shown in [9] that the optimal WLC test with respect to the criterion in (9)–(10), for a mean-Gaussian change, leads to the use of an FMA test. Inspired by this result and the idea of windowed solution, we propose the use of the FMA test for any general *transient* detection problem. The stopping time becomes:

$$\boxed{T_{\mathrm{FMA}} \doteq \inf\left\{n \geq m : S_n \geq h\right\}; S_n = \sum_{i=n-m+1}^{n} \mathrm{LLR}(i).} \tag{13}$$

Moreover, the use of the FMA stopping time is motivated by the fact, as we will show next, that we can obtain tight bounds for both the probability of missed detection and false alarm. On the contrary, the bounds available in the literature for these probabilities for the CUSUM and WLC are not that tight. This is a key factor for integrity algorithms, for which availability may considerably be improved.

4. PERFORMANCE BOUNDS FOR THE FMA STOPPING TIME

This section is devoted to theoretically investigating the statistical performance of the FMA stopping time T_{FMA}. That is, to determine the worst-case probability of missed detection $\mathbb{P}_{\mathrm{md}}(T_{\mathrm{FMA}})$ and false alarm for a given duration m_α, $\mathbb{P}_{\mathrm{fa}}(T_{\mathrm{FMA}})$. The exact computation of these probabilities is very complicated, and thus we focus on proposing some appropriate bounds instead.

4.1. Probability of missed detection

Applying the Bayes rule in (9) we have

$$\begin{aligned}
\mathbb{P}_{\mathrm{md}}(T_{\mathrm{FMA}}) &= \sup_{v>m} \mathbb{P}_v\left(T_{\mathrm{FMA}} > m+v-1 | T_{\mathrm{FMA}} \geq v\right) \\
&= \sup_{v>m} \frac{\mathbb{P}_v\left(T_{\mathrm{FMA}} > m+v-1\right)}{\mathbb{P}_v\left(T_{\mathrm{FMA}} \geq v\right)} \\
&= \sup_{v>m} \frac{\mathbb{P}_v\left(\bigcap_{n=m}^{m+v-1}\{S_n < h\}\right)}{\mathbb{P}_v\left(\bigcap_{n=m}^{v-1}\{S_n < h\}\right)},
\end{aligned} \tag{14}$$

where the last equality follows from the definition of T_{FMA} in (13). Due to the windowed behavior of T_{FMA} we assume that $v > m$. Now, letting the event $\mathcal{A}_n = \{S_n < h\}$, it is clear that $\mathcal{A}_{v-1}$ and $\mathcal{A}_{m+v-1}$ are independent because they do not share any samples, thus

$$\mathbb{P}_v\left(\bigcap_{n=m}^{m+v-1} \mathcal{A}_n\right) \leq \mathbb{P}_v\left(\mathcal{A}_{m+v-1}\right) \mathbb{P}_v\left(\bigcap_{n=m}^{v-1} \mathcal{A}_n\right), \tag{15}$$

since in the left side we evaluate more events than in the right side. Applying (15) to (14) we have that

$$\begin{aligned}
\mathbb{P}_{\mathrm{md}}\left(T_{\mathrm{FMA}}\right) &\leq \sup_{v>m} \mathbb{P}_v\left(S_{m+v-1} < h\right) \\
&= \mathbb{P}_1\left(S_m < h\right),
\end{aligned} \tag{16}$$

where the equality follows because S_{m+v-1} is identically distributed under $\mathbb{P}_v$.

It is worth mentioning that the inequality of (16) gives us an upper bound that can be easily computed as the cumulative distribution function (cdf) of $S_m = \sum_{i=1}^{m} \mathrm{LLR}(i)$ when the change is present. Particularly, for the C/N$_0$ monitoring problem presented in Section 2, from (4) we see that S_m follows a Gaussian distribution with mean $m\mu_{\mathrm{c}}$ and variance $m\sigma_{\mathrm{c}}^2$, so that

$$\mathbb{P}_{\mathrm{md}}\left(T_{\mathrm{FMA}}\right) \leq \Phi\left(\frac{h - m\mu_{\mathrm{c}}}{\sqrt{m}\sigma_{\mathrm{c}}}\right), \tag{17}$$

where $\Phi(x)$ is the cdf of the *standard* normal distribution and μ_{c} and σ_{c}^2 are given by (3).

4.2. Probability of false alarm

Let $V_l = \mathbb{P}_\infty(l \leq T_{\mathrm{FMA}} < l + m_\alpha)$ for $l \geq m$; then in a similar way as in [9] it is easy to verify that $\{V_l\}_{l\geq m}$ is a non-increasing sequence. Hence, we can write

$$\begin{aligned}
\mathbb{P}_{\mathrm{fa}}\left(T_{\mathrm{FMA}}\right) = \sup_{l\geq m} V_l &= \mathbb{P}_\infty\left(m \leq T_{\mathrm{FMA}} < m + m_\alpha\right) \\
&= 1 - \mathbb{P}_\infty\left(T_{\mathrm{FMA}} \geq m + m_\alpha\right),
\end{aligned} \tag{18}$$

with

$$\mathbb{P}_\infty\left(T_{\mathrm{FMA}} \geq m + m_\alpha\right) = \mathbb{P}_\infty\left(\bigcap_{n=m}^{m+m_\alpha-1} \{S_n < h\}\right). \tag{19}$$

It is worth remarking that S_n is the addition of different LLR values, and hence it is clearly a monotonically increasing function of these values. Since these LLR values are independent and S_n is identically distributed, under $\mathbb{P}_\infty$, from Theorem 5.1 of [14] we have

$$\mathbb{P}_{\mathrm{fa}}\left(T_{\mathrm{FMA}}\right) \leq 1 - \left[\mathbb{P}_\infty\left(S_m < h\right)\right]^{m_\alpha}, \tag{20}$$

which gives us an upper bound that can be easily computed as the cdf of S_m when the change is not present. Particularly, for C/N$_0$ monitoring, from (4), the S_m follows a Gaussian distribution with mean $-m\mu_{\mathrm{c}}$ and variance $m\sigma_{\mathrm{c}}^2$, and thus

$$\mathbb{P}_{\mathrm{fa}}\left(T_{\mathrm{FMA}}\right) \leq \alpha = 1 - \left[\Phi\left(\frac{h + m\mu_{\mathrm{c}}}{\sqrt{m}\sigma_{\mathrm{c}}}\right)\right]^{m_\alpha}. \tag{21}$$

5. NUMERICAL RESULTS

This section is firstly intended to assess the goodness of the theoretical results in Section 4 by making use of numerical simulations. Secondly, the proposed FMA stopping time is compared with other approaches in the literature. This is done by considering the following parameters motivated by integrity in GNSS applications: $m = 6$, $m_\alpha = 60$ (due to a sampling rate of 1 s and equivalent values of 6 s and 1 min. for m and m_α, respectively), and by the C/N$_0$-based multipath detection in [12]: $\mu_0 = 10^{4.4}$, $\mu_1 = 10^{3.7}$ and $\sigma^2 = (\Delta/3)^2$, with $\Delta = \mu_0(10^{0.3} - 1)$. All the simulations are obtained with 10^5 Monte-Carlo runs. It is worth pointing out that although the presented results assume a mean-Gaussian change, the theoretical results presented in Section 4.1 are valid for any kind of change, since they are obtained without any assumption on the distribution of x_n, except for the particular expression of (17) and (21).

5.1. CASE 1: Goodness of performance bounds

Here, we assess the goodness of the proposed theoretical bounds in Section 4 with numerical simulations of the worst-case probability of missed detection $\mathbb{P}_{\mathrm{md}}(T_{\mathrm{FMA}})$ and false alarm for a given duration m_α, $\mathbb{P}_{\mathrm{fa}}(T_{\mathrm{FMA}})$, of the FMA stopping time in (13). In addition, we also assess the match between the bounds available in the literature

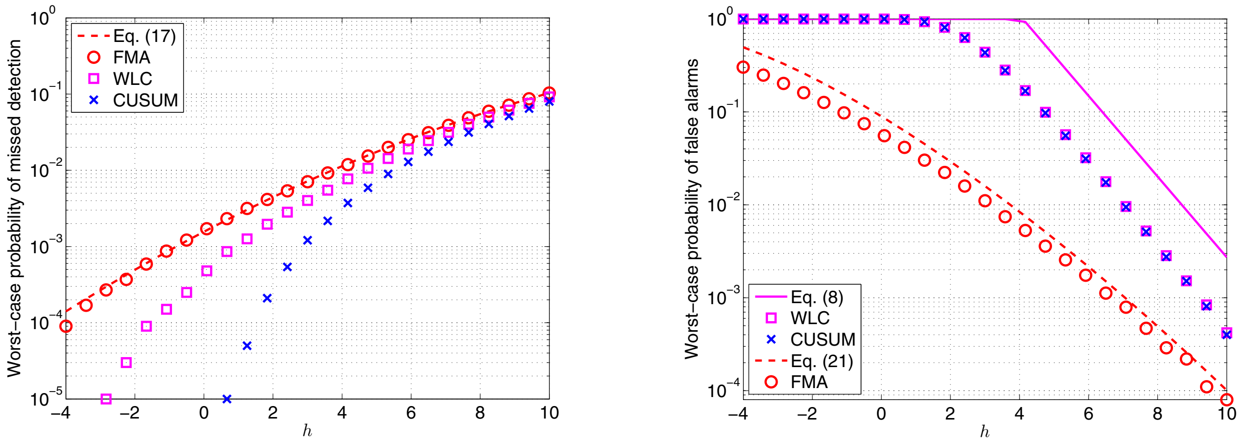

Fig. 1. (Left) Monte-Carlo simulations of the probabilities of missed detection $\mathbb{P}_{\mathrm{md}}(T)$ and false alarm $\mathbb{P}_{\mathrm{fa}}(T)$ (right) as functions of the threshold h, with their respective upper bounds in (17) and (8), (21), for the FMA, CUSUM and WLC stopping times.

for the CUSUM and the WLC stopping times in (6) and (12), and the simulation results.

The probability of missed detection $\mathbb{P}_{\mathrm{md}}$ for the different analyzed stopping times is shown in the left plot of Fig. 1 as a function of the threshold h. It is compared with the upper bound given by (17), which is an upper bound valid for all the analyzed stopping times (see [9] and [11] for the WLC and CUSUM, respectively). On the other hand, the simulated probability of false alarm as a function of the threshold h is shown in the right plot of Fig. 1. It is compared with the upper bound given by (21). Moreover, we present the upper bound given by (7)–(8), which holds for both the CUSUM and WLC stopping times. Therefore, it follows from the results in Fig. 1 that both proposed bounds for the FMA stopping time T_{FMA} are close to the results of numerical simulations for a wide range of values for the threshold h. These results confirm the practical application of the proposed bounds for bounding the probabilities of missed detection and false alarm of the FMA stopping time. Moreover, we see that the bounds available in the literature for the CUSUM and the WLC are not as tight as our bound for the FMA.

5.2. CASE 2: Probability minimization criterion evaluation

Let us finally compare the FMA stopping time, in the sense of the optimality criterion in (9)–(10), with those stopping times currently available in the literature of *transient* change detection. Fig. 2 shows the simulated probability of missed detection $\mathbb{P}_{\mathrm{md}}(T)$ as a function of the simulated false alarm probability $\mathbb{P}_{\mathrm{fa}}(T)$ for each analyzed stopping time. The obtained upper bound in (17) is also shown, taking into account the fixed threshold (as function of $\alpha = \mathbb{P}_{\mathrm{fa}}(T)$) from (21) (when using T_{FMA}); and from (8) (when using T_{C} or T_{WL}). It can be concluded from Fig. 2 that the FMA outperforms, in the sense of the optimality criterion in (9)–(10), the other stopping times, including the Shewhart test, which apparently loses its optimal properties for $m > 1$. Moreover, it is worth pointing out that the bounds will also impact the real performance of the methods because the threshold h is in practice fixed using the bounds, with the aim of fixing certain desired performance. Therefore, the availability of tight bounds is important not only for a theoretical study but also for setting the threshold in practice and provide a level of performance that is close to the desired one.

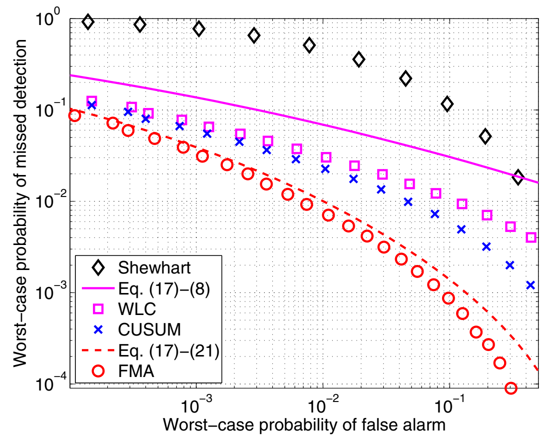

Fig. 2. Probability of missed detection $\mathbb{P}_{\mathrm{md}}(T)$ as a function of the probability of false alarm $\mathbb{P}_{\mathrm{fa}}(T) = \alpha$ for the analyzed stopping times, with the upper bound in (17) with h fixed from (21) and (8).

6. CONCLUSIONS

This work has introduced the problem of *transient* change detection. We have proposed the use of an FMA test inspired by the fact that it is the optimal windowed CUSUM for the case of a mean-Gaussian change. This optimality is based on the criterion of minimizing the probability of missed detection with a constraint on the false alarms. Based on this criterion, the statistical performance of the FMA stopping time has been investigated and compared by numerical simulations to different methods available in the literature. We have applied these results to GNSS signal integrity. This has been done in the setting of signal integrity in GNSS by considering the case of multipath detection based on the C/N$_0$ calculated within the GNSS receiver. These experiments have confirmed the match between theoretical and simulated results for the statistical performance of the proposed solution. Moreover, it has also been shown that the proposed solution outperforms those methods available in the literature of *transient* change detection, in the setting of signal integrity.

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