Chirp Interference Mitigation in Snapshot GNSS Receivers Using the Time-Frequency Ridge-Assisted Fractional Fourier Transform

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***Abstract*—Global Navigation Satellite Systems (GNSS) are an integral part of modern technology, providing essential positioning, navigation, and timing (PNT) services for various applications. These systems face escalating threats from sophisticated radio frequency (RF) interference, particularly linear frequency-modulated (LFM) chirp signals, which exploit rapid time-frequency variations and high bandwidth occupation to disrupt positioning, navigation, and timing services. These interference signals pose critical risks by overwhelming GNSS receivers and degrading their position accuracy. Effective mitigation techniques are therefore sought in order to protect the reliability of GNSS receivers and applications. Novel mitigation methods such as Fractional Fourier Transform (FrFT)-based techniques emerge as potential candidates to suppress chirp-type interferences, but face significant computational challenges for their practical implementation. This paper addresses this limitation by introducing a structured analytical approach that establishes a direct relationship between the characteristics of the chirp signal, specifically the chirp rates, and the optimal transformation parameters in the Fourier fractional domain. The proposed implementation is shown to achieve a robust interference mitigation with minimal computational overhead, making it suitable for most GNSS applications. The findings highlight the potential for improving the resilience of GNSS receivers under RF interference and suggest avenues for future research to extend this approach to more complex signal environments.**

***Index Terms*—Interference Mitigation, Fractional Fourier Transform, LFM, Anti-jamming Techniques**

I. Introduction

Global Navigation Satellite Systems (GNSS) have become a key part of modern technology, providing positioning, navigation, and timing services in various applications. However, radio frequency (RF) interference has become a significant threat to GNSS performance and reliability. Inside interference types, chirp signals are particularly challenging due to the nature of their time-frequency variation, which disrupts GNSS receivers across a wide bandwidth [1], [2].

Existing chirp interference mitigation strategies span across a wide range of signal processing approaches, each offering unique advantages and limitations when implemented in GNSS receivers. The filter bank approach presented by Rügamer et al. [3] provides a structured method for interference suppression by decomposing the received signal into multiple time-frequency segments. This technique demonstrates effectiveness in scenarios where interference exhibits predictable patterns across discrete frequency bands and time slots, allowing for targeted filtering operations. However, the method faces inherent challenges when confronting highly dynamic interfering signals that rapidly traverse different frequency bands, potentially requiring excessive filter bank complexity. Similarly, adaptive filtering techniques such as the Adaptive Notch Filter (ANF) proposed by Borio et al. [4] introduce sophisticated mechanisms that continuously update the filter parameters based on the signal statistics. This enables robust performance under varying interference conditions without requiring prior knowledge of interference characteristics. Despite these advantages, adaptive methods typically require substantial convergence periods and may struggle with abrupt changes in interference patterns, introducing transient vulnerability windows in the GNSS signal reception. Because of their required convergence time, adaptive methods may not fit well for their implementation into snapshot GNSS receivers, where a small piece of just a few ms-long signal needs to be processed. An alternative approach proposed in [5] jointly estimates GNSS parameters (time delay and Doppler shift) and mitigates constant modulus interferences by modelling interference phases with flexible von Mises priors. This enables robust handling of both linear and non-linear chirp interference without requiring prior knowledge of the interference structure, at the expense of significantly increasing the computational load.

More recently, Fractional Fourier Transform (FrFT) [6], [7] based techniques have emerged as powerful alternatives by leveraging transformation into domains between time and frequency, where chirp signals can be efficiently represented and subsequently mitigated. These implementations demonstrate superior performance for linear chirp interference but encounter computational bottlenecks when determining the optimal fractional-order parameters. Previous FrFT implementations by Sun et al. in [8], [9] have relied on exhaustive search methods in potential fractional orders, measuring the energy concentration at each iteration to identify the optimal transformation parameters. This approach introduces significant computational demands that seriously limit the implementation

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in resource-constrained GNSS receivers.

Our research addresses the fundamental computational challenge associated with FrFT-based interference mitigation by developing a direct relationship between the characteristics of the chirp signal and the optimal fractional domain parameter. Unlike previous implementations that need extensive transform computations across multiple fractional orders, our approach establishes an analytical framework that directly maps estimated chirp rates to the corresponding optimal transformation parameters. This development eliminates the need for iterative transform calculations and energy concentration measurements, substantially reducing the computational complexity while maintaining mitigation effectiveness. By implementing this direct parameter estimation technique, the enhanced FrFT method achieves comparable interference suppression performance to exhaustive search implementations while only requiring a fraction of the computational resources. This efficiency improvement represents a significant advancement toward practical implementation in GNSS receivers and resource-limited platforms, where processing capacity and power consumption constraints had previously limited the applicability of FrFT-based techniques. Our research transforms FrFT-based interference mitigation from a theoretically powerful but practically challenging approach into a viable technique for mainstream GNSS receiver implementations in diverse application domains, with emphasis on snapshot GNSS receivers.

The paper follows a structured approach to address the challenge of chirp interference in GNSS systems. It begins by introducing the signal model that forms the basis for the mitigation algorithm in Section II. Following this, the fundamental principles of the FrFT are exposed in Section III. After that, the fractional-order search using the time-frequency ridge (TFR) method is presented in Section IV. Then, the proposed interference mitigation method is described in Section V. To validate the efficacy of the proposed technique, a comprehensive simulation study is presented in Section VI, offering insights into the method's performance under various Linear Frequency Modulated (LFM) signals. The paper concludes in Section VII by summarizing the key findings, discussing the implications of the research, and suggesting potential avenues for future work in this domain.

II. GNSS Signal Model

Let us consider a GNSS received signal affected by an interference, leading to the following discrete-time baseband equivalent signal model:

$$x[n] = s[n] + v[n] + w[n], \tag{1}$$

where $v[n]$ represents the chirp interference signal and $w[n]$ is additive white Gaussian noise (AWGN). The GNSS signal, $s[n]$, is assumed negligible due to the low power received levels of GNSS signals. The chirp interference signal $v[n]$ is a LFM signal, defined as:

$$v[n] = A \cdot e^{-j\phi[n]} \tag{2}$$

where:

- A: Amplitude of the chirp signal,
- $\phi[n]$: Phase of the chirp signal, affected by a linear frequency shift.

The instantaneous frequency of the chirp is given by:

$$f_{\text{inst}}[n] = f_0 + \operatorname{mod}_B\left(B \cdot \frac{nT_s}{T_{\text{chirp}}}\right), \tag{3}$$

where:

- T_s: Sampling period,
- T_{chirp}: Chirp period,
- f_0: Starting frequency of the chirp,
- f_1: Ending frequency of the chirp,
- $B = f_1 - f_0$: Bandwidth of the chirp.

The noise component $w[n]$ is modelled as AWGN with zero mean and variance σ^2:

$$w[n] \sim \mathcal{N}(0, \sigma^2). \tag{4}$$

Considering a LFM chirp, its chirp rate is given by

$$k_{\text{chirp}} = \frac{B}{T_{\text{chirp}}}. \tag{5}$$

III. Fractional Fourier Transform

The Fractional Fourier Transform (FrFT) is a generalization of the classical Fourier Transform that provides a representation of signals in intermediate domains between time and frequency [6], [7]. It introduces a fractional order parameter, α, which determines the rotation angle in the time-frequency plane where the fractional transformation will be performed. This approach becomes interesting when dealing with linearly frequency modulated chirp signals, which exhibit time-varying features in both the time and frequency domains, but that become one-dimensionally time-varying when appropriately rotated in the time-frequency plane.

The continuous FrFT of a continuous-time signal $x(t)$ is defined as:

$$X_\alpha(u) = \int_{-\infty}^{\infty} x(t) K_\alpha(t,u) dt, \tag{6}$$

where the kernel $K_\alpha(t,u)$ is given by:

$$K_\alpha(t,u) = \begin{cases} A_\alpha e^{j\pi(t^2 c_\alpha - 2tuc_\alpha^{-1} + u^2 c_\alpha)}, & \alpha \neq k\pi \\ \delta(t-u), & \alpha = 2k\pi \\ \delta(t+u), & \alpha = (2k+1)\pi \end{cases} \tag{7}$$

with $A_\alpha = \sqrt{1 - jc_\alpha}$, $c_\alpha = \cot(\alpha)$, and k being an integer.

For discrete-time signals, the FrFT is computed using discrete algorithms that efficiently approximate its continuous-time counterpart. The Discrete Fractional Fourier Transform (DFrFT) preserves key properties of the continuous FrFT while operating on sampled data. The DFrFT used in this paper is the one defined in [6] and implemented in [7]. It is expressed as:

$$X_\alpha[k] = \sum_{n=0}^{N-1} x[n] K_\alpha[n,k] \tag{8}$$

where $K_\alpha[n,k]$ is a discretized version of the kernel. The FrFT is particularly suited for chirp signal analysis due to its ability to represent signals in rotated time-frequency domains. By selecting an appropriate fractional order α, chirp signals can be transformed into a compact representation in the fractional domain, enabling effective interference mitigation in GNSS systems. The relation between the chirp rate of the interfering signal and the α that maximizes the concentration of energy in a few bins of the DFrFT is defined by the kernel function with the following equation:

$$\hat{\alpha} = \frac{2\cot^{-1}(\hat{k}_{\text{chirp}} D)}{\pi} \tag{9}$$

where $\cot^{-1}$ is the inverse cotangent, $\hat{k}_{\text{chirp}}$ is the estimated chirp rate of the interfering signal and D is a normalization factor defined by $\frac{N}{F_s^2}$ that can be derived by the resolution of the DFrFT used.

IV. Fractional Order Definition

Until now, the methods used for searching the optimal fractional order are performed by a computationally demanding search. In [8], [9] and [10] the approach of using the FrFT as an interference mitigation method aims to optimize and improve the optimal α search, which is performed by iterating all possible α values and keeping the one that concentrates most of the energy in the corresponding fractional domain. The proposal in this paper intends to boost the optimal α search by estimating first the chirp rate, and using this estimate to determine the corresponding α. To do so, a time-frequency ridge (TFR) method is used. In this case, the time-frequency data points are obtained by using the Short Time Fourier Transform (STFT) on the received signal $x[n]$. Then, based on the extraction of the instantaneous frequency of the signal, the idea is to perform an estimation of the possible chirp rate inside the frequency variations.

A. Time-Frequency Ridge (TFR) Extraction Algorithm

The TFR algorithm is able to identify and extract the most prominent energy ridges in the time-frequency representation of a signal. In our case, the ridges obtained from the STFT, which correspond to the instantaneous frequency components of the chirp interference. The procedure for extracting such ridges follows a forward-backward greedy optimization approach [11], [12]. The algorithm operates using this inputs:

- $\mathbf{T}$: Time-frequency matrix of size $M \times N$, where M is the number of frequency bins and N is the number of time samples.
- $\mathbf{f}$: Frequency vector corresponding to the rows of $\mathbf{T}$.
- $P \geq 0$: Penalty parameter that discourages large frequency jumps between adjacent time steps.

In a simplified manner, for each time step $j = 1, 2, ..., N$, the algorithm identifies the frequency bin i that maximizes the energy function:

$$E_{i,j} = T(i,j) - P \cdot |f_i - f_{i-1}|^2 \tag{10}$$

where $T(i,j)$ is the energy at frequency bin i and time sample j , and the penalty term penalizes large changes in frequency between adjacent time steps. The ridge frequency at time step j is given by:

$$i_j = \underset{i \in \{1,\ldots,M\}}{\arg\max} \left\{ T(i,j) - P \cdot |i - i_{j-1}|^2 \right\}, \tag{11}$$

where a maximization of the frequency bin i at every time period j is done, thus providing the sets of frequencies where the energy is located and extracted as:

$$f_{\text{ridge}}(j) = \mathbf{f}(i_j), \quad \forall j \in \{1, \ldots, N\}, \tag{12}$$

Note that in our environment, the algorithm is reduced to only estimating the main ridge on the time-frequency matrix and not to manage other ridges. A detailed description of the algorithm can be found in [13].

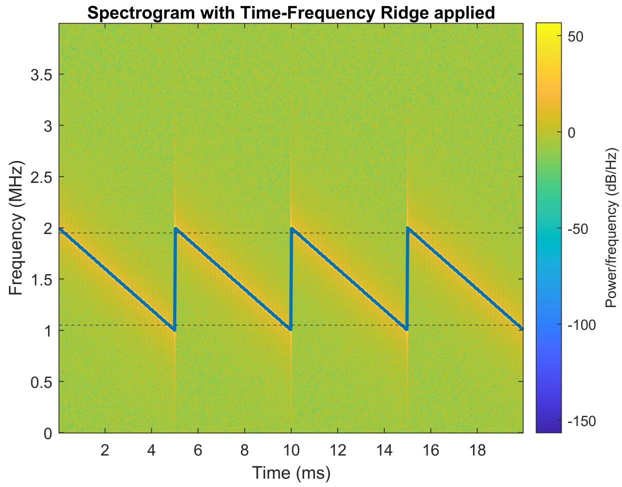

Fig. 1: Spectrogram of a linear chirp signal with the TFR method superposed. $T_{\text{chirp}} = 5$ ms, $BW = 1$ MHz, $f_0 = 1575.42$ MHz, JNR $= 40$ dB.

B. Chirp Period and Bandwidth Estimation

Once the instantaneous frequency is obtained, the chirp bandwidth and the chirp period can be estimated as follows:

- $\hat{B}$: for the bandwidth estimation we can take the difference between the smallest and the largest instantaneous frequencies.
- $\hat{T}_{chirp}$: the 90% of $\hat{B}$ can be used as a threshold to check the maximum peaks of the instantaneous frequency. The distance between such peaks will provide an estimate of the chirp period.

In Fig. 2 the obtained time-frequency variation is shown, where can be observed the ability of the method to estimate the frequency behaviour of the chirp signal. Then the estimate of the chipr rate, $\hat{k}_{chirp}$, is obtained through the relationship between the bandwidth and the chirp period.

$$\hat{k}_{\text{chirp}} = \frac{\hat{B}}{\hat{T}_{chirp}} \tag{13}$$

Using (9) we can directly obtain the estimation of α_{opt}.

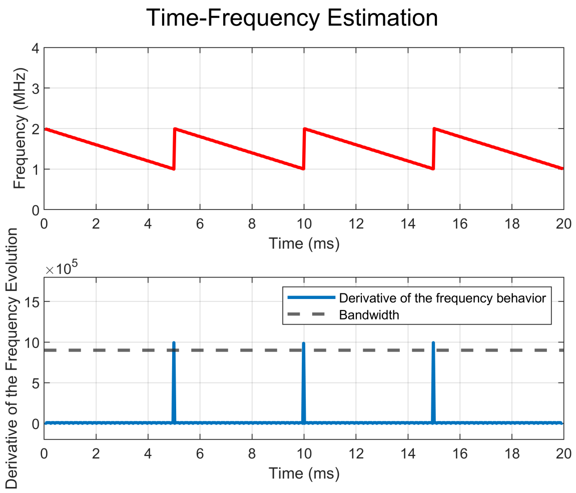

Fig. 2: Frequency estimation of the chirp signal and its derivative. TFR algorithm used. $k_{\text{chirp}} = 200\text{MHz/s}$

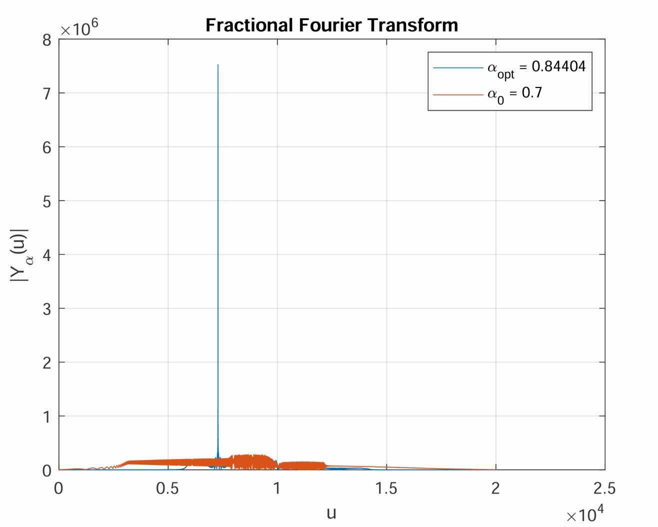

Fig. 3: DFrFT computed with different α on the same LFM signal. $k_{\text{chirp}} = 200\text{MHz/s}$.

V. Fractional Domain Mitigation Technique

Once an estimate for α is obtained, the mitigation needs to be applied on the fractional domain. Among these techniques, Pulse Blanking (PB) in the fractional domain emerges as a particularly effective approach for addressing high-power chirp interference in GNSS signals.

When a chirp interference signal is transformed into the appropriate fractional domain using the optimal rotation angle α, its energy becomes highly concentrated in a narrow region as seen in Fig. 3. This concentration effect occurs because the FrFT effectively aligns the transform domain with the chirp's time-frequency structure, resulting in a compact representation that resembles an impulse or narrowband signal rather than a wideband interferer. This property creates an ideal scenario for applying excision techniques like PB [9] or Notch filtering [8], which can effectively remove the interference while preserving most of the desired GNSS signal content.

The mitigation process follows a structured approach beginning with the transformation of the received signal into the fractional domain. After this transformation, the interference components appear as high-amplitude peaks in specific locations within the transformed signal representation. Then the inverse transform is performed to recover the signal without the interference component. The property of the sum of indexes of the DFrFT can be used to recover the signal, doing the transformation with an α order can be reversed by doing the transformation of $-\alpha$ order. Note that the DFrFT with fractional order 0 is recovering the signal in the time domain.

A. *Fractional filtering method and decision threshold*

The objective of the fractional domain transformation is to annihilate interference components in the received signal. To achieve this, we propose a hybrid approach combining pulse blanking with the complex signum non-linearity method from robust statistics [14]. For signals transformed into the fractional domain, the processing operation becomes:

$$X_{\alpha,\text{PB}}[k] = \begin{cases} \frac{X_\alpha[k]}{|X_\alpha[k]|}, & \text{if } |X_\alpha[k]| > \gamma \\ X_\alpha[k], & \text{otherwise} \end{cases} \quad (14)$$

where $X_\alpha[k]$ represents the k-th component of the signal in the fractional domain with rotation angle α, $|X_\alpha[k]|$ denotes its magnitude, and γ is the decision threshold. This operation effectively preserves the phase information of the affected components while reducing their amplitude to a normalized level.

The threshold γ was derived using a Weibull distribution fitted to empirical data collected under interference-free conditions (GNSS signal + background noise). The Weibull model effectively captured the amplitude statistics in the fractional domain, enabling computation of γ for a false alarm probability $P_{FA} = 10^{-7}$. This threshold serves as a decision boundary, enabling the detection algorithm to reliably distinguish between the chirp component and the GPS signal component on the fractional domain.

VI. Performance Analysis

The objective of this performance analysis is to comprehensively assess the effectiveness of the proposed FrFT mitigation technique when applied to chirp-like interferences in GNSS snapshot receivers. The analysis is focused on position accuracy, Carrier to Noise (C/N_0) density ratio, and also the time-frequency shape of the mitigated signal. The experimental methodology employs a structured test framework consisting of 100 sequential GNSS snapshots, systematically varying the interference characteristics to assess performance under different operational scenarios.

The GNSS signal is digitally generated as baseband I/Q samples using the Skydel GSG-8 simulator [15], operating at its native frequency of 12.5 MHz and 16-bit resolution. To align with the work requirements, the signal is resampled to 4 MHz using a decimation ratio of 8/25. After resampling, software-generated interference signal is superimposed onto the signal, maintaining the 16-bit integer format as the original Skydel-generated samples.

To comprehensively evaluate the technique's performance against chirp signals with varying frequency dynamics, the chirp period is progressively reduced every 10 snapshots throughout the test sequence. This methodical reduction in the chirp period enables evaluation of the mitigation technique's robustness against increasingly rapid frequency variations, representing progressively more challenging interference scenarios. The experimental setup is characterized by the following parameters:

- **Signal Characteristics:** GPS L1 C/A code signals with AWGN noise
- **Jammer-to-Noise Ratio (JNR):** Maintained at 40 dB across all test scenarios, representing a strong interference condition
- **Interference Bandwidth:** 1 MHz with initial frequency on the GNSS signal band (1575.42 MHz)
- **Chirp Period Variation:** Starting at the interference-free scenario for snapshots 1-10, then 4 ms chirp period for snapshots 11-20 and then decreasing by 0.5 ms for each subsequent set of 10 snapshots, reaching 0.25 ms for the final set which is reduced by 0.25 ms.
- **Snapshot Duration:** 20 ms per snapshot, providing sufficient integration time for signal acquisition
- **Sampling Rate:** 4 MHz

For each snapshot, three processing approaches are applied and their results are recorded: (1) the proposed DFrFT-based mitigation with alpha parameter estimation, (2) an ANF implementation [4] representing a conventional approach, and (3) unmitigated processing to check the effects of interference on the receiver. In addition, reference measurements are collected under interference-free conditions to establish the baseline. The first analysis performed is the evaluation of the C/N_0 evolution across the defined scenarios, as illustrated in Fig. 4. The interference-free scenario serves as a baseline reference, maintaining a stable mean C/N_0 around 40–41 dB-Hz, indicative of good signal quality for the satellites in view.

When the chirp interference is introduced without mitigation, the C/N_0 significantly deteriorates, dropping below 10 dB-Hz for most satellites. This severe degradation is attributed to the high Jammer-to-Noise Ratio (JNR), which overwhelms the receiver and results in a loss of satellite acquisition. Applying the ANF-based mitigation partially restores signal quality, improving C/N_0 to approximately 20–25 dB-Hz. However, due to its limited convergence capability at low sampling frequencies, the ANF method leaves residual interference effects within the signal. In contrast, the proposed DFrFT-based mitigation method successfully recovers C/N_0 levels comparable to those observed in the interference-free scenario. This superior performance is due to the intrinsic capability of DFrFT to effectively eliminate chirp interference components without adversely affecting the GPS signal.

Fig. 5 presents the corresponding analysis of 3-D position error across these scenarios, corroborating the findings from the C/N_0 evaluation. Initially, during the first 10 snapshots corresponding to an interference-free baseline period, all scenarios exhibit identical positioning performance with minimal errors. After introducing the interference at snapshot 11 onwards, significant differences emerge:

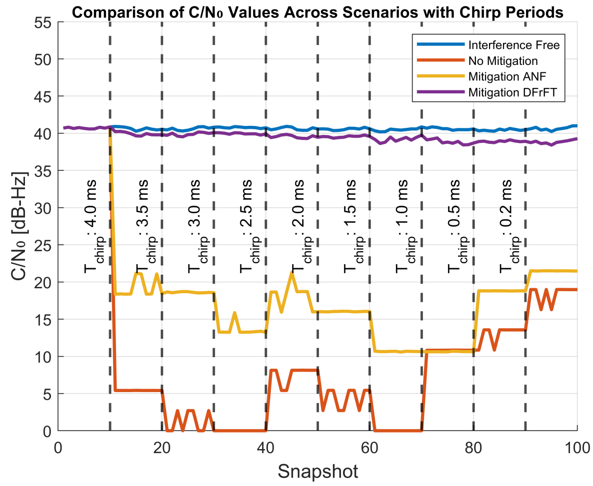

Fig. 4: Comparison of mean C/N_0 across interference mitigation scenarios. Vertical dashed lines denote transitions in interference chirp periods (every 10 snapshots). Four scenarios were found, interference-free, no mitigation applied, mitigation with ANF, and mitigation with the proposed method using DFrFT.

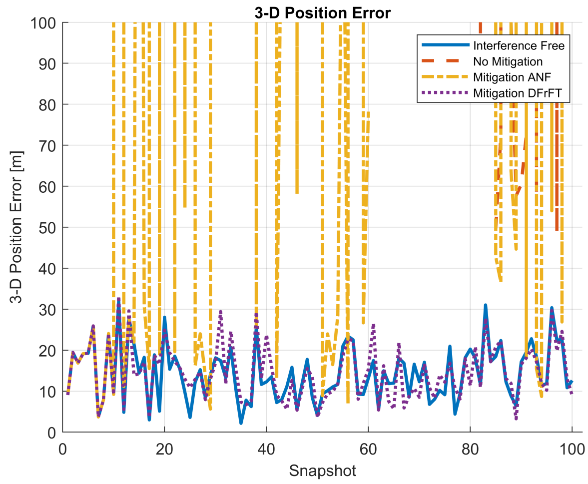

Fig. 5: Comparison of 3-D Position Error across interference mitigation scenarios. Four scenarios found, interference free, no mitigation applied, mitigation with ANF and mitigation with proposed method using DFrFT.

- In the no-mitigation scenario, position computation rapidly deteriorates due to severe signal degradation caused by interference. The receiver becomes unable to reliably track satellites or compute accurate positions, resulting in large errors or complete loss of position.
- The ANF mitigation method partially restores positioning capability compared to no mitigation; however, it still exhibits substantial errors and instability throughout subsequent snapshots. This behaviour aligns with previously observed interference effects in the C/N_0 analysis. Although ANF reduces interference impact, its limited

adaptability and slow convergence prevent full recovery of accurate positioning performance.
- Finally, the proposed DFrFT-based method closely matches the positioning accuracy observed in the interference-free scenario throughout all snapshots. The position error remains consistently low and stable, demonstrating the effectiveness of erasing the interference component on the fractional domain.

These results demonstrate that the proposed DFrFT-based mitigation approach significantly outperforms traditional ANF methods in terms of both recovered signal quality (C/N_0) and GNSS positioning accuracy under strong chirp interference scenarios.

VII. Conclusions

This paper presented a novel approach to address the computational complexity associated with FrFT-based interference mitigation in GNSS. The primary objective was to develop a method for directly finding the optimal fractional rotation parameter, thereby reducing the computational burden of traditional exhaustive search techniques. The proposed analytical framework successfully establishes a direct relationship between the chirp signal characteristics and the optimal transformation parameter, achieving significant computational efficiency. The key findings of this research include the derivation of an analytical method to compute the optimal fractional order, α, which eliminates the need for iterative or exhaustive search processes. This advancement not only reduces computational complexity but also maintains high interference suppression effectiveness, as demonstrated through comparative evaluations. The results validate the robustness and practicality of the proposed method, making it a promising and efficient solution for GNSS applications. However, certain limitations of the method were identified. The approach assumes that the signal remains within the analysed bandwidth and does not account for cases where LFM signals fall outside this range. Additionally, the method relies on low sampling frequencies, which may hinder its ability to accurately capture the frequency behaviour of signals, potentially leading to suboptimal computations of α. Furthermore, the current framework is restricted to LFM signals and does not extend to other types of chirp signals.

Future work aims to address these limitations by extending the analytical framework to accommodate a broader class of chirp signals, such as quadratic and sinusoidal modulations. Moreover, further research will focus on analytically examining the method's limitations with respect to sampling frequency and exploring solutions to ensure robust performance under varying signal conditions. These extensions will enhance the applicability and versatility of the proposed approach in a set of different GNSS interference chirp-like scenarios. A promising direction for future exploration is the modification of the fractional kernel function within the FrFT framework. By adapting or redesigning the kernel function, it may be possible to enhance its sensitivity to other types of chirp signals, such as sinusoidal or quadratic modulations.

In conclusion, this study provides a significant step forward in developing computationally efficient FrFT-based interference mitigation techniques for GNSS receivers. By addressing key computational challenges while maintaining effectiveness, this work contributes to advancing GNSS receiver technology and lays a foundation for future innovations in interference suppression.

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